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$\frac{11}{2}$ Dectation for almost evaluate increases with it and $\frac{11}{2}$ $\frac{1}{12}$ Bootstrap for almost cyclostationary processes with jitter effect $\frac{1}{78}$

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21 ARTICLE INFO ABSTRACT 87

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Keywords: Almost cyclostationarity Bootstrap Jitter Significance frequency detection

₂₃ Article history: **Article history: Article history:** In this paper we consider almost cyclostationary processes with jitter effect. We propose a bootstrap ag ²⁴ ^{Available online xxxx} states that the Moving Block Bootstrap method to construct pointwise and simultaneous $_{90}$ ²⁵ *Keywords:*
the simulation study we showed how our results can be used to detect the significant frequencies of 26 Almost cyclostationarity and the autocovariance function. We compared the behavior of our approach for jitter effects caused by 27 Bousuap
perturbations from two distributions, namely uniform and truncated normal. Moreover, we present a 28 94 real data application of our methodology. confidence intervals for the Fourier coefficients of the autocovariance function of such processes. In

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1. Introduction

In this paper we consider almost cyclostationary (ACS) processes. They are generalizations of cyclostationary (CS) processes. riod *d*, if it has periodic mean and covariance function, i.e.,

 $E(X(t + d)) = E(X(t))$ and

 $Cov(X(t), X(s)) = Cov(X(t + d), X(s + d)).$

For more details we refer the reader to [\[1\].](#page--1-0)

Moreover, a process *X(t)* with finite second moments is called ACS, if its mean and autocovariance functions are almost periodic. Let us recall that almost periodic functions were introduced by Besicovitch in [\[2\].](#page--1-0) A function $f : \mathcal{R} \to \mathcal{R}$ is called *almost periodic* if for every $\varepsilon > 0$ there exists a number l_{ε} such that for any interval of length greater than l_{ε} , there is a number p_{ε} in this interval such that

$$
\sup_{t \in \mathcal{R}} |f(t + p_{\varepsilon}) - f(t)| < \varepsilon. \tag{1}
$$

uniform limits of trigonometric polynomials (see [\[2\]\)](#page--1-0). For more information on ACS processes we refer the reader to [\[3\].](#page--1-0)

To analyze ACS processes, Fourier analysis is often applied. Fourier expansions of the mean and the autocovariance function

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65 131 1051-2004/© 2017 Elsevier Inc. All rights reserved.

34 **1. Introduction are used to detect significant frequencies. Although, results estab-100** 35 101 lishing the estimators of the Fourier coefficients and their proper-³⁶ In this paper we consider almost cyclostationary (ACS) pro-
102 37 cesses. They are generalizations of cyclostationary (CS) processes. also a method to obtain the range of possible values of the con- 103 ³⁸ A process *X*(*t*) with finite second moments is called CS with pe- sidered parameters. Unfortunately, the asymptotic confidence in- 104 39 riod d, if it has periodic mean and covariance function, i.e.. The example constructed because the asymptotic variances of the street o 40 106 the estimators depend on the unknown parameters. Thus, to com- $F(Y(t+d)) = F(Y(t))$ and the pute confidence intervals resampling methods are used. They allow to the pute confidence intervals resampling methods are used. They allow 42 108 us to approximate the distribution of the statistics of interest. sidered parameters. Unfortunately, the asymptotic confidence in-

