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# Overcoming a fundamental time-domain performance limitation by nonlinear control[☆]

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ABSTRACT

It is well-known that fundamental performance limitations exist when using linear feedback control for linear systems. In this note, we present an example of a nonlinear control strategy that can achieve a time-domain performance specification that cannot be obtained by any linear controller. In particular, we present a variable-gain control approach that meets an overshoot requirement that cannot be met by any linear controller.



## 1. Introduction

It is well-known that fundamental performance limitations exist when using linear time-invariant (LTI) feedback controllers for LTI single-input-single-output (SISO) plants (Freudenberg, Middleton, & Stefanopoulou, 2000; Middleton, 1991; Seron, Braslavsky, & Goodwin, 1997). These fundamental limitations may relate to fundamental frequency-domain limitations, such as the waterbed effect or Bode's gain–phase relationship, or time-domain limitations, such as restrictions on rise-time, overshoot and settling time of the closed-loop system.

In order to overcome these fundamental limitations, related to the usage of linear feedback controllers, or balance related performance trade-offs in a more desirable manner, the use of nonlinear control strategies has been studied extensively in the literature. Examples are the works on reset control strategies (Beker, Hollot, & Chait, 2001; Clegg, 1958; Nešić, Teel, & Zaccarian, 2011; Zhao, Nešić, Tan, & Wang, 2013; Zheng, Chait, Hollot, Steinbuch, & Norg, 2000) split-path nonlinear filters (Foster, Gieseking, & Waymeyer, 1966; van Loon, Hunnekens, Heemels, van de Wouw, & Nijmeijer, in press), switched controllers (Feuer, Goodwin, & Salgado, 1997; Lau & Middleton, 2003), or variable-gain controllers (Chen, Lee, Peng, & Venkataramanan, 2003; Heertjes & Leenknegt, 2010; Hunnekens, van de Wouw, Heertjes, & Nijmeijer, 2015; Lin, Pachter, & Ban, 1998; van de Wouw, Pastink, Heertjes, Pavlov, & Nijmeijer, 2008; Zheng, Guo, & Wang, 2005), which all aim at improving closed-loop performance compared to that obtained by linear feedback controllers.

All these works contain interesting performance-improving results, and the benefits of several control strategies have also been validated on industrial applications (Chen et al., 2003; Heertjes & Leenknegt, 2010; Hunnekens et al., 2015; van de Wouw et al., 2008; Zheng et al., 2000, 2005). However, to the best knowledge of the authors, there exists only one example of a nonlinear control strategy that explicitly shows that certain performance specifications can be met that cannot be obtained by any linear controller. This example involves reset control, for which in Beker et al. (2001) and Zhao et al. (2013) it has been shown that certain fundamental time-domain limitations can be overcome by resetting controller states.

In this note, we present a second example of a nonlinear control strategy that can achieve performance specifications not attainable by any linear controller. More specifically, we will study

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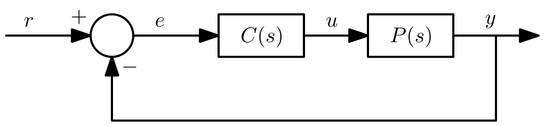


**Fig. 1.** Linear control scheme with linear plant $P(s)$ and controller $C(s)$.

a fundamental tradeoff for linear plants with a real unstable pole, which, given a certain rise-time specification, will exhibit a minimal amount of overshoot when controlled by any linear controller (Seron et al., 1997). Using a so-called phase-based variable-gain controller (Armstrong, Guitierrez, Wade, & Joseph, 2006; Xu, Hollerbach, & Ma, 1995), we show that this fundamental limitation can be overcome. In particular, we show that an overshoot specification can be attained that is not attainable by any linear feedback controller.

The remainder of this note is organized as follows. In Section 2, we briefly revisit a fundamental time-domain limitation for linear plants with an unstable real pole. In Section 3, we present the phase-based variable-gain control strategy and show, using a simulation example, that a time-domain specification can be met using this nonlinear control strategy that cannot be met by any linear feedback controller. Conclusions are presented in Section 4.

## 2. A fundamental time-domain limitation for linear systems

Consider the linear feedback configuration in Fig. 1, which consists of a linear time-invariant (LTI) single-input-single-output (SISO) plant $P(s)$, $s \in \mathbb{C}$, linear feedback controller $C(s)$, reference $r$, output $y$, tracking error $e := r - y$ and control action $u$. It is well-known that there exist fundamental performance limitations in the design of linear feedback controllers $C(s)$ for these linear SISO LTI plants $P(s)$, see e.g. Freudenberg et al. (2000), Middleton (1991) and Seron et al. (1997). The term *fundamental* relates to the fact that the performance limitations are independent of the design choices for the linear feedback controller $C(s)$.

