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On stability and application of extremum seeking control without steady-state oscillation[☆]

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ABSTRACT

To enhance the dynamic and static performance of the extremum seeking control (ESC) scheme, a novel fast ESC scheme without steady-state oscillation is proposed, in which the structure of the classic ESC scheme is adjusted to make the sinusoidal excitation signal amplitude locally exponentially converge to zero. The improved ESC scheme can speed up the convergence to shorten the seeking time dramatically, and enlarge the search area to avoid falling to local extrema effectively. It can also reduce the sinusoidal excitation signal amplitude to eliminate the adverse effects of the steady-state oscillation eventually. The rigorous stability analysis and proof of the improved ESC scheme are provided in detail, and the simulation results are presented to illustrate its effectiveness and superiority. Finally, the application of the improved ESC scheme to antilock braking systems (ABS) has been discussed through comparing with classical perturbation-based ESC scheme and sliding-mode-based ESC scheme to illustrate the practicability of the improved scheme.

1. Introduction

Extremum seeking is a kind of adaptive control which can drive and maintain the input and output of the controlled object to their respective extrema. Extremum seeking control will work without any explicit knowledge about the input–output characteristics as long as the extrema exist, which is its greatest advantage. Therefore, extremum seeking is a model independent control scheme. Though extremum seeking scheme has been developed for several decades, the first rigorous stability analysis of the classic extremum seeking scheme was published by Wang and Krstić (2000a,b). It appears that this paper renewed research interest in the theory of extremum seeking, and consequently the last decade has witnessed the significant development and numerous applications (Moase, Manzie, Nešić, & Mareels, 2010). Among them, Tan, Nešić, and Mareels (2008) studied how the different excitation signal in extremum seeking scheme affects the system performance. Lavretsky, Hovakimyan, and Calise (2003) pointed out that the amplitude of the excitation signal plays an important role in the system performance.

Extremum seeking scheme is essentially a method that achieves and maintains the function extremum by obtaining gradient information of the unknown function. Therefore, the extremum seeking scheme could easily converge to one of the local extrema if they exist. Committed to finding solutions to the problem, Tan, Nešić, Mareels, and Astolfi (2009) proposed a global extremum seeking scheme. A monotonically decreasing time function was designed to adjust the amplitude of the excitation signal, so that the searching arguments are expanded to get a certain ability to overcome the possible convergence to a local extremum. However, the performance of the scheme proposed by Tan et al. (2009) is not prominent because the excitation signal amplitude only varies with time. The same as the ordinary extremum seeking scheme, the improved scheme still has a large steady-state oscillation, which is undesirable, even not allowed for most practical systems.

In order to achieve the purpose of real-time optimality, the scheme is required to have a fast convergence rate. Krstić (1999) proposed a fast adaptive extremum seeking scheme to solve this problem. By introducing a dynamic compensator, the gain and phase margins of the feedback loop were improved to enhance the stability of the system and increase the response speed of the system. Compared with ordinary extremum seeking scheme it has better dynamic performance (Zuo, Hu, & Shi, 2006). However, the

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dynamic compensator design is dependent on the prior knowledge of the Wiener–Hammerstein model of the plant (Ariyur & Krstić, 2003), which brings inconvenience to its application.

As an application of extremum seeking control, the antilock braking systems with extremum seeking control have been investigated extensively (Dinçmen, Guvenc, & Acarman, 2014; Drakunov, Özgüner, Dix, & Ashrafi, 1995; Tunay, 2001; Yu & Özgüner, 2002; Zhang & Ordóñez, 2007). They all have reached the control purpose that the braking time is short and the tires are not locked during braking. However, most of them do bring additional oscillations in steady-state due to the perturbation signal and the sliding mode, respectively. Zhang and Ordóñez (2007) proposed an extremum seeking scheme based on numerical optimization through combining the numerical optimization algorithm with state regulation. The steady-state oscillation is successfully avoided when using the asymptotic state regulator numerical optimization based extremum seeking control. However, its real-time implementation will be affected when using a sophisticated optimization algorithm. Besides, the complex parameter adjustment of the scheme is not convenient for its application, too.

Motivated by the above study, we proposed a novel extremum seeking scheme in which the excitation signal amplitude can change adaptively with the extremum estimation error (Wang, Chen, & Zhao, 2014). The same as the scheme proposed by Tan et al. (2009), our scheme can avoid falling into local extrema effectively. The difference from the scheme proposed by Tan et al. (2009) is that our scheme can eliminate the steady-state oscillation due to the fact that the excitation signal amplitude will fast converge to zero with the decrease of the extremum estimation error. Furthermore it also has a strong adaptability to the extremum perturbation. In this paper, we improve the scheme further and give the rigorous stability proof of this improved scheme using Singular Perturbation Theory, Averaging Method, The Center Manifold Theorem and Lyapunov Method. We also give a simulation example to validate the characteristics of non-oscillating steady state of the improved scheme. Besides, the application of the improved scheme to antilock braking systems sufficiently illustrates the practicability of the scheme.