 43 $Cov(X(t), X(s)) = Cov(X(t+d), X(s+d)).$ One of the most popular resampling techniques is the bootstrap 109 A 110 110 110 method. It was introduced by Efron in [\[5\].](#page--1-0) The method was initially $_{45}$ for more details we refer the reader to [1]. $_{45}$ designed for independent and identically distributed data, but in 111 46 Moreover, a process $x(t)$ with finite second moments is called the late 1980s and beginning of the 1990s, there appeared modi-47 ACS, if its mean and autocovariance functions are almost periodic. θ fications dedicated for stationary time series (see [\[6\]](#page--1-0) and [\[7\]\)](#page--1-0). Fi- 113 $_{48}$ let us recall that almost periodic functions were introduced by anally, techniques for nonstationary processes have been developed 114 to 49 Besicovitch in [2]. A function $f: \mathcal{R} \to \mathcal{R}$ is called almost periodic if in the last 10 years. Methods dedicated to stationary or nonstation- $_{50}$ for every $\varepsilon > 0$ there exists a number l_{ε} such that for any inter- ary time series are designed to preserve the dependence structure 116 $_{51}$ val of length greater than l_{ε} , there is a number p_{ε} in this interval contained in the data. The idea is to randomly sample blocks of 117 52 118 observations and hence to keep inside of each block the depen-53 119 dence structure contained in the original data. Currently there exist $\frac{54}{20}$ $\frac{1}{20}$ three block bootstrap methods that can be applied to CS/ACS pro- 120 55 121 cesses. These are the Moving Block Bootstrap (MBB) introduced 56 Equivalently, the almost periodic functions can be defined as the independently in [6] and [7], the Generalized Seasonal Block Boot- 122 57 uniform limits of trigonometric polynomials (see [2]). For more in- strap (GSBB) proposed in [\[8\],](#page--1-0) and the Generalized Seasonal Tapered 123 58 formation on ACS processes we refer the reader to [3]. Block Bootstrap (GSTBB) proposed in [\[9\].](#page--1-0) All can be used for CS 124 59 To analyze ACS processes, Fourier analysis is often applied. processes, but since the GSBB and the GSTBB require knowledge of 125 60 Fourier expansions of the mean and the autocovariance function the period length, they cannot be applied to the ACS case. The first 126 61 127 bootstrap consistency result for CS/ACS processes was obtained in 62 128 2007 by Synowiecki in [\[10\].](#page--1-0) The author showed validity of the MBB 63 E-mail address: aedudek@agh.edu.pl (A.E. Dudek). (1298) controll for the overall mean of the ACS time series. Dudek et al. proved the independently in [\[6\]](#page--1-0) and [\[7\],](#page--1-0) the Generalized Seasonal Block Boot-

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¹ GSBB consistency for the overall mean and the seasonal means in The This paper we focus on the Fourier analysis of X. Generally, ⁶⁷ $2 \quad [8]$. The corresponding results for the GSTBB can be found in $[9]$. for a fully observed process X, the Fourier coefficients of the auto- 68 3 69 The applicability of the modified MBB for the Fourier coefficient of ⁴ the mean and the autocovariance functions of ACS time series was $\frac{1}{T}$ 5 proved in [\[11\].](#page--1-0) Fourier coefficient of the mean and the autocovari- $\frac{1}{r}$ $\frac{1}{r}$ 6 ance functions of CS time series were considered in [\[12\]](#page--1-0) (GSBB) $a(x, \tau) = \lim_{x \to \infty} \frac{1}{T} \int E(X(t)X(t+\tau)) \exp(-1\xi) ds$ (2) 72 ⁷ and [\[9\]](#page--1-0) (GSTBB). Moreover, Dehay and Dudek showed the MBB va- $\frac{9}{0}$ ⁸ lidity for Fourier coefficient of the mean and the autocovariance $\frac{1}{n}$. The contract of the state of the st ⁹ functions of ACS continues time process that is not fully observed 10 $(\text{see } [13] \text{ and } [14])$ $(\text{see } [13] \text{ and } [14])$ $(\text{see } [13] \text{ and } [14])$ $(\text{see } [13] \text{ and } [14])$. 26

