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Boundary observer design for hyperbolic PDE–ODE cascade systems ☆

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A B S T R A C T

The paper presents an observer design for a class of hyperbolic PDE–ODE cascade systems with a boundary measurement. The cascade systems consist of coupled PDEs, featuring one rightward and one leftward convecting first-order transport PDEs, and a set of ODEs, which enter the PDEs through the left boundary of the systems. The design, which is based on the Volterra integral transformation, relies only on a single sensor at the right boundary of the system. The observer consists of a copy of the plant plus output injection terms both in the PDEs and the ODEs. The observer is constructed in a collocated setup, which means both sensing and actuation are located at the same boundary. The observer gains are computed analytically by solving Goursat-type PDEs in terms of Bessel function of the first kind. The observer design is tested against a field scale flow-loop test experiment in Stavanger by Statoil Oil Company. The results show that the observer converges to the actual values and that the design can be used as a process monitoring tool in oil well drilling.

1. Introduction

1.1. Problem statement

We consider a boundary observer design for a class of semilinear hyperbolic PDE–ODE cascade systems which can be transformed into the following form:

$$\mathbf{w}_t(x,t) = \mathbf{\Sigma}(x)\mathbf{w}_x(x,t) + \boldsymbol{\Omega}(x)\mathbf{w}(x,t) + \mathbf{f}(\mathbf{w}(x,t),x) \quad (1)$$

$$w_1(0,t) = qw_2(0,t) + \mathbf{CX}(t) \quad (2)$$

$$w_2(1,t) = U(t) \quad (3)$$

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) \quad (4)$$

where $\mathbf{w} = [w_1 \; w_2]^\intercal$ and $\mathbf{w} : [0,1] \times [0,\infty) \to \mathbb{R}^2$.

The matrices $\mathbf{\Sigma}(x)$ and $\boldsymbol{\Omega}(x)$ are given by:

$$\mathbf{\Sigma}(x) = \begin{pmatrix} -\epsilon_1(x) & 0 \\ 0 & \epsilon_2(x) \end{pmatrix}, \qquad \boldsymbol{\Omega}(x) = \begin{pmatrix} 0 & \omega_1(x) \\ \omega_2(x) & 0 \end{pmatrix} \quad (5)$$

where $\epsilon_1(x), \; \epsilon_2(x) > 0$. The subscripts x and t denote partial derivatives with respect to x and t, respectively. The constant $q \neq 0$ and $U(t)$ is the control input. $\mathbf{X}(t)$ is an n-dimensional vector, $\mathbf{A}$ is an $n \times n$ matrix and $\mathbf{C}$ is an $1 \times n$ matrix. The function $\mathbf{f} : \mathbb{R}^2 \times [0,1] \to \mathbb{R}^2$ constitutes nonlinear terms. The objective of this paper is to design an observer for the cascade system (1)–(4) with only one boundary measurement at $x = 1$, i.e.,

$$y(t) = w_1(1,t). \quad (6)$$

This state observer problem was solved for the linear case without disturbance ($\mathbf{f} = 0, \mathbf{A} = 0$, and $\mathbf{C} = 0$) in Vazquez, Coron, and Krstic (2011) and for the linear case with disturbance in Aamo (2013). In Coron, Vazquez, Krstic, and Bastin (2013), the state feedback stabilization problem was solved for the quasilinear systems (without disturbance). The controllability of the quasilinear systems with nonlinear source has been studied in, e.g., Wang (2006).

The following assumptions are used in this paper:

Assumption 1. The first derivatives of the entries in $\mathbf{\Sigma}$ are continuously differentiable, i.e., $\epsilon_1, \epsilon_2 \in \mathbb{C}^1([0,1])$, while the entries in $\boldsymbol{\Omega}$ are continuous, i.e., $\omega_1, \omega_2 \in \mathbb{C}([0,1])$.

Assumption 2. The function $\mathbf{f}$ is twice continuously differentiable with respect to $\mathbf{w}$. Furthermore, $\mathbf{f}(0,x) = 0$ and $\frac{d\mathbf{f}}{dx}(0,x) = 0$.

Assumption 3. The pair $(\mathbf{A}, \mathbf{C})$ is observable.

Assumption 4. The control law U is continuous, i.e., $U \in \mathbb{C}([0,\infty))$.

