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Geometric target detection based on total Bregman divergence

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ABSTRACT

This paper develops a geometric detection approach based upon the total Bregman divergence on the Riemannian manifold of Hermitian Positive-Definite (HPD) matrices to realize target detection in a clutter. First of all, the radar received clutter data in each range cell in one coherent processing interval is modeled and mapped into an HPD matrix space, which can be described as a complex Riemannian manifold. Each point of this manifold is an HPD matrix. Then, a class of total Bregman divergences are presented to measure the closeness between HPD matrices. Based on these divergences, the medians for a finite collection of HPD matrices are derived. Furthermore, the three divergences, namely the total square loss, the total log-determinant divergence, and the total von Neumann divergence are deduced, and their corresponding geometric detection methods are designed. The principle of detection is that if a location has enough dissimilarity from the median estimated by its neighboring locations, targets are supposed to appear at this location. Numerical experiments and real clutter data are given to demonstrate the effectiveness of the proposed geometric detection methods.

1. Introduction

There is a growing need for effective detection of the target embedded in the presence of clutter, which is very meaningful for modern radars to improve their detection performances [1,2]. In particular, it is difficult to enhance the detection signal-to-clutter/noise ratio (SCR/SNR), when few pulses are available. In these situations, it seems to be important and challenge to achieve a satisfactory performance.

The classical fast Fourier transform (FFT) based constant false alarm rate (CFAR) detection algorithms [3] suffer from severe performance degradation with few pulses available owing to the poor Doppler resolution as well as the energy spread of the Doppler filter banks. A strategy to circumvent these drawbacks was proposed by Barbaresco [4–6]. As illustrated in Fig. 1, the data $\boldsymbol{R}_i$ in each range cell is a Hermitian positive-definite (HPD) matrix estimated by the sample data $\mathbf{z}$ according to its correlation coefficient. Then, calculate the distance between the covariance matrix $\boldsymbol{R}_D$ of the cell under test and the mean matrix $\bar{\boldsymbol{R}}$ of reference cells around the cell under test. Finally, the detection is made by comparing the distance between $\boldsymbol{R}_D$ and $\bar{\boldsymbol{R}}$ with a given threshold γ. In his work, a Riemannian geometry detection approach was devised on the complex Riemannian manifold of HPD matrices. The radar echo is modeled using an HPD matrix, and the Riemannian metric is exploited to measure the dissimilarity between two HPD matrices. This method has been used for the monitoring of wake vortex turbulences [7–9], and target detection in coastal X-band and HF surface wave radars. Real sea clutter experiments are given to prove that the Riemannian distance-based geometric detection approach has better detection performance than the classical FFT-CFAR detection algorithm [5]. Based on Barbaresco's work, Balaji utilized the Riemannian mean to estimate the covariance matrix in space-time adaptive processing. It has been found that the projection algorithm with the Riemannian mean can yield significant performance gains [10,11].

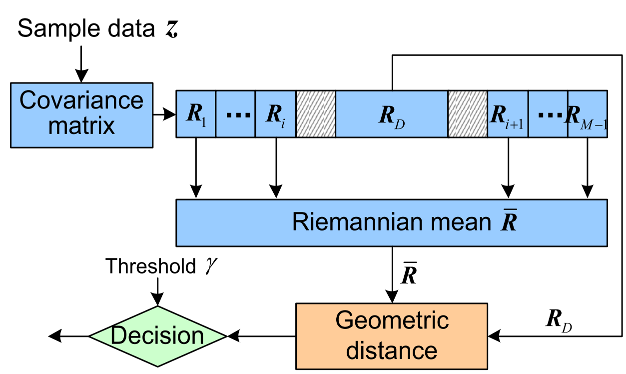

Fig. 1. Riemannian distance-based geometric detector [5].

