JID:YDSPR AID:2230 /FLA [m5G; v1.227; Prn:9/01/2018; 11:24] P.1 (1-7)

Digital Signal Processing ••• (••••) •••–•••

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Deconvolution using Fourier transform phase, ℓ_1 and ℓ_2 balls, and filtered variation

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Deconvolution
Fourier transform phase
ℓ_1 ball
Projection onto convex sets

ABSTRACT

In this article, we present a deconvolution software based on convex sets constructed from the phase of the Fourier Transform, bounded ℓ_2 energy and ℓ_1 energy of a given image. The iterative deconvolution algorithm is based on the method of projections onto convex sets. Another feature of the method is that it can incorporate an approximate total variation bound called filtered variation bound on the iterative deconvolution algorithm. The main purpose of this article is to introduce the open source software called projDeconv v2.

1. Motivation and significance

Deconvolution is the process of inverting the effects of filtering that reduces the image quality – mostly by blurring – in many image processing applications. The source of the blurriness may vary form camera motion to optical characteristics of the image capturing equipment. Therefore it may be crucial in many image processing problems to deconvolve an input image before further processing.

In this article we present a software that uses Projections onto Convex Sets (POCS) based method in order to correct highly blurred out of focus images. It may not be possible to estimate the parameters of blurring process in out of focus microscopic and Magnetic Particle Imaging (MPI) images. In such highly out-of focus images, well-known deconvolution algorithms are not very efficient. In these imaging problems it may not be possible to estimate a point spread function (psf) which is crucial for most deconvolution algorithms. However, in some cases it may be possible to assume that the psf is symmetric with respect to the origin. As a result it is possible to estimate the phase of the input from the observed image if the psf is symmetric. Our software exploits the symmetry characteristics of the psf and estimates the Fourier Transform phases from the blurry image. Software then uses iterative POCS method on Fourier phase, ℓ_1 and ℓ_2 balls, and Filtered Variation in order to perform deconvolution.

POCS based deconvolution was first developed by Trussel and Civanlar [19]. The method relies on iterative projections onto known convex properties of the image in spacial/frequency domains. Let the observed image y be the blurred version of the original image x_0 and h be the blurring function. In many problems $y[n_1, n_2]$ is also corrupted by noise. For a given image pixel $[n_1, n_2]$ we define a hyperplane as follow:

$$C_{n_1,n_2} = \left\{ x \middle| y[n_1, n_2] = \sum_{k_1,k_2} h[k_1, k_2]x[n_1 - k_1, n_2 - k_2] \right\} \tag{1}$$

Therefore the solution of $x_0[n_1, n_2]$ must be in the intersection of these hyperplanes:

$$x_0 \in \cap_{n_1,n_2} C_{n_1,n_2} \tag{2}$$

Projection onto C_{n_1,n_2} is essentially equivalent to making orthogonal projections onto hyperplanes because the convolution is a linear operation. Let x_i be the current iterate of the iterative deconvolution algorithm. Its orthogonal projection x_{i+1} onto C_{n_1,n_2} is given by:

$$x_{i+1} = x_i + \lambda \frac{y[n_1, n_2] - (h * x)[n_1, n_2]}{||h||^2} h, \quad i = 1, 2, 3, \ldots \tag{3}$$

where $\lambda = 1$ for orthogonal projection and x_1 is an estimate of x_0. POCS theory allows $0 < \lambda < 2$ for convergence [22,15]. Eq. (3) abuses the notation a little bit because size of the image x and the blur h may be different. The h vector should be padded with zeros before addition operation.

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https://doi.org/10.1016/j.dsp.2017.11.004
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Please cite this article in press as: O. Yorulmaz, A.E. Cetin, Deconvolution using Fourier transform phase, ℓ_1 and ℓ_2 balls, and filtered variation, Digit. Signal Process. (2018), https://doi.org/10.1016/j.dsp.2017.11.004

But making successive orthogonal projections onto the sets C_{n_1,n_2} may not be sufficient to reconstruct the original image x_0 because of the ill-posed structure of the problem and the noise. Therefore other closed and convex sets restricting the solution into a feasible set may be necessary. Trussel and Civanlar used ℓ_2 norm based regularizing sets in their POCS based deconvolution algorithm. In our algorithm, we use the phase of the Fourier Transform and ℓ_1-ball, filtered variation based sets in addition to ℓ_2-ball. In what follows we describe other closed and convex sets that can be used in deconvolution problems.