In this note, we focus on a fundamental time-domain limitation for plants $P(s)$ which have an unstable pole at $s = p > 0$. If the closed-loop system in Fig. 1 is subject to a unit step-reference $r(t) = 1$, for $t \in \mathbb{R}_{\geq 0}$ ($r(t) = 0$, $t < 0$), a certain fundamental limitation exists between the rise-time and amount of overshoot of the closed-loop system. In order to make the latter statement mathematically more precise, consider the following definitions of rise-time and amount of overshoot.

**Definition 1** (*Seron et al., 1997*)**.** The rise-time of the closed-loop system is defined as:

$$t_r := \sup_{\delta}\left\{\delta : y(t) \leq \frac{t}{\delta} \text{ for all } t \in [0, \delta]\right\}. \quad (1)$$

**Definition 2** (*Seron et al., 1997*)**.** The overshoot $y_{os}$ of the closed-loop system is defined as the maximum value by which the output $y(t)$ exceeds the final set-point value $r = 1$:

$$y_{os} := \sup_{t\geq 0}(-e(t)). \quad (2)$$

A graphical interpretation of the definition of rise-time and overshoot is given in Fig. 2. In words, this means that the rise time $t_r$ is defined as the largest value for which the response $y(t)$ is still below the line $t/t_r$, for all $t \leq t_r$.

Now, a fundamental time-domain limitation can be formulated in the result below.

**Corollary 3** (*Seron et al., 1997*)**.** *Suppose that $P(s)$ in Fig. 1 has a real pole at $s = p > 0$ in the open right-half-plane. If the closed-loop system is stabilized by any linear time-invariant controller $C(s)$, then its step-response $y(t)$ must exhibit overshoot, and satisfy the following inequality:*

$$y_{os} \geq \frac{(pt_r - 1)e^{pt_r} + 1}{pt_r} \geq \frac{pt_r}{2}. \quad (3)$$

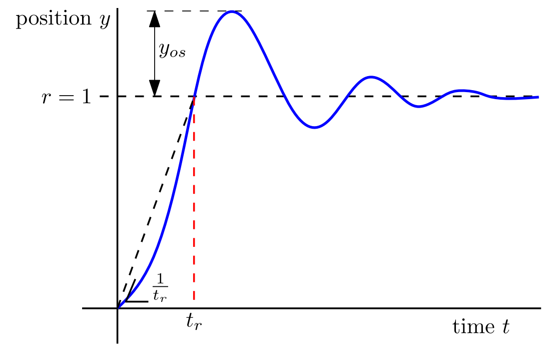


**Fig. 2.** Definition of rise-time $t_r$ according to (1) and overshoot $y_{os}$ according to (2).

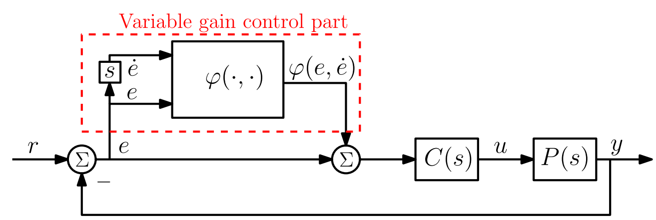


**Fig. 3.** Phase-based variable-gain control scheme with variable-gain element $\varphi(e, \dot{e})$.

**Proof.** The proof can be found in Seron et al. (1997).

Note that both the lower-bounds for the overshoot $y_{os}$ in (3) are monotonic in the rise time $t_r$. Therefore, Corollary 3 expresses the fact that if the closed-loop system is 'slow', i.e., it has a large rise time $t_r$, the step response will present a large amount of overshoot if there are open-loop unstable real poles. In practice, it is reasonable to assume that, a certain lower-bound for the rise-time of a closed-loop system with unstable real open-loop poles may exist, for example due to physical actuator constraints or bandwidth limitations in the system. This lower-bound for the rise-time results (via (3)) in an explicit lower bound on the amount of overshoot that the system will exhibit when using a linear feedback controller $C(s)$, no matter how the controller $C(s)$ is designed/tuned.

In Section 3, we present a type of nonlinear controller which can overcome this fundamental time-domain performance limitation.

## 3. A nonlinear controller overcoming a fundamental time-domain limitation

### 3.1. Phase-based variable-gain control

Consider the nonlinear control strategy as shown in Fig. 3, which represents a so-called variable-gain control (VGC) scheme. The term *variable-gain* controller is used since the controller configuration allows the use of a variable amount of controller gain through the function $\varphi(e, \dot{e})$. Here, we will focus on a *phase-based* variable-gain controller, which applies additional gain based on information on the error $e$ and time-derivative of the error $\dot{e}$, see e.g. Armstrong et al. (2006) and Xu et al. (1995), as opposed to *magnitude-based* variable-gain control, which modulates the gain based only on the magnitude of the error $e$, see Heertjes and Leenknegt (2010), Hunnekens et al. (2015) and van de Wouw et al. (2008).