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1.2. Motivation and previous works

Physical systems which can be modeled and transformed into the first-order hyperbolic PDE–ODE cascade systems (1)–(4) have attracted considerable attention in research communities because these systems can be used to model various processes such as road traffic (Goatin, 2006), gas flow pipeline (Gugat & Dick, 2011), and flow of fluids in transmission lines (Hasan & Imsland, 2014; White, 2007) and in open channels (Coron, Andrea-Novel, & Bastin, 2007). A typical problem is to estimate the states and the parameters of the systems using a limited number of measurements. In many cases, the only reliable measurement is located at the boundary. These estimated states and parameters are in turn used in a feedback control algorithm that automates the control input to maintain a desired state trajectory. Observer design for PDE–ODE cascade systems has been studied for many types of coupling such as an ODE and a diffusion PDE (Krstic, 2009a; Susto & Krstic, 2010; Tang & Xie, 2011), an ODE and a hyperbolic PDE (Bekiaris-Liberis & Krstic, 2011; Hasan, Krstic, & Aamo, 2015; Krstic & Smyshlyaev, 2008a), and an ODE and a wave PDE (Bekiaris-Liberis & Krstic, 2010; Krstic, 2009b).

The results in this paper employ the backstepping method, and in particular build on the results of Aamo (2013); Coron et al. (2013); Vazquez et al. (2011). The backstepping method has been successfully used as control and state estimation designs for many PDEs such as the parabolic-type equation (Jadachowski, Meurer, & Kugi, 2014; Meurer, 2013), the Ginzburg–Landau equation (Aamo, Smyshlyaev, & Krstic, 2005), and the Schrödinger equation (Krstic, Guo, & Smyshlyaev, 2011). The idea is to use a Volterra integral transformation to transform the original system into a target system (Krstic & Smyshlyaev, 2008b). The stability of the target system is usually known beforehand. For some cases, the gains for both the controller and the observer, can be computed analytically in terms of the Bessel function (Smyshlyaev & Krstic, 2005) or the Marcum Q-function (Vazquez & Krstic, 2014).

The applicability of the results obtained in the present paper are demonstrated on a problem from the oil and gas industry in Section 4. Backstepping has found several applications in oil and gas, including the gas coning problem Hasan, Foss, and Sagatun (2013); Hasan, Sagatun, and Foss (2010), flow in porous media (Hasan, Foss, & Sagatun, 2012), slugging control (Di Meglio, Vazquez, Krstic, & Petit, 2012), the lost circulation and kick problem (Hasan, 2014a, 2015; Hauge, Aamo, & Godhavn, 2013), and the heave problem (Anfinsen & Aamo, 2015; Hasan, 2014b).

1.3. Contribution of this paper

The contribution of this paper is an observer design for a class of hyperbolic PDE–ODE cascade systems with a boundary measurement. We employ a composition of two transformations, one Volterra-based backstepping transformation of the PDE observer state, and one transformation of the transformed PDE observer state with a spatially scaled shift based on the ODE observer state. The observer consists of the plant plus output injection terms, where the gains are found explicitly in terms of Bessel functions of the first kind. The stability of the target system is studied using a Lyapunov functional. Two cases are considered, linear ($\mathbf{f} = 0$) and semilinear ($\mathbf{f} \neq 0$). In the linear case we show that the observer error system is globally exponentially stable in the $\mathbb{L}^2$-norm, while in the semilinear case we show the observer error system is locally exponentially stable in the $\mathbb{H}^2$-norm. The observer design is tested against a field scale flow-loop test experiment in Stavanger by Statoil Oil Company.

1.4. Organization of the paper

The paper is organized as follows. Section 2 contains preliminary definitions and notations used throughout the paper. The observer designs for both linear and semilinear cases are presented in Section 3. In Section 4, a real case application of oil well drilling where we estimate the flow, the pressure, and the downhole rate under lost circulation is presented. Finally, Section 5 contains conclusions and recommendations.

2. Preliminary definitions

For a vector $\boldsymbol{\gamma}(x) \in \mathbb{R}^2$ with components $\gamma_1(x)$ and $\gamma_2(x)$, we denote $|\boldsymbol{\gamma}(x)| = |\gamma_1(x)| + |\gamma_2(x)|$, and we define $\|\boldsymbol{\gamma}\|_\infty = \sup_{x\in[0,1]} |\boldsymbol{\gamma}(x)|$, $\|\boldsymbol{\gamma}\|_{\mathbb{L}^1} = \int_0^1 |\boldsymbol{\gamma}(\xi)|\,d\xi$, and $\|\boldsymbol{\gamma}\|_{\mathbb{L}^2} = \left(\int_0^1 \boldsymbol{\gamma}(\xi)^\intercal \boldsymbol{\gamma}(\xi)\,d\xi\right)^{1/2}$. Furthermore, we define the following norms:

$$\|\boldsymbol{\gamma}\|_{\mathbb{H}^1} = \left(\|\boldsymbol{\gamma}\|^2_{\mathbb{L}^2} + \int_0^1 \boldsymbol{\gamma}_x(\xi)^\intercal \boldsymbol{\gamma}_x(\xi)\,d\xi\right)^{1/2} \tag{7}$$

$$\|\boldsymbol{\gamma}\|_{\mathbb{H}^2} = \left(\|\boldsymbol{\gamma}\|^2_{\mathbb{H}^1} + \int_0^1 \boldsymbol{\gamma}_{xx}(\xi)^\intercal \boldsymbol{\gamma}_{xx}(\xi)\,d\xi\right)^{1/2}. \tag{8}$$