1.1. Fourier transform phase

In many deconvolution problems [19,2] the blurring function is symmetric with respect to origin, i.e.,

$$h[n_1,n_2]=h[-n_1,-n_2] \tag{4}$$

For example, this condition is satisfied by Gaussian blurs, disk shaped blurs and some motion blur kernels [5]. Such kernels do not distort the Fourier phase of the input image. This means that the phase of the observed image and the original image are related and the phase of the original image can be estimated from the observed image. Any iterative deconvolution algorithm can take advantage of this relation by performing orthogonal projections onto the phase set during iterations [18,21]. In the absence of noise:

$$Y(e^{j\omega})=H(e^{j\omega})X_0(e^{j\omega}) \tag{5}$$

If the blur is symmetric $H(e^{j\omega})$ is real. When $H(e^{j\omega})>0$ for some ω_1,ω_2,

$$\angle Y(e^{j\omega})=\angle X_0(e^{j\omega}). \tag{6}$$

When $H(e^{j\omega})<0$ for some ω_1,ω_2,

$$\angle Y(e^{j\omega})=\angle X_0(e^{j\omega})+\pi. \tag{7}$$

As a result we can determine the phase of X_0 from $Y(e^{j\omega})$. The set of images with a given Fourier Transform phase is a closed and convex set [22,12]. Therefore we introduce the following set for deconvolution problems

$$C_\phi=\left\{x|\angle X(e^{j\omega})=\angle X_0(e^{j\omega})\right\} \tag{8}$$

which is the set of images whose Fourier transform phase is equal to a given phase $\angle X_0(e^{j\omega})$.

Projection onto C_ϕ is obtained in the Fourier domain. Let $x_{\phi i}$ be the projection of x_i. The Fourier transform $X_{\phi i}$ of $x_{\phi i}$ is given by

$$X_{\phi i}(e^{j\omega})=|X_i(e^{j\omega})|e^{j\angle X_0(\omega)} \tag{9}$$

where the phase of X_i is replaced by the given phase $\angle X_0(\omega)$. The spatial domain image $x_{\phi i}[n_1,n_2]$ will be $F^{-1}[X_{\phi i}(e^{j\omega})]$ where F^{-1} is the inverse Fourier Transform operation. Recently, the subspace or sparsity constraints were imposed to reduce the search space [9,1]. The phase set is also a subspace which reduces the search space.

1.2. Total variation

Total Variation (TV) is a widely used cost function in image processing [14,5,13,17]. Bounded TV set was introduced by [3,2] for denoising problems:

$$C_{TV}=\{x|TV(x)\le\epsilon\} \tag{10}$$

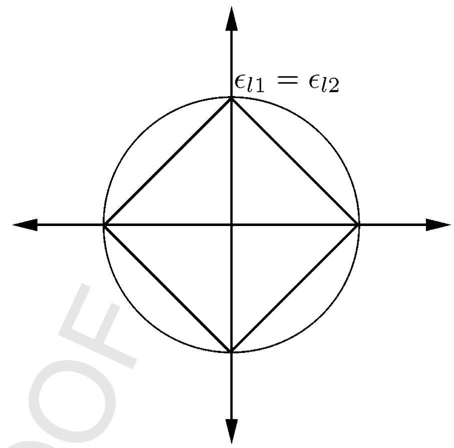

Fig. 1. ℓ_1 and ℓ_2 balls.

which is the set of images whose TV is below a given ϵ. The set C_{TV} is also a closed and convex set. Therefore it can be used in any Projection onto Convex Sets (POCS) based deconvolution problem.

In this article we introduce the bounded filtered variation (FV) set, which is based on the following cost function:

$$FV(x)=\sum_{n_1,n_2}|x[n_1,n_2]-(g*x)[n_1,n_2]| \tag{11}$$

where g is a low-pass filter. Any low-pass filter can be used in Eqn. (11). In 1-D TV function $g[n]$ is simply equal to $\delta[n-1]$ because 1-D TV function is $TV(x)=\sum_n|x[n]-x[n-1]|$. The bounded FV set is defined as follows:

$$C_{FV}=\{x|FV(x)\le\epsilon\} \tag{12}$$

It can be shown that C_{FV} is also a closed and convex set.

We perform projections onto the set C_{FV} in an approximate manner in two steps. Let x_k be the current image that we want to project onto the set C_{FV}. The image x_k is divided into its low-pass and high-pass components $x_{k,lo}=x_k*g$ and $x_{k,hi}=x_k-x_{k,lo}$ using the low-pass filter g, respectively. We project the high-pass filtered component onto the ℓ_1-ball and obtain:

$$x_{k,hp}=P_{\ell_1}(x_{k,hi}) \tag{13}$$

where P_{ℓ_1} represent the orthogonal projection operation onto the ℓ_1-ball. Finally, we combine the low-pass component of the image with $x_{k,hp}$ to obtain the approximate projection onto the set C_{FV}:

$$x_{k+1}=x_{k,lo}+x_{l,hp} \tag{14}$$

where x_{k+1} contains the low-pass components of the original image but its high-pass components are regulated by the ℓ_1-ball.

1.3. Bounded energy set

Both ℓ_1-ball and ℓ_2-ball are well known sets used in image reconstruction problems. The ℓ_1-ball is

$$C_{\ell_1}=\left\{\sum_{n_1,n_2}|x[n_1,n_2]|\le\epsilon\right\} \tag{15}$$

and the ℓ_2-ball is

$$C_{\ell_2}=\left\{\sum_{n_1,n_2}|x[n_1,n_2]|^2\le\epsilon^2\right\} \tag{16}$$

In Fig. 1 ℓ_1 and ℓ_2 balls in R^2 are shown.