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Unified coprime array with multi-period subarrays for direction-of-arrival estimation ☆

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ABSTRACT

The original coprime array is a sparse geometry with a pair of one-period subarrays, where the interelement spacing and number of sensors are coprime so that no elements overlap except at the zeroth position. In this paper, we unify the coprime array by extending two subarrays from one period to multiple periods. Each subarray can have an arbitrary number of periods and therefore more than one element of the unified structure coincide. The resulting configuration treats the nested array as well as the existing coprime structures, such as the original coprime array, the extended one and the coprime array with compressed inter-element spacing (CACIS) as its special cases. The new coprime array has a lower peak side-lobe level than the nested array and the CACIS. An improved structure based on the multi-period coprime array is further proposed to improve the degrees-of-freedom (DOF) of the unified geometry with one of the subarrays displaced by a certain number of spacings. Although the improved configuration does not reach the same DOF as the CACIS and the coprime array with displaced subarrays (CADiS), it can acquire a low peak side-lobe level with the DOF not being decreased too much. Different from CADiS, we focus on the analysis and proper choice of the displacement to acquire more consecutive lags. The properties of coarray of the proposed array design strategy are provided as well. Finally, the direction-of-arrival estimation, physical aperture and peak side-lobe level performance of the unified structures are examined through numerical simulations.

1. Introduction

Sparse array is a kind of non-uniform linear structure proposed first in 1950's for degrees-of-freedom (DOF) enhancement [1]. By exploiting the vectorized covariance matrix and considering it as a virtual array with larger aperture than the traditional uniform linear array (ULA), the spatial sampling techniques are able to detect more sources than the number of sensors, and therefore widely used in array signal applications [2–4]. A typical sparse structure that provides the largest possible virtual array is the minimum redundancy array (MRA) [5]. It has been shown that, by constructing an augmented covariance matrix [6], MRA is the optimum geometry achievable for a given number of sensors as each spacing presents only once. However, the augmented covariance matrix is not positive definite. The MRA is proved to exist only when the number of sensors is less than five. Besides, there are no analytical expressions for the element locations and the virtual array [7], leading to much difficulties in further array optimization and analysis.

To overcome this shortcoming, the nested array (NA) [8] and the coprime array [9], which are easy to express and attractive for a higher number of DOFs, have been proposed recently. By recovering the information in the covariance matrix instead of the impinging signal, the two sparse arrays successfully increase the number of DOFs from $O(N)$ and $O(M+N)$ to $O(N^2)$ and $O(MN)$. More sources can therefore be detected by performing the MUSIC based direction-of-arrival (DOA) estimation [3] on the consecutive lags and the sparse recovery algorithm [10,11] on the unique lags of the difference coarray. The coprime theory has been experimentally validated in an acoustic implementation [12,13]. It verified the DOF advantage and analyzed the beampatterns of the coprime array experimentally for the first time. To further increase the number of DOFs, some more efficient geometries are developed. The nested MRA [14], composed of multiple identical MRA, is capable of providing $O(M^2N^2)$ DOF using only MN physical sensors. The extended nested [15] and coprime array [16] utilize the fourth-order coarray concept to increase the number of consecutive lags and generate a larger virtual ULA. A generalized coprime

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array [17] designated as the coprime array with compressed interelement spacing (CACIS) can unify the NA and the coprime array by introducing an integer factor that compresses the interelement spacing of one subarray. The number of DOFs can be dramatically increased if the factor gets large. In [18], another improved configuration named the coprime array with displaced subarrays (CADiS) was provided to acquire more unique lags. Such a coprime array replaces the two subarrays co-linearly, thereby utilizing the negative lags more efficiently. Furthermore, some algorithms are proved effective in DOF enhancement by combining multi-frequency operation [19,20]. Based on the coprime array concept, many applications are developed to avoid spatial aliasing and obtain better performance [21–23].

Although the geometries mentioned above provide a higher number of DOFs, there are still some limitations. NA has dense sensors, leading to more severe mutual coupling effect than the coprime array. For CACIS, some characteristics of coarray, such as the position of holes, are not completely or precisely provided. CADiS cannot obtain a higher number of continuous lags, which are useful for the MUSIC based DOA estimation. For solving the problem of NA, super nested array [24,25] was proposed to reduce the mutual coupling. The analysis and compensation algorithms were investigated in [26] for non-uniform arrays DOA estimation. However, the above researches mainly focus on the DOF improvement and mutual coupling reduction. The peak side-lobe level (PSL), which is important for false peak detection and noise rejection, needs to be considered as well in the array design. Adhikari et al. [27] proposed an extended coprime array to achieve the same low PSL of the coarray beam pattern as the ULA. However, the new array is based on the coprime array with a coprime pair of M and $N = M + 1$. The total number of sensors, given by $2cN - 1 - \lceil c \rceil$ with c as the extension factor, is usually so large that it cannot be applied in practice.

