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Compressive sensing meets time–frequency: An overview of recent advances in time–frequency processing of sparse signals

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ABSTRACT

Compressive sensing is a framework for acquiring sparse signals at sub-Nyquist rates. Once compressively acquired, many signals need to be processed using advanced techniques such as time–frequency representations. Hence, we overview recent advances dealing with time–frequency processing of sparse signals acquired using compressive sensing approaches. The paper is geared towards signal processing practitioners and we emphasize practical aspects of these algorithms. First, we briefly review the idea of compressive sensing. Second, we review two major approaches for compressive sensing in the time–frequency domain. Thirdly, compressive sensing based time–frequency representations are reviewed followed by descriptions of compressive sensing approaches based on the polynomial Fourier transform and the short-time Fourier transform. Lastly, we provide brief conclusions along with several future directions for this field.

1. Introduction

Time–frequency analysis provides a framework for a descriptive analysis of non-stationary signals whose models are not available or easily constructed [1], [2]. For such signals, time or frequency domain descriptions typically do not offer comprehensive details about changes in signal characteristics [3]. The main issue with the time domain representation is that it provides no details about the frequency content of those signals, and sometimes, even the time content can be difficult to interpret [4]. The frequency domain on the other hand provides no easily understood timing details about the occurrence of various frequency components [5]. In other words, timing details are buried within the phase spectrum of a signal, which is the most common reason for only analyzing the amplitude spectrum of a signal obtained via the Fourier transform. To combine timing and spectral into a joint representation, a time variable is typically introduced into a Fourier-based analysis to obtain two-dimensional, redundant representations of non-stationary signals [6]. Such representation provide a description of spectral signal changes as a function of time, that is, the description of time-varying energy concentration changes along the frequency axis. In an ideal case, these two-dimensional signal representations would combine instantaneous frequency spectrum with global temporal behavior of a signal [7], [8], [9], [10], [11], [12], [13], [14], [15].

Time–frequency analysis is often employed in the analysis of complex non-stationary signals (e.g., physiological signals [16], [17], [18], [19], [20], [21], [22], [23], mechanical vibrations [24], [25], [26], [27], audio signals [28], [29], [30], radar signals [31], [32], [33], [34], [35], [36]). However, continuously monitoring such signals for an extended period of time can impose heavy burdens on data acquisition and processing systems, even when sampling these non-stationary signals at low sampling rates. To avoid these data acquisition and processing burdens, compressive sensing aims to compress signals during a data acquisition process, rather than afterwards [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51], [52].

In this paper, we review recent advances that combine the ideas of time–frequency and compressive sensing analyses. Section 2 reviews the main ideas behind compressive sensing. In Section 3, we introduce the main approaches to obtain compressed samples in the time–frequency domain. Several different approaches are presented here including compressive sensing in the ambiguity domain, but also compressive sensing of non-stationary signals using time–frequency dictionaries. We also reviewed compressive sensing approaches that relied on the short-time Fourier transform and the polynomial Fourier transform. Compressive sensing based time–frequency representations are described in Section 4. Conclusions and future directions are provided in Section 5.

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2. Compressive sensing

In traditional signal processing, the Shannon–Nyquist sampling theorem mandates that any signal needs to be sampled at least twice the highest frequency present in the signal to be able to accurately recover information present in the signal. The traditional sampling approach can yield a large number of samples, and compressive strategies are often used immediately after sampling in order to reduce storage requirements or transmission complexities. While this has been a prevailing approach for many years, it is clearly a redundant approach as most of acquired samples are disregarded immediately after sampling. To avoid these redundant steps, compressive sensing has been proposed and it postulates a signal can be recovered using a fewer number of samples than required by the Shannon–Nyquist theorem [38], [39], [53], [40], [54] [55], [56], [57], [58], [59], [60], [61], [62], [63].

The main idea behind compressive sensing is to combine sensing and compression steps into a single step during a data acquisition process [39], [40], [42], [64], [65]. Compressive sensing approaches typically acquire signals at sub-Nyquist rates (e.g., one tenth of the Nyquist rate) and signals can be accurately recovered from these samples with *a certain probability* [39]. These approaches work very well for K-sparse signals, i.e., signals that can be represented by K bases in an N-dimensional space. In other words, compressive sensing approaches will acquire $M \ll N$ samples that will encode a K-sparse signal of dimension N by computing a measurement vector $\mathbf{y}$ of a signal vector $\mathbf{s}$ [66], [67], [68], [69]:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{s} \tag{1}$$

where $\mathbf{\Phi}$ represents an $M \times N$ sensing matrix [40]. The signal vector $\mathbf{s}$ can be recovered from sparse samples by utilizing a norm minimization approach:

