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Brief paper

# Optimality and flexibility in Iterative Learning Control for varying tasks ☆

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ABSTRACT

Iterative Learning Control (ILC) can significantly enhance the performance of systems that perform repeating tasks. However, small variations in the performed task may lead to a large performance deterioration. The aim of this paper is to develop a novel ILC approach, by exploiting rational basis functions, that enables performance enhancement through iterative learning while providing flexibility with respect to task variations. The proposed approach involves an iterative optimization procedure after each task, that exploits recent developments in instrumental variable-based system identification. Enhanced performance compared to pre-existing results is proven theoretically and illustrated through simulation examples.



## 1. Introduction

ILC enables a significant performance enhancement of batch-repetitive processes. In ILC the command signal is iteratively updated from one experiment (trial) to the next. Typical ILC algorithms generate a control signal that exactly compensates for the trial-invariant exogenous disturbances during a specific task. ILC has been thoroughly researched, including convergence analysis (Moore, 1993; Norrlöf & Gunnarsson, 2002), and robustness to model uncertainty (Ahn, Moore, & Chen, 2007; Bristow & Alleyne, 2008) and disturbances (Ghosh & Paden, 2002; Saab, 2005). In addition, many successful applications have been reported, including wafer scanners (de Roover & Bosgra, 2000; Mishra, Coaplen, & Tomizuka, 2007) and printing systems (Bolder, Oomen, Koekebakker, & Steinbuch, 2014).

ILC can perfectly compensate for non-varying disturbances, but is consequently very sensitive to varying disturbances. These varying disturbances include measurement noise and also changing reference trajectories. As a result, a learned signal corresponds to a specific reference signal and a change in this signal potentially leads to performance deterioration (Gao & Mishra, 2014; Heertjes & van de Molengraft, 2009; Hoelzle, Alleyne, & Wagoner Johnson, 2011; Phan & Frueh, 1996). To overcome this drawback, several solutions to enhance the extrapolation properties of ILC have been developed. In Hoelzle et al. (2011), the extrapolation properties are enhanced by constructing the task such that it consists of a set of basis tasks. This provides extrapolation to tasks consisting of a finite set of elementary tasks. A more general approach is to parameterize the command signal in a set of basis functions (Oh, Phan, & Longman, 1997; Phan & Frueh, 1996). Such an approach allows for arbitrary tasks. Examples include polynomial basis functions (Bolder et al., 2014; Gao & Mishra, 2014; Heertjes & van de Molengraft, 2009; van de Wijdeven & Bosgra, 2010) for which the associated optimization problem has an explicit analytic solution (Gunnarsson & Norrlöf, 2001). These polynomial approaches have clear advantages from an optimization perspective, since global optimality can be guaranteed and the implementation and computation is generally inexpensive and fast.

Recently, rational basis functions have been introduced in ILC in Bolder and Oomen (2015). These rational basis functions are more general than polynomial basis functions since the latter are recovered as a special case. In the rational case, an analytic solution can be retained if the poles are pre-specified (Heuberger, Van den Hof, & Wahlberg, 2005). To enable enhanced performance the poles are also optimized in Bolder and Oomen (2015), where the non-convex optimization problem is solved using a similar algorithm as in Steiglitz and McBride (1965). In Bolder and Oomen (2015),

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fast convergence to a stationary point and increased performance is reported. In addition, the algorithm is reported to be less sensitive to local minima when compared to a Gauss–Newton type of algorithm as shown in, for example, (Bohn & Unbehauen, 1998). However, in the present paper both a theoretical and numerical analysis are presented that reveal that the stationary point of the iteration is not necessarily a minimum of the objective function, which in fact has also been observed in related system identification algorithms (Whitfield, 1987).

Although important contributions have been made to enhance extrapolation capabilities of ILC through basis functions, presently available optimization algorithms suffer from the problem of non-optimality or poor convergence properties. The aim of this paper is to develop a new approach that guarantees that the stationary point of the iterative solution is always an optimum. As a consequence, increased performance is achieved compared to pre-existing approaches. The proposed approach is related to instrumental variable system identification. Note that the instrumental variable approach in Boeren, Oomen, and Steinbuch (2015) is essentially different in that it deals with an estimation problem and not an ILC problem.

The contributions of this paper are threefold. First, a new iterative solution algorithm for rational basis functions in ILC is proposed which constitutes the main contribution of this paper. Second, non-optimality of the pre-existing approach for rational basis functions in ILC is established. Third, it is shown by two simulation examples that (i) the proposed approach outperforms the pre-existing approach, and (ii) ILC with basis functions outperforms standard ILC for varying reference tasks. Since the proposed approach has close connections to instrumental variable-based system identification, the simulation study may be of interest to instrumental variable based system identification.

