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#### $^{11}$  Teamer Vaisor energy methods for signal and image analysis: A review  $\frac{1}{12}$  Teager–Kaiser energy methods for signal and image analysis: A review  $\frac{1}{78}$

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### <sup>19</sup> ARTICLE INFO ABSTRACT <sup>85</sup>

*Article history:* Available online xxxx *Keywords:* Teager–Kaiser energy operator Higher order differential operators Multi-dimensional energy operator Cross- $\Psi_{\text{\tiny B}}$ -energy operator AM–FM Model Signal demodulation

<sup>21</sup> Article history: **External 20** This paper provides a review of the Teager–Kaiser (TK) energy operator and its extensions for signals <sup>87</sup> 22 Available online xxxx **and the set of our and images processing.** This class of operators possesses simplicity and good time-resolution and is very a<sup>88</sup> 23 89 efficient in instantaneously estimating AM–FM signals and images. We point out the importance of the 24 Texty with the point of energy from the point of view of the generation of the signal. More precisely, we emphasize 90 25 Higher order differential operators of the importance of analyzing signals from the point of view of the energy of the system needed to 91<br>25 Higher order differential operators 26 Multi-dimensional energy operator **notable 20 Froduce them. We show how this class of TK** energy operators can be used to estimate useful features <sub>92</sub> <sub>27</sub> Cross-V<sub>B</sub>-energy operator **the system of the state of the state of the state and frequency domains such as instantaneous frequency, <sub>93</sub>** 28 Importance of the higher derivative order of TK energy operator in terms of demodulation for both mono and signal demodulation 29 29 29 and multi-dimensional signals. Most of the developed tools around TK energy operators deal with real 30 96 and complex-valued signals and some of them extended to multi-dimensional case. Due to their low <sup>97</sup> 31 31 Complexity and their instantaneous-adapting nature, the class of TK energy operators offers valuable 32 **88 processing tools for time and frequency analysis.** The same of the same of the set of the se second-order moment frequency, coherence function or spatial envelope and phase. We also show the

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#### **1. Introduction**

40 Signals may represent a broad variety of phenomena. In many In their work on non-linear speech modeling, Herbert and  $\frac{106}{100}$ 41 applications, signals are directly related to physical quantities cap- Shushan Teager pointed out the dominance of modulation as a  $_{107}$ 42 turing energy and power in a physical system. The concept of process in the speech production [8,9]. Based on the leager's work, the 43 signal energy is of primary importance in the design of continuous kaiser proposed an energy measure that includes both the ampli-  $\frac{109}{100}$ 44 and discrete domain systems  $[1-7]$ . This work is interested in sig-<br> $\frac{1}{2}$  ture and the frequency of the signal  $\frac{1}{3}$ . This work is interested in sig-45 hals provided by sensors and thus, to the energy associated with the referred to as the Teager–Kaiser (TK) energy operator. Using the the fi 46 these signals. In the real world, we always transmit signals with conventional view of the energy, it is easy to see that two tones  $\frac{112}{12}$ 47 finite total energy  $0 < E_x < +\infty$  (or with finite average power) at 10 Hz and 1000 Hz of unit-amplitude have the same energy.  $113$ 48 representing the amount of energy contained in signal  $x(t)$ . The However, the energy required to produce the signal of 1000 Hz is  $114$ 49 quantity  $E_x$  should be independent of the method used to calcu-<br> $\frac{1}{2}$  and the man that in the independent of the method used to calcu-<br> $\frac{1}{2}$  and the train that in the independent of the method used to calcu-<br>50 116 late it. Engineers refer to such signals as having finite total energy,  $51$  although  $E_x$  is not necessarily the *physical* energy of the signal  $x(t)$ . Inighlights the concept of signal energy from the point of view of 117 52 For example, the total energy of the source system modeled as a content the sensition of the signal and emphasizes the importance of an-53 mass suspended by a spring of a constant stiffness required to pro-<br>53 mass suspended by a spring of a constant stiffness required to pro-54 120 duce a simple undamped harmonic oscillation is calculated by the 55 sum of the kinetic energy of the mass and the potential energy in the direction and its different extensions as tools for three frequency and 121 56 the spring. By studying the second order differential associated to the dialysis of signals, and for estimation of envelope and phase for the 122 57 this harmonic oscillator, it is easy to show that a simple sinusoidal the line is presented. This class of operators is inustrated on simulation and the state 58 varies as a function of both amplitude and oscillation frequency of the lated and real data. 59 the signal  $x(t)$ , which is quite different from simple squaring of the  $\frac{125}{25}$   $\frac{125}{25}$ 

38 **1. Introduction** signal magnitude,  $x^2(t)$ . It is this source modeling that is used for  $104$ 39 105 characterizing *x(t)* by amplitude and frequency.

