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Brief paper

Using exponential time-varying gains for sampled-data stabilization and estimation ☆

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ABSTRACT

This paper provides exponential stability results for two system classes. The first class includes a family of nonlinear ODE systems while the second consists of semi-linear parabolic PDEs. A common feature of both classes is that the systems they include involve sampled-data states and a time-varying gain. Sufficient conditions ensuring global exponential stability are established in terms of Linear Matrix Inequalities (LMIs) derived on the basis of Lyapunov–Krasovskii functionals. The established stability results prove to be useful in designing exponentially convergent observers based on sampled-data measurements. It is shown throughout simulated examples from the literature that the introduction of time-varying gains is beneficial to the enlargement of sampling intervals while preserving the stability of the system.
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1. Introduction

Designing sampled-data observers and controllers has been a hot topic in recent years, see e.g. Fridman (2010) and reference list therein. In this regard, a long standing issue is how to enlarge the sampling intervals (Heemels, Johansson, & Tabuada, 2012) while ensuring global exponential stability. In a recent paper (Cacace, Germani, & Manes, 2014), it has been shown that the introduction of time-varying gains in a specific class of observers improves their exponential convergence properties in presence of measurement delay. Presently, these properties are investigated in presence of measurement sampling. To this end, we consider two classes of sampled-data systems and analyze their exponential stability. The considered classes are respectively consisting of nonlinear globally Lipschitz ODEs and semi-linear parabolic PDEs. A common feature of both classes is that the systems they include are allowed to involve a time-varying gain of the form $e^{-\eta(t-t_k)}$ with $\eta > 0$ a tuning parameter, where t_k $(k = 0, 1, \ldots)$ are sampling instants. It turns out that, the first family, including ODE systems, is a generalization of that dealt with in Cacace et al. (2014). For both classes of systems, we establish sufficient conditions for global exponential stability in terms of Linear Matrix Inequalities (LMIs) derived from Lyapunov–Krasovskii functionals. Then, it is shown that these stability results are useful in designing sampled-data observers with time-varying gains. As the established LMIs conditions involve both the tuning parameter η and the maximum sampling interval h, these parameters can then be used to improve the observer convergence properties. Actually, it is checked through several simulated examples that the utilization of the above time-varying gain entails significant enlargement of the maximum sampling interval, compared with the constant gain case (corresponding to $\eta = 0$). It is worth noting that, the present theoretical stability results can also be used in sampled-data control design improving exponential stability properties and enlarging sampling intervals. A part of the present results, namely those concerning ODEs, have been presented in our conference paper (Ahmed-Ali et al., 2015).

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The paper is organized as follows: in Section 2, the first stability result, concerning nonlinear ODE systems, is stated and applied to observer design; in Section 3, the second stability result, concerning semi-linear PDE systems, is stated and applied to observer design; a conclusion and reference list end the paper. Some technical proofs are appended.

Notations and preliminaries

Throughout the paper the superscript T stands for matrix transposition, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space with vector norm $|.|$, $\mathbf{R}^{n\times m}$ is the set of all $n \times m$ real matrices, and the notation $P > 0$, for $P \in \mathbf{R}^{n\times n}$, means that P is symmetric and positive definite. In Symmetric matrices, symmetric terms are denoted $*$; $\lambda_{\min}(P)$ (resp. $\lambda_{\max}(P)$) denotes the smallest (resp. largest) eigenvalue. The notation $(t_k)_{k\geq 0}$ refers to a strictly increasing sequence such that $t_0 = 0$ and $\lim_{k\to\infty} t_k = \infty$. The sampling periods are bounded i.e. $0 < t_{k+1} - t_k < h$ for some scalar $0 < h < \infty$ and all $k = 0, 1, \ldots, \infty$. We also define the variable $\tau(t) = t - t_k$, $t \in [t_k, t_{k+1})$. $\mathcal{H}^1(0, l)$ is the Sobolev space of absolutely continuous functions $z : (0, l) \to \mathbf{R}$ with the square integrable derivative $\frac{d}{dx}$. Given a two-argument function $u(x, t)$, its partial derivatives are denoted $u_t = \frac{\partial u}{\partial t}$, $u_{xx} = \frac{\partial^2 u}{\partial x^2}$.

2. Sampled-data globally Lipschitz nonlinear ODEs

2.1. System description and stability result

We are considering a class of sampled-data nonlinear systems described by the following equation:

$$\dot{x}(t) = A_0 x(t) + A_1 e^{-\eta(t-t_k)} x(t_k) + \phi(x(t)), \quad t \in [t_k, t_{k+1}) \tag{1}$$

where $x(t) \in \mathbf{R}^n$; the scalar $\eta \geq 0$; A_0, A_1 are constant matrices with appropriate dimensions. As in Bar Am and Fridman (2014), the function ϕ is supposed to be class $\mathcal{C}^1$ with uniformly bounded Jacobian ϕ_x, satisfying $\phi(0) = 0$ and