In Bolder and Oomen (2015) a different iterative solution for rational basis functions in ILC is provided. In this paper it is theoretically proven and illustrated through simulation examples that this pre-existing approach is non-optimal by construction and is outperformed by the proposed approach. Related to the present paper, preliminary results can be found in van Zundert, Bolder, and Oomen (2015). This paper significantly extends this earlier research on several aspects. First of all, a more general basis parameterization is considered. Second, a proof for the optimal proposed approach is presented. Third, solutions of ILC with polynomial basis functions and standard norm-optimal ILC are recovered as special cases. Fourth, the non-optimality of the pre-existing approach is mathematically proven, motivating the use of the novel approach. Fifth, a numerical simulation example on convergence is presented to provide insight into both approaches. Finally, a simulation example is presented demonstrating the enhanced extrapolation properties with respect to norm-optimal ILC and ILC with polynomial basis functions.

The outline of this paper is as follows. In Section 2, the problem considered in this paper is introduced. The proposed approach is presented in Section 3. In Section 4, the proposed approach is compared with the pre-existing approach (Bolder & Oomen, 2015). Moreover, non-optimality of the pre-existing approach is established. The two iterative approaches are compared by use of a simulation example in Section 5, demonstrating that the proposed approach outperforms the pre-existing approach on a complex industrial system. In Section 6, a simulation example is presented revealing the benefit of using basis functions in ILC. Section 7 contains conclusions.

*Notation* In this paper, systems are discrete-time, linear, time-invariant (LTI), single-input, single-output (SISO). Systems are generally rational in complex indeterminate $z$ and indicated in boldface with the argument $z$, for example $\mathbf{H}(z)$. Let $x(k)$ denote a signal $x$ at time $k$. Let $h(l)$ be the impulse response of the system

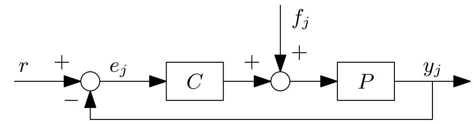


**Fig. 1.** Block diagram of closed-loop system under consideration.

$\mathbf{H}(z)$. The output $y(k)$ of the response of $\mathbf{H}(z)$ to input $u$ is given by $y(k) = \sum_{l=-\infty}^{\infty} h(l)u(k-l)$. Let $N \in \mathbb{Z}^+$ denote the trial length, i.e. the number of samples per trial. Assuming $u(k) = 0$ for $k < 0$ and $k > N-1$, then the input–output relation can be recast as

$$\underbrace{\begin{bmatrix} y[0] \\ y[1] \\ \vdots \\ y[N-1] \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} h(0) & h(-1) & \dots & h(1-N) \\ h(1) & h(0) & \dots & h(2-N) \\ \vdots & \vdots & \ddots & \vdots \\ h(N-1) & h(N-2) & \dots & h(0) \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} u[0] \\ u[1] \\ \vdots \\ u[N-1] \end{bmatrix}}_{u},$$

with $u, y \in \mathbb{R}^N$ the input and output, respectively. Let $\|x\|_W := x^\top W x$, where $x \in \mathbb{R}^N$ and $W = W^\top \in \mathbb{R}^{N\times N}$. $W$ is positive definite ($W \succ 0$) iff $x^\top W x > 0, \forall x \neq 0$ and positive semi-definite ($W \succeq 0$) iff $x^\top W x \geq 0, \forall x$.

To facilitate presentation, occasionally transfer functions are assumed causal to enable a direct relation between infinite and finite time. This is standard in ILC (Norrlöf & Gunnarsson, 2002) and not a restriction on the presented results. For instance, the approach in Boeren et al. (2015, Appendix A) may be adopted.

## 2. Problem formulation

In this section the considered problem is defined by describing the system, introducing norm-optimal ILC, and highlighting the limitations of standard norm-optimal ILC. Finally, the contributions are listed explicitly.

### 2.1. System description

The control setup is shown in Fig. 1. Here $P = \frac{B_0}{A_0}$, $B_0, A_0 \in \mathbb{R}[z]$, is the rational system and $C$ an internally stabilizing feedback controller. The closed-loop system is assumed to operate batch-repetitive, i.e. the same process of fixed length $N$ is repeated over and over. A single execution is referred to as a trial. The aim is to determine the feedforward $f_{j+1}$ for trial $j+1$ such that the output $y_{j+1}$ follows the trial-invariant reference $r$, i.e. minimizes the error $e_{j+1} = r - y_{j+1}$.

The error for trial $j$ is given by

$$e_j = Sr - SPf_j \tag{1}$$
$$= \tilde{r} - Jf_j, \tag{2}$$

with sensitivity $S := (I + PC)^{-1}$, process sensitivity $J := SP$, and $\tilde{r} := Sr$. The error for trial $j+1$ is given by

$$e_{j+1} = \tilde{r} - Jf_{j+1}. \tag{3}$$

Eliminating $\tilde{r}$ from (3) by using (2) yields the trial-to-trial dynamics

$$e_{j+1} = e_j + J\left(f_j - f_{j+1}\right), \tag{4}$$

which are optimized in norm-optimal ILC.

### 2.2. Norm-optimal ILC

Norm-optimal ILC is an important class of ILC in which the feedforward signal $f_{j+1}$ for the next trial is determined by minimizing a performance criterion as in Definition 1.