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Adaptive filtering for the identification of bilinear forms

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ABSTRACT

Bilinear systems are involved in many interesting applications, especially related to the approximation of nonlinear systems. In this context, the bilinear term is usually defined in terms of an input–output relation (i.e., with respect to the data). Recently, a different approach has been introduced, by defining the bilinear term with respect to the impulse responses of a spatiotemporal model, which resembles a multiple-input/single-output (MISO) system. Also, in this framework, the Wiener filter has been studied to address the identification problem of these bilinear forms. Since the Wiener filter may not be always very efficient or convenient to use in practice, we propose in this paper an adaptive filtering approach. Consequently, we develop and analyze some basic algorithms tailored for the identification of bilinear forms, i.e., least-mean-square (LMS), normalized LMS (NLMS), and recursive-least-squares (RLS). Simulations performed in the context of system identification (based on the MISO system approach) support the theoretical findings and indicate the appealing performance of these algorithms.

1. Introduction

Bilinear systems have been found to be popular in a wide range of domains [1], being addressed in the literature in different ways and contexts. Most often, they are related to the approximation of nonlinear systems. It is known that a bilinear model can approximate a large class of nonlinear systems via a finite sum of the Volterra series expansion between the inputs and outputs of the system. Hence, a bilinear system can be considered among the simplest recursive nonlinear systems. On the other hand, bilinear systems behave similarly (to some extent) to linear models, which further simplify the analysis [2]. Therefore, they provide a good compromise between the accuracy of nonlinear systems and the tractability of linear systems.

Due to their practical features, the bilinear systems have been involved in many interesting applications, e.g., [3–17] and the references therein. Among these, we can mention system identification [4], [5], [14], digital filter synthesis [6], prediction problems [7], channel equalization [8], echo cancellation [9], chaotic communications [12], active noise control [13], [17], neural networks [16], etc. Many of these works were concerned with developing adaptive filtering algorithms for nonlinear systems modeled as bilinear systems. Nevertheless, in all these frameworks, the bilinear term is defined with respect to the data, i.e., in terms of an input–output relation.

In this work, we focus on a different approach by defining the bilinear term with respect to the impulse responses of a spatiotemporal model, in the context of multiple-input/single-output (MISO) systems. Recently, in [18], this problem has been addressed from a system identification perspective and two forms of the Wiener filter (namely direct and iterative) were developed in this context. However, the Wiener filter may not be always convenient to use in practice, due to some well-known limitations (e.g., matrix inversion, estimation of the statistics, etc.). Consequently, the next natural step is to analyze this framework in terms of an adaptive filtering approach, which represents the main motivation behind this paper. In this context, we focus on the most popular adaptive algorithms, i.e., least-mean-square (LMS), normalized LMS (NLMS), and recursive-least-squares (RLS).

Similar frameworks can be found in [19–24], in the context of particular applications, e.g., channel equalization [19], target detection [23], and nonlinear acoustic echo cancellation [20–22], [24]. However, most of these works were not associated or analyzed in conjunction with bilinear forms. Usually, they were referred as cascaded systems (e.g., similar to the Hammerstein model [25]) or joint adaptation processes, while the resulted algorithms were derived mostly in an ad-hoc manner, without analyzing their convergence features in a more general framework. The main goal of this paper is to perform a first step toward the development and analysis of adaptive filters tailored for the identification of bilinear forms, under the general framework of a MISO system identifica-

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tion problem. The overall approach can be interpreted (to some extent) as a multidimensional adaptive filtering technique.

The framework and the algorithms proposed in this paper could be used in the context of different applications related to the identification of such bilinear forms. For example, nonlinear acoustic echo cancellation represents an appealing choice, since the general configuration related to this application can be interpreted as a particular case of our proposed model; basically, in this context, the general scheme reduces to a Hammerstein model [20–22], [24]. Also, we can link our contribution to multichannel interference cancellation, e.g., in the context of adaptive noise cancellation (where the noise signal is picked up by a microphone array [26]) or multichannel dereverberation. In addition, the particular form of the MISO system could be exploited for the identification of block-sparse systems [27]. For the sake of generality, we do not focus in this paper on one particular application, but on the capabilities and features of the proposed algorithms in the general framework of system identification.

The rest of the paper is organized as follows. In Section 2, the proposed signal model with bilinear forms is introduced. Next, the LMS algorithm tailored for bilinear forms is presented in Section 3, together with its convergence analysis. Section 4 is dedicated to the NLMS algorithm for bilinear forms; in this context, a variable step-size version is also presented, aiming to achieve a proper compromise between the main performance criteria (i.e., convergence rate versus misadjustment). In Section 5, the RLS algorithm for bilinear forms is introduced, targeting a faster convergence rate as compared to its LMS-based counterparts. Simulation results are presented in Section 6, in the context of system identification (from a MISO system perspective). Finally, Section 7 concludes this paper and outlines some perspectives for future works.

2. Signal model with bilinear forms

In the proposed approach, the bilinear term is defined with respect to the impulse responses of a spatiotemporal model, in the context of MISO systems. Consequently, the signal model is

$$\begin{aligned} d(n) &= \mathbf{h}^T \mathbf{X}(n)\mathbf{g} + w(n) \\ &= y(n) + w(n), \end{aligned} \tag{1}$$

where $d(n)$ is the zero-mean desired (or reference) signal at the discrete-time index n, $\mathbf{h}$ and $\mathbf{g}$ are the two impulse responses of the system of lengths L and M, respectively, the superscript ^{T} is the transpose operator,

$$\mathbf{X}(n) = \begin{bmatrix} \mathbf{x}_1(n) & \mathbf{x}_2(n) & \cdots & \mathbf{x}_M(n) \end{bmatrix} \tag{2}$$

is the zero-mean multiple-input signal matrix of size $L \times M$,

$$\mathbf{x}_m(n) = \begin{bmatrix} x_m(n) & x_m(n-1) & \cdots & x_m(n-L+1) \end{bmatrix}^T \tag{3}$$

is a vector containing the L most recent samples of the mth ($m = 1, 2, \ldots, M$) input signal, $y(n) = \mathbf{h}^T\mathbf{X}(n)\mathbf{g}$ is the bilinear form, and $w(n)$ is the zero-mean additive noise. It is assumed that all the signals are real valued, and $\mathbf{X}(n)$ and $w(n)$ are uncorrelated. A simple block diagram of this model is illustrated in Fig. 1(a).