11 77 In the following we extend applicability of the bootstrap ¹² method to the ACS processes, which are observed in instants that $\hat{\tau}_{m}(1, \tau) = \frac{1}{N} \int_{-\infty}^{\infty} Y(t) Y(t + \tau) \exp(-i1t) dt$ (3) ⁷⁸ 13 are randomly disturbed. This effect is called jitter. It appears in $T \int T^{(1)}(x) dx = T \int T^{(2)}(x) dx$ ¹⁴ many signal analysis problems [\[15–17\],](#page--1-0) e.g., receiver design in \sim 0 ¹⁵ telecommunication, audio applications, optical encoders, etc. Jitters is consistent and asymptotically normal (see, e.g. [4.22]). ¹⁶ in clock signals are typically caused by noise or other disturbances $\frac{16}{10}$ restimate the Fourier coefficients of the autocovariance func-¹⁷ in the system. Contributing factors include thermal noise, power tion of X in our case, we need to use some approximation. The ⁸³ ¹⁸ supply variations, loading conditions, device noise, and interfer-
18 supply variations, loading conditions, device noise, and interfer-
10 problem is caused by the fact that we need to know values. ence coupled from nearby circuits.

²⁰ When acquiring a signal, the jitter of the sampling clock can be $\frac{1}{2}$ mate τ by the nearest multiple of h I et k_r be the nearest integer ⁸⁶ ²¹ voluntary or involuntary. For example, in the case of compressed $\frac{1}{10} \frac{\tau}{h}$ We have 22 88 sensing it is essential to achieve a random signal acquisition. In ²³ practice this operation can be performed by adding a random jitter $\tau \in \Gamma_{\leq b}$, $\tau = 1$ ²⁴ on the clock signal. In other applications, the jitter can be undergo- $h = 2 \binom{n}{k}$ $h = 2$ ²⁵ ing. This is the case for the angular acquisitions of vibratory signals $\frac{1}{1}$ $\frac{$ 26 issued from rotating machinery. In this situation, angular sampling $\sum_{n=1}^{\infty}$ issued from rotating machinery. In this situation, angular sampling $\sum_{n=1}^{\infty}$ issued from rotating machinery. In this situation, a ²⁷ is sensitive to hardware imperfections (optical encoder precision, $\frac{1000 \text{ eV}}{1000 \text{ eV}} = \frac{1000 \text{ eV}}{1000 \text{ eV$ 28 electrical perturbation, etc.) (see [\[18\]\)](#page--1-0). The angular sampled sig-
28 electrical perturbation, etc.) (see [18]). The angular sampled sig-
29 electrical perturbation, etc.) (see [18]). The angular sampled sig-29 95 nal quantification step or sampling frequency determination is not ³⁰ identical to the time domain [\[19,20\].](#page--1-0) Angular sampling is also not $\frac{n}{1}$, $\frac{n}{2}$ ³¹ adapted to study time domain signals like impulse response. Some $\bar{a}(\lambda, \tau) = -\sum b_k(\lambda, \tau)$, (4) ⁹⁷ ³² of the imperfections can be viewed as non-uniform sampling or $\begin{array}{c} u_{k=1} \end{array}$ $\begin{array}{c} u_{k=1} \end{array}$ ³³ random jitter. In these voluntary or involuntary circumstances, it where the state of t ³⁴ appears appropriate to develop CS signal analysis tools sampled in the state of the sta 35 the presence of jitter. $\widetilde{L}(t, z) = \widetilde{L}(t, z) - \widetilde{L}(t + H) \cdot \widetilde{L}(t + H) \cdot \widetilde{L}(t + H)$ and $\widetilde{L}(t + H) = \widetilde{L}(t + H) \cdot \widetilde{L}(t + H)$