In their work on non-linear speech modeling, Herbert and Shushan Teager pointed out the dominance of modulation as a process in the speech production [\[8,9\]](#page--1-0). Based on the Teager's work, Kaiser proposed an energy measure that includes both the amplitude and the frequency of the signal [\[3\]](#page--1-0). This measure is often referred to as the Teager–Kaiser (TK) energy operator. Using the conventional view of the energy, it is easy to see that two tones at 10 Hz and 1000 Hz of unit-amplitude have the same energy. However, the energy required to produce the signal of 1000 Hz is much greater than that for the 10 Hz signal [\[3\]](#page--1-0). Using TK definition of energy, the two tones show different energy. This definition highlights the concept of signal energy from the point of view of the generation of the signal and emphasizes the importance of analyzing signals from the energy aspect of the system needed to produce them. In the following, the theory behind TK energy operator and its different extensions as tools for time, frequency and TF analysis of signals, and for estimation of envelope and phase for images is presented. This class of operators is illustrated on simulated and real data.

#### 60 126 **2. Teager–Kaiser energy operator**

63 129 erating on continuous-time signal *x(t)* is given by [\[3\]](#page--1-0) In its continuous form, TK energy operator, noted  $\Psi_c$ , when op-

61 127

66 and the contract of the con

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$$
\Psi_c[x(t)] \triangleq \left(\frac{dx(t)}{dt}\right)^2 - x(t)\left(\frac{d^2x(t)}{dt^2}\right) \n= \dot{x}^2(t) - x(t)\ddot{x}(t)
$$
\n(1)

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12 and under realistic conditions,<sup>1</sup>  $\Psi_c[x(t)]$  approximately estimates  $\tau_c$  and  $\tau_c$  and 13 the energy of the source producing the oscillation  $x(t)$ . Using,  $\frac{1}{2}$  Multiresolution Tegger-Kaiser energy operator 14 for example, the backward approximation of the time derivatives and the theory of the time derivatives of the time derivatives 15 (Eq. (1)), the discrete-time counterpart of  $\Psi_c$  becomes To resolve two closely spaced tones (AM signal) an extended  $\frac{81}{2}$ where  $\dot{x}(t)$  and  $\ddot{x}(t)$  are the first and the second derivative of  $x(t)$ with respect to time  $t$  respectively. When  $\Psi_\mathfrak{c}$  is applied to signals generated by a simple harmonic oscillator (mass-spring oscillator of constant stiffness), it can track the oscillator's energy (per half unit mass) which is proportional to the squared product of the

$$
\Psi_d[x(n)] = \frac{x^2(n-1) - x(n)x(n-2)}{(\Delta t)^2}
$$
\nversion of 1K operator has been developed by introducing a lag  
\n
$$
\Psi_d[x(n)] = \frac{x^2(n-1) - x(n)x(n-2)}{(\Delta t)^2}
$$
\n(2) parameter, k, in the expression (3) as follows [17]

 $\frac{20}{21}$  frequency. Equation (2) is scaled and centered yielding

$$
\Psi_d[x(n)] = x^2(n) - x(n-1)x(n+1)
$$
\n(3)

<sup>25</sup> is still a very local property of  $x(t)$ . This input–output relation <sup>26</sup> is called by Kaiser "Teager's algorithm" [\[3\]](#page--1-0). For a tone  $x(n) = \sqrt{\Psi_{d_k}[x(n)]} \approx (A \sin(\omega k))^2$  (7)  $^{92}$ 27  $A\cos(\omega n + \phi_0)$ , TK operator yields