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The geometric detection method has similar scheme with the FFT-CFAR processing. The differences between them lie in three ways: (1) the observation data in each cell is an HPD matrix, and not the FFT value of sample data; (2) the distance measures used in the geometric detector is the Riemannian distance or divergence measure, and not the Euclidean distance; and (3) the averaging process in geometric detector is geometric mean of a set of HPD matrices, not the arithmetic mean of scalar number. These differences imply that the geometric detector performs on the HPD matrix space, in other words, the different geometry considered in detection. Our previous work [12] had provided a further proof of the superiority of the geometric detection method compared to the FFT-CFAR. In particular, an alternative measure, the Kullback–Leibler divergence, was used as the distance between HPD matrices. Moreover, the detection performance of the Kullback–Leibler divergence-based geometric approach is better than that of the Riemannian distance-based geometric approach [12]. It brings out a viewpoint that different measure used in this geometric detector results in different performance.

In addition to the Riemannian metric, a lot of divergences structure can be used as the measurements on the Riemannian manifold of HPD matrices. There are many widely used divergences. The square loss function has been applied to regression analysis; the Kullback–Leibler divergence [13] has been used for measuring the dissimilarity between two probability density functions; and the Bhattacharyya divergence is exploited for Diffusion Tensor Magnetic Resonance Image (DT-MRI) segmentation [14,15]. Recently, in [16], the authors have defined a class of divergence, namely the total Bregman divergence, which has many perfect property. Based on this divergence, the l_1-norm t-center is derived. The total Bregman divergence has been applied to DT-MRI analysis [16] and shape retrieval [17].

In this paper, we extend the definition of total Bregman divergence to the HPD matrix. The three divergences, namely the total square loss, the total log-determinant divergence, and the total von Neumann divergence, are defined to measure the dissimilarity between two HPD matrices. According to these divergences, the medians for a finite set of HPD matrices are derived. Furthermore, we employ these divergences and their corresponding medians to devise the geometric detector. Experimental results show that these divergences-based geometric detection methods have better performance than the Riemannian distance-based geometric method as well as the FFT-CFAR algorithm.

The rest of this paper is organized as follows: Section 2 gives a concise description about how to construct HPD covariance matrices from the original radar observation data; the extended definition of the total Bregman divergence for HPD matrix is presented in Section 3; the total Bregman divergence-based medians, in particular, the total square loss median, total log-determinant divergence median, and total von Neumann divergence median, are derived in Section 4; results obtained from simulated data and real clutter data are presented in Section 5; Section 6 concludes our work.

1.1. Notation

A lot of notations are adopted as follows. We use math italic for scalars x, uppercase bold for matrices $\boldsymbol{A}$, and lowercase bold for vectors $\boldsymbol{x}$. The conjugate transpose operator is denoted by the symbol $(\cdot)^H$. $tr(\cdot)$ and $\det(\cdot)$ are the trace and the determinant of the square matrix argument, respectively. $\boldsymbol{I}$ denotes the identity matrix, and $\mathbb{C}(n)$, $\mathbb{H}(n)$ are the sets of n-dimensional vectors of complex numbers and of $n \times n$ Hermitian matrices, respectively. The Frobenius norm of the matrix $\boldsymbol{A}$ is denoted by $\|\boldsymbol{A}\|_F$. For any $\boldsymbol{A} \in \mathbb{H}(n)$, $\boldsymbol{A} > 0$ means that $\boldsymbol{A}$ is an HPD matrix, and denoted by $\mathbb{P}(n)$. Finally, $\mathbb{E}(\cdot)$ denotes the statistical expectation.

2. Signal modeled using HPD matrix

For the radar received complex clutter data $\boldsymbol{z} = \{z_1, z_2, \dots, z_n\}$ in each cell in one coherent processing interval (CPI), where n is the length of data, assuming $\boldsymbol{z}$ is a complex circular multivariate Gaussian distribution, $\boldsymbol{z} \sim CN(\boldsymbol{0}, \boldsymbol{R})$, with zero mean and covariance matrix $\boldsymbol{R}$ [5],

$$\boldsymbol{p}(\boldsymbol{z}|\boldsymbol{R}) = \frac{1}{\pi^n \det(\boldsymbol{R})} \exp\left\{-\boldsymbol{z}^H \boldsymbol{R}^{-1} \boldsymbol{z}\right\} \tag{1}$$