36 102 The paper is organized as follows. In Section 2 the problem is $\mathcal{L}_{K}(x, y) = X(x, y) = X(x, y) + \mathcal{L}_{K}(x, y) + \mathcal{L}_{K}(y, y) + \mathcal{L}_{K}(y, y) + \mathcal{L}_{K}(y, y)$ ³⁷ formulated and the considered assumptions are presented. In addi-
and $0 < k < n$ and $0 < k + k< n$. Note that for fixed $h > 0$, the ³⁸ tion, the estimators of the Fourier coefficients are introduced and $\frac{1}{2}$ $\frac{1}{2}$ ¹⁰⁵ 105
¹⁰⁵ 105 their asymptotic properties are discussed. Section [3](#page--1-0) is dedicated to the part subsection we discuss the assumptions that we used ⁴⁰ the bootstrap method. The MBB approach adapted to our problem to derive our results 41 107 is presented and its consistency for the Fourier coefficients of the ⁴² autocovariance function is shown. Finally, the construction of the $\frac{1}{2}$, *Accumptions* ⁴³ bootstrap pointwise and the simultaneous confidence intervals are the state of the stat ^{[4](#page--1-0)4} provided. Section 4 is devoted to the alternative bootstrap tech-
¹¹⁰ 45 nique that can be used in the considered problem. In Section [5,](#page--1-0) $\frac{1}{11}$ in the sequence constrains are asset. ⁴⁶ a simulation study is presented in which the performance of the $\frac{1}{2}$ is a sempled at a constant rate greater than the Nuguist rate $\frac{112}{2}$ 47 proposed bootstrap method is verified. Finally, in Section [6](#page--1-0) the real $\frac{1}{2}$ is the star by $\frac{1}{2}$ (by in small appeals to such the star by $\frac{1}{2}$ single star by $\frac{1}{2}$ such the star by $\frac{1}{2}$ such the 48 114 data vibratory gear vibration signal is analyzed and the obtained

51 **2. Problem formulation** that the contract of the contract **2. Problem formulation**

54 is uniformly almost cyclostationary (UACS), i.e., for any $s \in \mathbb{R}$ $\begin{array}{c} E(A(t) \ge 0) \ge 0 \le t \le 0 \le t \le 0 \end{array}$ $K(t + t_2) \wedge (t + t_3)$ is althost periodic in *t* difficulties with the set $K(t_1, t_2)$ is althost periodic in *t* difficulties with the set $K(t_1, t_2)$ is althost periodic in *t* difficulties with the set of k_1 is π , 56 = E(*X*(*t*)*X*(*t* + *τ*)) is almost periodic in *t* uniformly in *τ*. The spectrum $\binom{r}{i}$, $\binom{r}{2}$, $\binom{r}{3}$ varying in $\binom{r}{1}$

57 In the following, we use notation introduced in [\[21\].](#page--1-0) (vi) $X(t)$ is α -mixing and $\sum_{k=1}^{\infty} k \alpha_X^{(n+4)}(k) < \infty$.

 58 The process $X(t)$ is not observed continuously but only in in-59 stants $t_k = kh + U_k$, $h > 0$. Hence one observes the discrete time Condition (ii) prevents permutation of observations caused by the 125 ⁶⁰ process $X_k = \{X(kh + U_k), k \in \mathcal{Z}\}$. Random variables U_k are inde-
itter effect, i.e., we assume that the instants of the observations of ⁶¹ pendent and identically distributed (iid) and are independent of *X*. are perturbed but the order of the observations is unchanged. As- ¹²⁷ 62 They can be considered as random errors. In fact we assume that sumption (iii) denotes that for each τ there is a finite number 128 63 each instant in which the process is observed is disturbed. In the of non-zero coefficients $a(\lambda, \tau)$ or equivalently a finite number of 129 ⁶⁴ sequel we assume that *h* > 0 is fixed and small enough to avoid significant frequencies λ . This condition is not necessary but allows 130 65 aliasing. In [\[18\]](#page--1-0) a similar model was considered with U_k being iid us to simplify the presentation of the results. Finally, to obtain the 131 process $X_k = \{X(kh + U_k), k \in \mathcal{Z}\}\$. Random variables U_k are indeeach instant in which the process is observed is disturbed. In the random variables from the standard normal distribution.

In this paper we focus on the Fourier analysis of *X*. Generally, for a fully observed process *X*, the Fourier coefficients of the autocovariance function are of the form

$$
a(\lambda, \tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} E(X(t)X(t+\tau)) \exp(-i\lambda s) ds \tag{2}
$$

For a continuous time observation of a UACS process *X* the estimator

$$
\widehat{a}_T(\lambda, \tau) = \frac{1}{T} \int_0^T X(t)X(t + \tau) \exp(-i\lambda t)dt
$$
 (3)

is consistent and asymptotically normal (see, e.g., [\[4,22\]\)](#page--1-0).