$$
\Psi_d\left[\chi(n)\right] \approx A^2 \omega^2\tag{4}
$$

the form

$$
x(n) = r^n \cos(\omega n + \phi_0) \tag{5}
$$

36 The operator  $\Psi_d$  [x(n)] approximates the product of squared am-<br> $M_d = \frac{L}{L} \frac{1}{L} + 0.5$  |;  $M_s = \frac{1}{L} \frac{1}{L} + 0.5$  | 37 plitude and frequency of  $x(n)$  and can be termed as frequency-<br> $L J1 + J2$   $L J1 + J2$ 38 weighted energy. The non-linear operators  $\Psi_c$  and  $\Psi_d$  were devel-<br>where [.] is the floor function and *i* ∈ N∗. Larger values of *i* pro-39 oped by leager [9,10] and introduced by Kaiser [3,11]. Note that duce greater signal enhancement, but can introduce longer delay  $105$  $\Psi_d$   $\Psi_d$  [*x*(*n*)] is independent of the initial phase of *x*(*t*), symmetric [\[17\]](#page--1-0). We illustrate the interest of  $\Psi_{d_k}$  [*x*(*n*)] operator using a signal 106 41 and capable of responding very quickly to changes in amplitude  $\frac{1}{2}$  of two closely spaced tones  $x(t)$ . 42 and frequency of  $x(t)$  [\[3\]](#page--1-0). Furthermore, it is robust even when  $x(n)$ 43 passes through zero, as no division operation is required. The op-<br> $y(t) = \sin(2\pi f_0 t) + \sin(2\pi f_0 t) + \eta(t)$  (0)  $y(t) = \cos(2\pi f_0 t) + \eta(t)$ 44 erator  $\Psi_d[x(n)]$  offers excellent time resolution because only three  $\cdots$ ,  $\cdots$ weighted energy. The non-linear operators  $\Psi_c$  and  $\Psi_d$  were developed by Teager [\[9,10\]](#page--1-0) and introduced by Kaiser [\[3,11\]](#page--1-0). Note that  $\Psi_d$  [*x*(*n*)] is independent of the initial phase of *x*(*t*), symmetric samples  $(x(n-1), x(n), x(n+1))$  are required for the energy computation at each time instant  $n \Delta t$ , so it has good adaptability to the instantaneous changes in  $x(t)$ . This why TK operator is well adapted, for example, for measuring formant modulation. The response of  $\Psi_d\left[\chi(n)\right]$  is nearly instantaneous and this operator can be easily implemented in DSP processors, due to its low computation cost and extremely low requirements of memory storage.

The TK operator has been discussed from the mathematical types of combination (e.g. linear, algebraic) of input signals and classified the root and pre-constant signals of the operator [\[12\]](#page--1-0). The matrix framework of the operator has been introduced in [\[13\]](#page--1-0) by interpreting it as the determinant of a Toeplitz matrix containing the signal and its derivatives:

$$
\Psi_c[x(t)] = \begin{vmatrix} \dot{x}(t) & x(t) \\ \ddot{x}(t) & \dot{x}(t) \end{vmatrix}; \quad \Psi_d[x(n)] = \begin{vmatrix} x(n) & x(n+1) \\ x(n-1) & x(n) \end{vmatrix}
$$

<sup>1</sup>  $\frac{1}{\sqrt{1-\frac{1}{2}}}\int \frac{dx(t)}{dx(t)}$   $\frac{d^2x(t)}{dt}$  The determinant is time-invariant for a signal with constant fre- $\frac{q}{dt}$   $\frac{q}{dt}$   $\frac{q}{dt^2}$   $\frac{q}{dt^2$ 3 by adding delayed *x*(*n*) up to *x*[*n*  $\pm$  (*M* − 1)], the determinant is 69 4 70 also time-invariant [\[13,14\]](#page--1-0). This time-invariant property can be ex-<sup>5</sup> where  $\dot{y}(t)$  and  $\ddot{y}(t)$  are the first and the second derivative of  $y(t)$  ploited using probe tones to uncover the amplitudes and frequen- $\frac{6}{2}$  with respect to time t respectively When W is applied to signals cies of multiple sinusoids [\[13\]](#page--1-0). Using this matrix framework, the  $\frac{72}{2}$  $\frac{7}{2}$  generated by a simple barmonic oscillator (mass enting oscillator) output of the TK operator is interpreted as the measured energy  $\frac{73}{2}$  $\frac{8}{100}$   $\frac{1}{200}$   $\frac{1$  $\frac{9}{2}$  become serious which is the second second analogonal set  $\frac{1}{2}$  energy matrix, a notion analogous to that seen in quantum me-10 The mass, which is proportional to the squared product of the chanics [\[15\]](#page--1-0). An extension of this matrix framework to generalized 76 11 oscillation amplitude and frequency. For narrowband signal  $x(t)$   $\frac{100}{100}$   $\frac{1000}{100}$  and  $\frac{1000}{100}$   $\frac{1000}{100}$   $\frac{1000}{100}$   $\frac{1000}{100}$   $\frac{1000}{100}$   $\frac{1000}{100}$   $\frac{1000}{100}$   $\frac{1000}{100}$  TK operator has been introduced in [\[16\]](#page--1-0).