The two impulse responses, i.e., $\mathbf{h}$ and $\mathbf{g}$, correspond to the temporal and spatial parts of the system, respectively. It is easy to verify that for every fixed $\mathbf{h}$, $y(n)$ is a linear function of $\mathbf{g}$, and for every fixed $\mathbf{g}$, it is a linear function of $\mathbf{h}$. Therefore, $y(n)$ is bilinear in $\mathbf{h}$ and $\mathbf{g}$ [28].

It can be noticed that the reference signal from (1) can be expressed as

$$\begin{aligned} d(n) &= \sum_{m=1}^{M} g_m \mathbf{h}^T \mathbf{x}_m(n) + w(n) \\ &= \sum_{m=1}^{M} y_m(n) + w(n), \end{aligned} \tag{4}$$

which illustrates the processing line of each input signal, as shown in Fig. 1(b). On the other hand, based on the vectorization operation (i.e., conversion of a matrix into a vector [28]), the matrix $\mathbf{X}(n)$ of size $L \times M$ can be rewritten as a vector of length ML:

$$\begin{aligned} \mathrm{vec}\left[\mathbf{X}(n)\right] &= \begin{bmatrix} \mathbf{x}_1^T(n) & \mathbf{x}_2^T(n) & \cdots & \mathbf{x}_M^T(n) \end{bmatrix}^T \\ &= \widetilde{\mathbf{x}}(n). \end{aligned} \tag{5}$$

Consequently, the output signal $y(n)$ can be expressed as

$$\begin{aligned} y(n) &= \mathbf{h}^T \mathbf{X}(n)\mathbf{g} \\ &= \mathrm{tr}\left[\left(\mathbf{h}\mathbf{g}^T\right)^T \mathbf{X}(n)\right] \\ &= \mathrm{vec}^T\left(\mathbf{h}\mathbf{g}^T\right)\mathrm{vec}\left[\mathbf{X}(n)\right] \\ &= \left(\mathbf{g}\otimes\mathbf{h}\right)^T \widetilde{\mathbf{x}}(n) \\ &= \mathbf{f}^T \widetilde{\mathbf{x}}(n), \end{aligned} \tag{6}$$

where $\mathrm{tr}[\cdot]$ denotes the trace of a square matrix, $\otimes$ is the Kronecker product, and $\mathbf{f} = \mathbf{g}\otimes\mathbf{h}$ is the spatiotemporal impulse response (of length ML), which is simply the Kronecker product between the two individual impulse responses $\mathbf{g}$ and $\mathbf{h}$. Hence, the signal model in (1) results in

$$d(n) = \mathbf{f}^T \widetilde{\mathbf{x}}(n) + w(n), \tag{7}$$

which can be seen as a particular form of a MISO system, as depicted in Fig. 1(c). In the general case of a MISO system, $\mathbf{f}$ has ML different elements. On the other hand, in this bilinear context, $\mathbf{f} = \mathbf{g}\otimes\mathbf{h}$ is formed with $M + L$ different elements only even though it is of length ML.

A particular case of this system is the Hammerstein model [25], which is illustrated in Fig. 1(d). In this context, there is a single input signal, $x(n)$, which passes through a cascade of two systems, i.e., a nonlinear block and a linear system. In this case, the reference signal is similar to (1), but the mth ($m = 1, 2, \ldots, M$) column of the input signal matrix $\mathbf{X}(n)$ has a particular form, i.e., $\begin{bmatrix} x^m(n) & x^m(n-1) & \cdots & x^m(n-L+1) \end{bmatrix}^T$. Based on this model, the cascaded adaptive filters were developed in the context of different applications, like nonlinear acoustic echo cancellation [20–22], [24]. However, they were not associated with bilinear forms or analyzed in a more general framework, like we target in this paper.

Based on the equivalent model in (7), the variance of $d(n)$ is

$$\begin{aligned} \sigma_d^2 &= E\left[d^2(n)\right] \\ &= \left(\mathbf{g}\otimes\mathbf{h}\right)^T \mathbf{R}\left(\mathbf{g}\otimes\mathbf{h}\right) + \sigma_w^2, \end{aligned} \tag{8}$$

where $E[\cdot]$ denotes mathematical expectation, $\mathbf{R} = E\left[\widetilde{\mathbf{x}}(n)\widetilde{\mathbf{x}}^T(n)\right]$ is the covariance matrix of $\widetilde{\mathbf{x}}(n)$, and $\sigma_w^2 = E\left[w^2(n)\right]$ is the variance of $w(n)$. As a result, the signal-to-noise ratio (SNR) of the MISO system is

$$\mathrm{SNR} = \frac{\left(\mathbf{g}\otimes\mathbf{h}\right)^T \mathbf{R}\left(\mathbf{g}\otimes\mathbf{h}\right)}{\sigma_w^2}. \tag{9}$$

The covariance matrix $\mathbf{R}$ consists of M^2 submatrices of size $L \times L$, i.e.,