19 ence coupled from nearby circuits.
 $X(t)X(t+\tau)$. Since τ is not always a multiple of *h*, we approxi-To estimate the Fourier coefficients of the autocovariance function of *X* in our case, we need to use some approximation. The problem is caused by the fact that we need to know values mate τ by the nearest multiple of *h*. Let k_{τ} be the nearest integer to *τ /h*. We have

$$
\frac{\tau}{h}-\frac{1}{2}
$$

Finally, $U_{k,\tau} = U_{k+k_{\tau}} + k_{\tau}h - \tau$ is a time perturbation for the time moment $kh + \tau$.

We observe a sample $\{X(kh + U_k) : 1 \le k \le n\}$. The estimator of *a*($λ$, $τ$) is defined as follows

$$
\widetilde{a}(\lambda,\tau) = \frac{1}{n} \sum_{k=1}^{n} \widetilde{b}_k(\lambda,\tau),\tag{4}
$$

where

$$
\widetilde{b}_k(\lambda, \tau) = X(kh + U_k)X((k + k_{\tau})h + U_{k + k_{\tau}}) \exp(-i\lambda kh) \tag{5}
$$

and $0 \le k \le n$ and $0 \le k + k_{\tau} \le n$. Note that for fixed $h > 0$, the time series $\widetilde{b}_k(\lambda, \tau)$ is ACS.

In the next subsection we discuss the assumptions that we used to derive our results.

2.1. Assumptions

In the sequel the following conditions are used:

- (i) *X* is sampled at a constant rate greater than the Nyquist rate with time step $h > 0$ (*h* is small enough to avoid aliasing);
- 49 results are discussed in Section [7.](#page--1-0) (ii) the random perturbations U_k are iid from some distribution on *(*−*h/*2*,h/*2*)*;
- 50

50 $\text{in} \{n/2, n/2\}$,

51 **116 116**

51 **116 116** (iii) set $\Lambda = {\lambda \in \mathbb{R} : a(\lambda, \tau) \neq 0 \text{ for some } \tau \in \mathbb{R}}$ is finite;
- 52 **118**

52 **118**

52 **118**
- 53 Let *X* = {*X(t)*, *t* ∈ R} be a zero-mean real-valued process that $\begin{bmatrix} (V) & A & B & A \ B & C & C \end{bmatrix}$ and the function *(t a m* m), $\begin{bmatrix} (V \cap V) & (V \cap V) \end{bmatrix}$ = {*X* + } $\begin{bmatrix} (V \cap V) & (V \cap V) \end{bmatrix}$ = {*X* + } $\begin{bmatrix} (V$ (v) *X* has almost periodic fourth moments, i.e. for each $t \in \mathbb{R}$, $E{X(t)^4} < \infty$; the function $(t, \tau_1, \tau_2, \tau_3) \mapsto E{X(t)X(t + \tau_1) \times \tau_2}$ $X(t + \tau_2)X(t + \tau_3)$ is almost periodic in *t* uniformly with respect to τ_1 , τ_2 , τ_3 varying in \mathbb{R} ;

(vi)
$$
X(t)
$$
 is α -mixing and $\sum_{k=1}^{\infty} k\alpha_X^{\frac{1}{\gamma+4}}(k) < \infty$.

66 random variables from the standard normal distribution.
66 asymptotic normality of $\tilde{a}(\lambda, \tau)$, a mixing condition *(vi)* is needed. ¹³² Condition *(ii)* prevents permutation of observations caused by the are perturbed but the order of the observations is unchanged. Assumption *(iii)* denotes that for each *τ* there is a finite number significant frequencies *λ*. This condition is not necessary but allows us to simplify the presentation of the results. Finally, to obtain the

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