$$\phi_x^T(x)\phi_x(x) \leq M \quad \forall x \tag{2}$$

for some positive constant $n \times n$-matrix M. Using Jensen's inequality it is readily checked that (2) implies the following inequality:

$$\int_0^1 \phi_x^T(sx)ds \int_0^1 \phi_x(sx)ds \leq M.$$

Remark 1. Eq. (1) may represent a networked control system described by

$$\dot{x}(t) = A_0 x(t) + \phi(x(t)) + Bu(t),$$

with the communication network placed between the sensor and the controller (but no network is placed between the controller and the actuator). Assuming that the discrete-time state measurements $x(t_k)$ are transmitted through the communication network from the sensor to controller, consider the state-feedback,

$$u(t) = e^{-\eta(t-t_k)} K x(t_k), \quad t \in [t_k, t_{k+1}),$$

where K is a gain and $\eta > 0$ is a scalar. It turns out that the resulting closed-loop system fits Eq. (1) with $A_1 = BK$.

As in Cacace et al. (2014), introduce the following change of coordinates $z(t) = e^{\eta t} x(t)$ with $\eta > 0$. Then one gets

$$\dot{z}(t) = \eta z(t) + A_0 z(t) + A_1 z(t_k) + \left[\int_0^1 \phi_x(sx(t))ds\right] e^{\eta t} x(t), \quad t \in [t_k, t_{k+1}) \tag{3}$$

which is rewritten as follows:

$$\dot{z}(t) = (\eta I_n + A_0) z(t) + A_1 z(t_k) + \left[\int_0^1 \phi_x(sx(t))ds\right] z(t), \quad t \in [t_k, t_{k+1}). \tag{4}$$

Following Fridman (2010), consider the following Lyapunov–Krasovskii functional for (4):

$$V(t) = \bar{V}(t) + V_X(t) \tag{5}$$

with

$$\bar{V}(t) = z^T(t)Pz(t) + (t_{k+1} - t)\int_{t_k}^t \dot{z}^T(s)U\dot{z}(s)ds,$$
$$P > 0,\ U > 0,\ t \in [t_k, t_{k+1})$$

and

$$V_X(t) = (t_{k+1} - t)\xi^T \begin{bmatrix} \frac{X + X^T}{2} & -X + X_1 \\ * & -X_1 - X_1^T + \frac{X + X^T}{2} \end{bmatrix} \xi,$$

where $\xi(t) = col\{z(t), z(t_k)\}$, X and X_1 are $n \times n$ matrices. The positiveness of (5) is ensured if the following LMI holds (Fridman, 2010):

$$\begin{bmatrix} P + h\frac{X + X^T}{2} & hX_1 - hX \\ * & -hX_1 - hX_1^T + h\frac{X + X^T}{2} \end{bmatrix} > 0. \tag{6}$$

Using the definition of $z(t)$, we can see that the exponential stability of system (1) is guaranteed if:

$$\dot{V}(t) + 2\alpha V(t) \leq 0 \quad t \in [t_k, t_{k+1}) \tag{7}$$

for some scalar $\alpha \in (-\eta, 0]$ (note that the scalar α is allowed to be negative). Indeed, if (7) is satisfied one has,

$$\dot{V}(t) \leq -2\alpha V(t) \Longrightarrow |z(t)| \leq \left(\frac{\sqrt{V_{|t=0}}}{\sqrt{\lambda_{\min}(P)}}\right) e^{-\alpha t}.$$

Then, using the fact that $z(t) = e^{\eta t} x(t)$, one gets:

$$|x(t)| \leq \left(\frac{\sqrt{V_{|t=0}}}{\sqrt{\lambda_{\min}(P)}}\right) e^{-(\eta+\alpha)t}.$$

From the above inequality, one sees that the exponential convergence is guaranteed if $\eta + \alpha > 0$. Since the parameter η is positive and free, it is sufficient to let $\alpha \in (-\eta, 0]$ for ensuring an exponential convergence with a decay rate $\eta + \alpha$. In the following proposition, it is shown that the property (7), and resulting exponential stability with a decay rate $\eta + \alpha > 0$, are actually ensured under well established sufficient conditions, expressed in terms of LMIs.

Proposition 1. *Consider the system (1) with possibly varying sampling-intervals subject to $t_{k+1} - t_k \leq h$ with some scalar $h > 0$. Given $\eta > 0$ and $\alpha \in (-\eta, 0]$, let there exist $n \times n$ matrices $P > 0, U > 0, X, X_1, P_2, P_3, T, Y_1, Y_2$ and a scalar $\lambda > 0$ that satisfy the LMI (6) and the following LMIs:*

$$\Psi(0) \triangleq \begin{bmatrix} \Phi_{11} - X_\alpha & \Phi_{12} + X_{\tau(t)} & \Phi_{13} + X_{1\alpha} & P_2^T \\ * & \Phi_{22} + hU & \Phi_{23} - X_{1\tau(t)} & P_3^T \\ * & * & \Phi_{33} - X_{2\alpha} & 0 \\ * & * & * & -\lambda I_n \end{bmatrix}_{\Big|\tau(t)=0} < 0 \tag{8}$$

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