with the covariance matrix $\boldsymbol{R}$ given by [5],

$$\boldsymbol{R} = \mathbb{E}\left[\boldsymbol{z}\boldsymbol{z}^H\right] = \begin{bmatrix} r_0 & \bar{r}_1 & \cdots & \bar{r}_{n-1} \\ r_1 & r_0 & \cdots & \bar{r}_{n-2} \\ \vdots & \ddots & \ddots & \vdots \\ r_{n-1} & \cdots & r_1 & r_0 \end{bmatrix},$$
$$r_k = \mathbb{E}[z_i \bar{z}_{i+k}], \quad 0 \le k \le n-1, \ 1 \le i \le n \tag{2}$$

where $r_k = \mathbb{E}[z_n \bar{z}_{n+k}]$ is called the correlation coefficient and $\bar{z}$ denotes the complex conjugate of z. $\boldsymbol{R}$ is a Toeplitz HPD matrix with $\boldsymbol{R}^H = \boldsymbol{R}$. It is well known that the stationary Gaussian processes have both ergodicity and strict stationarity. According to the ergodicity, the correlation coefficient r_k of data $\boldsymbol{z}$ can be calculated by averaging over time instead of its statistical expectation, as

$$\hat{r}_k = \frac{1}{n} \sum_{n=0}^{n-1-|k|} z(n)\bar{z}(n+k), \quad |k| \le n-1 \tag{3}$$

The pulses data in each cell in one CPI are modeled by equations (1) and (2), and the information of target or clutter can be represented by its covariance matrix. Through parameterization using an HPD matrix, the radar echo $\boldsymbol{z} = \{z_1, z_2, \dots, z_n\}$ can be mapped into an n dimensional parameter space.

$$\boldsymbol{\psi} : \mathbb{C}(n) \to \mathbb{P}(n), \quad \boldsymbol{z} \to \boldsymbol{R} \in \mathbb{P}(n) \tag{4}$$

Here $\mathbb{P}(n)$ forms a differentiable Riemannian manifold [18,19] with nonpositive curvature [20]. HPD matrix manifold is a closed, self-dual convex cone, and served as a canonical higher-rank symmetric space [21]. An excellent overview for HPD manifold is provided in [22].

3. Total Bregman divergence on the Riemannian manifold of HPD matrices

In this section, the geometry of space of HPD matrices is described first; and then we extend the definition of total Bregman divergence to the HPD matrix.

3.1. Geometry of the space of HPD matrices

Let $\mathbb{H}(n) = \{\boldsymbol{A}, \boldsymbol{A}^H = \boldsymbol{A}\}$ denotes the space of all $n \times n$ Hermitian matrices. For $\boldsymbol{A} \in \mathbb{H}(n)$, $\boldsymbol{A} > 0$ if the quadratic form $\boldsymbol{x}^H \boldsymbol{A}\boldsymbol{x} > 0$, $\forall \boldsymbol{x} \in \mathbb{C}^n$. The subset of $\mathbb{H}(n)$ consisting of all positive-definite matrices is a convex symmetric cone, which is denoted by [23]

$$\mathbb{P}(n) = \left\{\boldsymbol{A} \in \mathbb{H}(n), \ \boldsymbol{A} > 0\right\} \tag{5}$$

The exponential of any Hermitian matrix is a positive-definite Hermitian matrix, and the principal logarithm of any positive-definite Hermitian matrix is a Hermitian matrix [23].

It can be noted that $\mathbb{P}(n)$ is a differentiable manifold of dimension $n(n+1)/2$ whose tangent space $\mathbb{T}_{\boldsymbol{A}}$ at any of its points $\boldsymbol{A}$ is identified with $\mathbb{H}(n)$. The infinitesimal arclength

$$ds := \left(tr\left(\boldsymbol{A}^{-1} d\boldsymbol{A}\right)^2\right)^{1/2} = \left\|\boldsymbol{A}^{-1/2} d\boldsymbol{A} \boldsymbol{A}^{-1/2}\right\|_F \tag{6}$$