#### *2.1. Multiresolution Teager–Kaiser energy operator*

<sup>16</sup> version of TK operator has been developed by introducing a lag  $\frac{82}{16}$ <sup>17</sup>  $\Psi_d[x(n)] = \frac{x(n-1) - x(n)x(n-2)}{(4x)^2}$  (2) parameter, k, in the expression (3) as follows [\[17\]](#page--1-0) To resolve two closely spaced tones (AM signal), an extended

$$
\frac{19}{20} \quad \text{where } x(n) = x(n\Delta) \text{ with } \Delta t = 1/f_s \text{ and } n \in \mathbb{N}; \ f_s \text{ is the sampling} \qquad \Psi_{d_k}[x(n)] = x^2(n) - x(n-k)x(n+k) \tag{6}
$$

21 **11** Exercise Control (2) is search and economic yielding  $k \in \mathbb{N}$  is a resolution parameter. For a dual tones with fre- $\psi_d[x(n)] = x^2(n) - x(n-1)x(n+1)$  (3) quencies  $f_1$  and  $f_2$ , an optimal choice of *k* parameter enhances  $\frac{88}{3}$  $\begin{array}{c} 23 \\ 24 \end{array}$  the difference  $(f_1 - f_2)$  or the sum frequency  $(f_1 + f_2)$  [\[17\]](#page--1-0). For a <sup>24</sup> The energy operator spans three adjacent samples of  $x(t)$  and sinusoidal input  $x(n) = A \cos(\omega n)$ , the operator output is given by sinusoidal input  $x(n) = A \cos(\omega n)$ , the operator output is given by

$$
\Psi_{d_k}[x(n)] \approx (A\sin(\omega k))^2 \tag{7}
$$

where  $k = \pi/2\omega$  ( $\omega = 2\pi f/f_s$ ) gives the maximum output. Thus, <sup>94</sup> <sup>29</sup>  $\Psi_a$ [x(n)]  $\approx A^2 \omega^2$  each frequency has its own optimum resolution parameter which <sup>95</sup>  $\frac{30}{20}$  maximizes the relation (7). The resolution parameters  $M_d$  and  $M_s$  and  $\frac{96}{20}$ <sup>31</sup> with  $\omega < \pi/4$ . This property is also valid for damped sinusoids of are optimally chosen to enhance respectively the sum and the <sup>97</sup>  $\frac{32}{2}$  the form the form the state of the optimum lag  $\frac{98}{2}$ 33  $(5 - 5) M - 1/5 = 5$  99  $\begin{bmatrix} 5 \ 3^4 \ \end{bmatrix}$   $\begin{bmatrix} x(n) - r^n \cos(\omega n + d_0) \end{bmatrix}$  (5) parameters for  $(f_1 - f_2)$ ,  $M_d$ , and  $(f_1 + f_2)$ ,  $M_s$ , are given by [\[17\]](#page--1-0) are optimally chosen to enhance respectively the sum and the

$$
\begin{array}{ll}\n\text{35} & \text{The operator } \Psi_d \left[ x(n) \right] \text{ approximates the product of squared am-} \\
\text{36} & \text{Poisson} \left[ \frac{0.5f_s}{f_1 + f_2} + 0.5 \right]; \\
\text{37} & \text{plitude and frequency of } x(n) \text{ and can be termed as frequency-} \\
\end{array} \qquad\n\begin{array}{ll}\n\text{38} & \text{201} \\
\text{49} & \text{21} \\
\text{501} & \text{22} \\
\text{61} & \text{23} \\
\text{71} & \text{24} \\
\text{82} & \text{25} \\
\text{93} & \text{26} \\
\text{103} & \text{27} \\
\text{104} & \text{28} \\
\text{105} & \text{201} \\
\text{106} & \text{202} \\
\text{108} & \text{203} \\
\text{109} & \text{205} \\
\text{100} & \text{208} \\
\text{101} & \text{201} \\
\text{102} & \text{201} \\
\text{103} & \text{202} \\
\text{104} & \text{203} \\
\text{105} & \text{204} \\
\text{106} & \text{205} \\
\text{107} & \text{206} \\
\text{108} & \text{208} \\
\text{109} & \text{209} \\
\text{100} & \text{200} \\
\text{101} & \text{201} \\
\text{102} & \text{201} \\
\text{103} & \text{201} \\
\text{104} & \text{201} \\
\text{105} & \text{201} \\
\text{106} & \text{201} \\
\text{107} & \text{201} \\
\text{108} & \text{201} \\
\text{109} & \text{201} \\
\text{1000} & \text{202} \\
\text{101} & \text{203} \\
\
$$

of two closely spaced tones, *x(t)*:

$$
x(t) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t) + n(t)
$$
\n(9)