To improve the PSL and provide a complete analysis on the well-known sparse arrays under a unified framework, in this paper, we construct a new coprime array with each subarray extended and then analyze the characteristics of the difference coarray. If the original coprime array is the union of two one-period subarrays, the proposed structure can be viewed as a sparse array with K and L periods in each subarray. We refer to it as coprime array with multi-period subarrays (CAMpS). As a result, it treats all the existing coprime arrays mentioned above and the NA as its special cases when the number of periods and the interelement spacing of each subarray are set as special values. Then, based on the unified geometry, we analyze and compare the DOF, physical aperture (PA) and PSL performances of the NA, CACIS and CAMpS completely. The theoretical analysis reveals that the proposed multi-period coprime arrays can acquire a lower PSL at the cost of DOF reduction. It is also pointed out that the cost of the NA and CACIS's high DOF is the increase of PSL. To remedy the disadvantage of the CAMpS, at last, we propose an improved geometry with one subarray displaced by a certain number of spacings. It is referred to as coprime array with displaced multi-period subarrays (CADMpS) in this paper. In this case, the negative lags in the cross difference set of CAMpS can move into the positive range. As a result, the proposed structure can acquire more consecutive lags and therefore detect more sources than the CAMpS. The performance of DOA estimation and beam patterns of the geometries under the unified framework are evaluated and examined by simulations.

The rest of the paper is organized as follows. The coprime array and the MUSIC based DOA estimation approach are reviewed in Section 2. The unified coprime configuration with multi-period subarrays, i.e., CAMpS, is described and analyzed in Section 3. The analytical expressions of the consecutive coarray lags, coarray aperture and hole positions are provided as well in this section. Three parameters characterizing the sparse array are compared in Section 4. The advantage and disadvantage of the existing sparse arrays and the multi-period coprime array are analyzed and clarified. An improved structure with one of the subarrays displaced a certain number of spacings, i.e., CADMpS, and its difference coarray structure are provided in Section 5. The performances of the unified configurations and the improved structure are demonstrated and compared through numerical examples in Section 6. Section 7 concludes the paper.

2. Coprime array and DOA estimation

As shown in Fig. 1, a coprime array is a sparse array comprising two ULAs with the first sensors collocated [9]. One has the interelement spacing of Md and the number of sensors of N, where M and N are coprime and $d = \lambda/2$ is the unit spacing with λ as the wavelength. The other has the interelement spacing of Nd and the number of sensors of M. Here, we assume $M < N$. As such, the union of the two ULAs has sensors located at

$$S = \{mNd, 0 \le m \le M-1\} \cup \{nMd, 0 \le n \le N-1\}. \quad (1)$$

Assume that Q narrowband uncorrelated signals imping on the array from directions $\{\theta_1, \theta_2, \ldots, \theta_Q\}$. Then the received signal is

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad (2)$$

where $\mathbf{s}(t) = [s_1(t), \ldots, s_Q(t)]^T$ is the signal vector with the power of the qth source as σ_q^2, $q = 1, \ldots, Q$, $\mathbf{n}(t)$ is a zero-mean white Gaussian noise vector with covariance matrix of $\sigma_n^2\mathbf{I}$ and $\mathbf{A} = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_Q)]$ is the array manifold matrix whose elements are defined by $\mathbf{a}(\theta_q) = [1, e^{j2\pi x_2 d \sin\theta_q/\lambda}, \ldots, e^{j2\pi x_{N+M-1} d \sin\theta_q/\lambda}]^T$ $(1 \le q \le Q)$, with $x_i d \in S, 1 \le i \le M+N-1$ as the position of the ith sensor. $(\cdot)^T$ denotes the transpose of a matrix or a vector. $\mathbf{I}$ is the $(M+N-1) \times (M+N-1)$ identity matrix.

If we assume that the noise is uncorrelated with the sources, then the correlation matrix of the data can be expressed as

$$\mathbf{R}_{xx} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \sum_{q=1}^{Q} \sigma_q^2 \mathbf{a}(\theta_q)\mathbf{a}^H(\theta_q) + \sigma_n^2\mathbf{I}, \quad (3)$$

where $E[\cdot]$ is the expectation operator and $(\cdot)^H$ is the conjugate transpose of a matrix or a vector.

Traditional DOA estimation techniques can provide satisfactory results for ULA by making use of $\mathbf{R}_{xx}$ obtained from a group of snapshots of $\mathbf{x}(t)$. For sparse array, it does not work because the uniform spatial sparse sampling is likely to introduce spatial aliasing. To overcome this problem, it is shown in [8] that vectorizing $\mathbf{R}_{xx}$ is an effective way. The result

$$\mathbf{z} = \mathrm{vec}(\mathbf{R}_{xx}) = \mathbf{B}\mathbf{p} + \sigma_n^2\tilde{\mathbf{1}}_n, \quad (4)$$

where $\mathbf{B} = [\mathbf{a}^*(\theta_1) \otimes \mathbf{a}(\theta_1), \ldots, \mathbf{a}^*(\theta_Q) \otimes \mathbf{a}(\theta_Q)]$, $\mathbf{p} = [\sigma_1^2, \ldots, \sigma_Q^2]^T$ and $\tilde{\mathbf{1}}_n = \mathrm{vec}(\mathbf{I})$ with $\mathrm{vec}(\cdot)$ as the vectorization operator stacking all columns of a matrix into a vector, can be viewed as the received data from a virtual array whose sensors are located at the difference positions of the original sparse array. $\mathbf{a}^*(\theta)$ is the

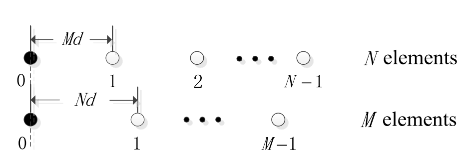

Fig. 1. The original coprime array with one-period subarrays.