This paper is organized as follows. The problem formulation is given in Section 2. The main results are stated in Section 3. In Section 4 we discuss the improved scheme in detail. Section 5 consists of simulation example and application to ABS. Finally, a brief summary of the full text is given in Section 6. Some lemmas which are used in proof and auxiliary results are presented in the Appendix.

2. Problem formulation

Consider a general single input and single output (SISO) nonlinear model as follows:

$$\dot{x} = f(x, u), \qquad y = h(x), \tag{1}$$

where $f : R^n \times R \to R^n$ and $h : R^n \to R$ are continuously differentiable, x is the state, u is the input and y is the measurable output. Suppose there exists a family control laws of the following form:

$$u = \alpha(x, \theta), \tag{2}$$

where $\theta \in R$ is a scalar parameter. The closed-loop system

$$\dot{x} = f(x, \alpha(x, \theta)), \tag{3}$$

then has an equilibrium point parameterized by θ. For simplicity, θ is assumed to be scalar and (1), (2) is assumed to be SISO. Multi-dimensional parameter situations could be acquired by extending the results of the case of SISO. We make the following assumptions about the closed-loop system, which are the same as (Wang & Krstić, 2000a,b).

Assumption 1. There exists a smooth function $l : R \to R^n$, such that $f(x, \alpha(x, \theta)) = 0$, if and only if $x = l(\theta)$.

Assumption 2. For each $\theta \in R$, the equilibrium $x = l(\theta)$ of the system (3) is locally exponentially stable.

This assumption means that a control law can be designed for local stabilization, independent of the modeling knowledge of either $f(x, u)$ or $l(\theta)$.

Assumption 3. There exists $\theta^* \in R$ such that

$$\begin{aligned}&(h \circ l)'(\theta^*) = 0,\\&(h \circ l)''(\theta^*) < 0.\end{aligned} \tag{4}$$

Assumption 3 is made to ensure the function $y = h(l(\theta))$ has a maximum at $\theta = \theta^*$. Without loss of generality, the minimum case would be treated identically by replacing y by $-y$. Let $g(\theta) = (h \circ l)(\theta)$ represent a cost function of the maximum seeking problem.

Remark 1. In this paper we only investigate local stability of the extremum seeking control scheme without steady-state oscillation (ESCWSSO), which can be seen from the above assumptions. The global study will be a topic of future research in our subsequent work.

3. Main results

In this section we will investigate stability of a novel ESCWSSO. This novel scheme is an improvement of the perturbation-based extremum seeking scheme. The analysis of the proposed scheme lends itself to an understanding of this improvement. The proposed ESCWSSO is shown in Fig. 1.

The scheme shown in Fig. 1 introduces two new design parameters ω_l, r instead of the excitation signal amplitude in the classic extremum seeking scheme. ω_l is the cutoff frequency of the low-pass filter and r is a constant gain for adjusting the speed of convergence of the scheme. Note that a design constraint on r is that the signal a should be positive. For details, see the Discussions (Section 4). In addition, m is the low-frequency component in y.

The closed-loop system shown in Fig. 1 can be written as

$$\begin{aligned}&\dot{x} = f(x, \alpha(x, \hat{\theta} + a\sin\omega t)),\\&\dot{\hat{\theta}} = k(y - m)\sin\omega t,\\&\dot{a} = -\omega_l a + r\omega_l(y - m),\\&\dot{m} = -\omega_h m + \omega_h y.\end{aligned} \tag{5}$$

Introduce the new following coordinates

$$\begin{aligned}&\tilde{\theta} = \hat{\theta} - \theta^*,\\&\tilde{m} = m - h \circ l(\theta^*).\end{aligned} \tag{6}$$

and we get

$$\begin{aligned}&\dot{x} = f(x, \alpha(x, \theta^* + \tilde{\theta} + a\sin\omega t)),\\&\dot{\tilde{\theta}} = k(h(x) - h \circ l(\theta^*) - \tilde{m})\sin\omega t,\\&\dot{a} = -\omega_l a + r\omega_l(h(x) - h \circ l(\theta^*) - \tilde{m}),\\&\dot{\tilde{m}} = -\omega_h\tilde{m} + \omega_h(h(x) - h \circ l(\theta^*)).\end{aligned} \tag{7}$$

For further analysis of Eq. (7), the design parameters are selected as

$$\begin{aligned}&\omega_h = \omega\omega_H = \omega\delta\omega'_H = O(\omega\delta),\\&\omega_l = \omega\omega_L = \omega\varepsilon\delta\omega'_L = O(\omega\varepsilon\delta),\\&k = \omega K = \omega\delta K' = O(\omega\delta).\end{aligned} \tag{8}$$