45 samples  $(x(n-1), x(n), x(n+1))$  are required for the energy com-<br>where  $t \in [0, 1]$ ,  $n(t)$  is a white Gaussian noise of signal to noise 111 46 putation at each time instant  $n\Delta t$ , so it has good adaptability to ratio (SNR) of 26 dB,  $f_1 = 435$  Hz,  $f_2 = 400$  Hz and  $f_s = 8$  kHz. 112 47 the instantaneous changes in  $x(t)$ . This why TK operator is well the parameter *i* is set to 2. Using relations (8) we get  $M_d = 5$  and 113 48 114 *Ms* = 19 (Figs. [1\(](#page--1-0)a)–(c)). To identify the frequency components at 49 sponse of  $\Psi_d$ [x(n)] is nearly instantaneous and this operator can  $f_1-f_2=$  35 Hz and  $f_1+f_2=$  835 Hz, a frequency analysis of 115 50 be easily implemented in DSP processors, due to its low computa- $\Psi_{d_k}$ [x(n)] and  $\Psi_d$ [x(n)] operators is performed by calculating their 116 51 tion cost and extremely low requirements of memory storage. power spectral density (PSD).  $\Psi_{d_k}$  [x $(n)$ ] operator is applied to  $x(t)$  117 52 The TK operator has been discussed from the mathematical with  $k=1$  (classical TK operator),  $k=M_d$  and  $k=M_s$ . Both  $\Psi_d$  118 53 point of view. For example, it has been characterized algebraically and  $\Psi_{d\varsigma}$  operators reveal a strong frequency component at the dif-<br> <sup>54</sup> by giving expressions of the outputs corresponding to different ference frequency of 35 Hz, but not corresponding component at <sup>120</sup> <sup>55</sup> types of combination (e.g. linear, algebraic) of input signals and the summation frequency of 835 Hz. Note that the spectral peak <sup>121</sup> <sup>56</sup> classified the root and pre-constant signals of the operator [12]. at 35 Hz is more prominent in the case  $k = 5$  compared to result 122 <sup>57</sup> The matrix framework of the operator has been introduced in [13] provided by TK operator  $(k = 1)$ . This result shows the interest to 123 <sup>58</sup> by interpreting it as the determinant of a Toeplitz matrix contain-<br>Combine TK operator with a lag parameter  $k \neq 1$ , even in the pres-<sup>59</sup> ing the signal and its derivatives: 60 126 of *-<sup>d</sup>*<sup>19</sup> shows a clear peak at 835 Hz and none at 35 Hz. The use  $\begin{bmatrix} \dot{x} & \dot{x} \\ \dot{x}(t) & x(t) \end{bmatrix}$   $\begin{bmatrix} x(t) & x(t) \\ x(t) & \dot{x}(t) \end{bmatrix}$   $\begin{bmatrix} x(t) & x(t) \\ x(t) & \dot{x}(t) \end{bmatrix}$   $\begin{bmatrix} x(t) & x(t) \\ x(t) & \dot{x}(t) \end{bmatrix}$   $\begin{bmatrix} x(t) & x(t) \\ x(t) & \dot{x}(t) \end{bmatrix}$  $\epsilon_2$   $\mathcal{F}_{\epsilon}[X(t)] = |\ddot{\chi}(t) - \dot{\chi}(t)|$ ,  $\mathcal{F}_{d}[X(t)] = |\chi(n-1) - \chi(n)|$  quency. This example confirms the findings reported in [\[17\]](#page--1-0) and 128 63 129 show that changing the lag parameter *k* can result in enhancing 64 130 the summation or difference frequencies of two component tones.  $\frac{65}{1}$  The amplitude and the frequency of the signal do not vary too fast (rate of Furthermore, noise with SNR = 26 dB does not degrade the promi-<sup>66</sup> change) or too greatly (range of value) with time compared to carrier frequency. Thence of the summation or the difference frequency component. To 132 and  $\Psi_{d_5}$  operators reveal a strong frequency component at the difference frequency of 35 Hz, but not corresponding component at

The amplitude and the frequency of the signal do not vary too fast (rate of change) or too greatly (range of value) with time compared to carrier frequency.

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