For a 2×2 matrix $\mathbf{D}$, we denote:

$$|\mathbf{D}| = \max\left\{|\mathbf{D}v|;\ v \in \mathbb{R}^2,\ |v| = 1\right\}. \tag{9}$$

For the kernel matrices $\mathbf{K}$, we denote:

$$\|\mathbf{K}\|_\infty = \sup_{(x,\xi)\in\mathscr{T}} |\mathbf{K}(x,\xi)| \tag{10}$$

where $\mathscr{T} = \{(x,\xi) : 0 \le \xi \le x \le 1\}$. For $\boldsymbol{\gamma} \in \mathbb{H}^2([0,1])$ and positive constants c_1, c_2, c_3, c_4, c_5, and c_6, recall the following well-known inequalities (Coron et al., 2013):

$$\|\boldsymbol{\gamma}\|_{\mathbb{L}^1} \le c_1\|\boldsymbol{\gamma}\|_{\mathbb{L}^2} \le c_2\|\boldsymbol{\gamma}\|_\infty \tag{11}$$

$$\|\boldsymbol{\gamma}\|_\infty \le c_3\left(\|\boldsymbol{\gamma}\|_{\mathbb{L}^2} + \|\boldsymbol{\gamma}_x\|_{\mathbb{L}^2}\right) \le c_4\|\boldsymbol{\gamma}\|_{\mathbb{H}^1} \tag{12}$$

$$\|\boldsymbol{\gamma}_x\|_\infty \le c_5\left(\|\boldsymbol{\gamma}_x\|_{\mathbb{L}^2} + \|\boldsymbol{\gamma}_{xx}\|_{\mathbb{L}^2}\right) \le c_6\|\boldsymbol{\gamma}\|_{\mathbb{H}^2}. \tag{13}$$

3. Observer design

We consider first the linear case, $\mathbf{f} = 0$. The semilinear design is found by utilizing the result from the linear design. We assume that we can measure $w_1(x,t)$ at $x = 1$, and design an observer to estimate both $\mathbf{w}$ and $\mathbf{X}$.

3.1. Linear system

We design the collocated observer as a copy of the plant plus output injection, that is

$$\hat{\mathbf{w}}_t = \boldsymbol{\Sigma}(x)\hat{\mathbf{w}}_x + \boldsymbol{\Omega}(x)\hat{\mathbf{w}} + \mathbf{p}(x)\left(w_1(1,t) - \hat{w}_1(1,t)\right) \tag{14}$$

$$\hat{w}_1(0,t) = q\hat{w}_2(0,t) + \mathbf{C}\hat{\mathbf{X}}(t) \tag{15}$$

$$\hat{w}_2(1,t) = U(t) \tag{16}$$

$$\dot{\hat{\mathbf{X}}}(t) = \mathbf{A}\hat{\mathbf{X}}(t) + e^{\mathbf{A}d}\mathbf{L}\left(w_1(1,t) - \hat{w}_1(1,t)\right) \tag{17}$$

where $d = \int_0^1 \frac{d\chi}{\epsilon_1(\chi)}$. Defining error functions as $\tilde{\mathbf{w}} = \mathbf{w} - \hat{\mathbf{w}}$ and $\tilde{\mathbf{X}} = \mathbf{X} - \hat{\mathbf{X}}$, the error dynamics is given by

$$\tilde{\mathbf{w}}_t = \boldsymbol{\Sigma}(x)\tilde{\mathbf{w}}_x + \boldsymbol{\Omega}(x)\tilde{\mathbf{w}} - \mathbf{p}(x)\tilde{w}_1(1,t) \tag{18}$$

$$\tilde{w}_1(0,t) = q\tilde{w}_2(0,t) + \mathbf{C}\tilde{\mathbf{X}}(t) \tag{19}$$

$$\tilde{w}_2(1,t) = 0 \tag{20}$$

$$\dot{\tilde{\mathbf{X}}}(t) = \mathbf{A}\tilde{\mathbf{X}}(t) - e^{\mathbf{A}d}\mathbf{L}\tilde{w}_1(1,t) \tag{21}$$