$$\min ||\mathbf{s}||_0 \text{ subject to } \|\mathbf{y} - \mathbf{\Phi}\mathbf{s}\|_2 < \xi \tag{2}$$

where ξ is measurement noise, $||s||_0$ represents the number of nonzero entries of $\mathbf{s}$ and $\|\bullet\|_2$ is the Euclidean norm. However, it should be mentioned that it is not guaranteed that eqns. (1) and (2) will provide an accurate representation of sparse signals. In some applications that are sensitive to small changes such as medical diagnostic applications, it is almost mandatory to achieve almost perfect recovery of these sparse signals, otherwise compressive sensing schemes are not useful at all in medical diagnostic applications. To reach these almost perfect reconstructions of sparse signals, compressive sensing can be performed in other domains (i.e., other than the time domain), which yields a new reformulation of the compressive sensing approach proposed in (1) as [64], [67]:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{s} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{x}. \tag{3}$$

Here, $\mathbf{x}$ is the vector of expansion coefficients representing the sparse representation of the signal $\mathbf{s}$ in the basis $\mathbf{\Psi}$. A very good example of this change is representing a single sinusoid in the frequency domain. This transformation would enable us to represent such a sinusoid with by a two-sparse vector. In this paper, this change of the domain is achieved by representing a signal in the time–frequency domain, rather than using its time-domain samples.

It should be understood that the compressive sensing approach proposed by eqn. (3) affects the sparsity in the transform domain, which then inherently affects the number of measurements needed to reconstruct a signal. This is assessed using the so-called coherence measure between the matrices $\mathbf{\Phi}$ and $\mathbf{\Psi}$ [70], [71], [72], [73]:

$$\mu(\mathbf{\Phi}, \mathbf{\Psi}) = \sqrt{N} \max |\langle \phi_k, \psi_j \rangle| \tag{4}$$

where N is the signal length, ϕ_k is the kth row of $\mathbf{\Phi}$, and ψ_j is the jth row of $\mathbf{\Psi}$. Smaller values of the coherence measure typically denote that a smaller number of random measurements is needed to accurately reconstruct a signal.

3. Time–frequency based compressive sensing

The time–frequency domain represent an ideal domain to sparsely represent nonstationary signals for several different reasons. First, it is very difficult to represent nonstationary signals sparsely either in time or frequency domains. For example, a frequency modulated signal is concentrated along its instantaneous frequency in the time–frequency domain, and most of other values are equal to zero. But, its frequency domain representation has many non-zero components, and its time domain representation typically has many (large) amplitude changes that can be difficult to compress. Therefore, such a frequency modulated signal or any other signal with complex non-stationary structures should be compressively sampled in the time–frequency domain, as their representations are often sparse in the time–frequency domain [74], [75], [76]. Second, recent advances in computational resources enabled fast manipulations of large matrices, which are required for compressive sensing of nonstationary signals in the time–frequency domain [77].

In this section, we will overview two major approaches for compressive sensing of nonstationary signals in the time–frequency domain. We will begin with compressively sampling a nonstationary signal in the ambiguity domain as proposed in [78] with understanding that this approach is only applicable for quadratic time–frequency representations. A more general approach is to utilize time–frequency dictionaries to obtain a sparse time–frequency representation of a nonstationary signal, which can be then used to compressively sensed such a signal.

3.1. Compressive sensing in the ambiguity domain

As mentioned in the previous paragraph, the ambiguity domain provides a suitable framework to compressively sampled nonstationary signals. To achieve representations in the ambiguity domain, we can start with the Wigner–Ville distribution, $WVD(t, f)$, and take the two-dimensional Fourier transform of it to obtain the ambiguity domain representation [1], [79]:

$$A_x(\nu, \tau) = \mathcal{F}_{2D}\{WVD(t, f)\} \tag{5}$$

where $\mathcal{F}_{2D}$ is the forward and inverse two-dimensional Fourier operator. The ambiguity domain offers an opportunity to suppress or completely remove cross-terms, which plague the quadratic time–frequency representations, as cross-terms are typically displaced from the origin in the ambiguity domain, and the auto-terms are typically centered around the origin. Therefore, low-pass filtering by multiplying the ambiguity representation of the signal, $A_x(\nu, \tau)$, by a kernel function $k(\nu, \tau)$:

$$\mathcal{A}_x(\nu, \tau) = A_x(\nu, \tau)k(\nu, \tau). \tag{6}$$

However, it should be mentioned here that compressive sensing approaches here are mostly used to obtain enhanced time–frequency signal energy localization in the time–frequency domain. Specifically, we compressively sample the ambiguity domain representation of the signal in order to obtain a very sparse time–frequency domain signal representation. This is achieved by solving the l_1-norm minimization problem to obtain a sparse time–frequency distribution $\widehat{\Upsilon}_x(t, f)$:

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