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Bootstrap for almost cyclostationary processes with jitter effect

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ABSTRACT

In this paper we consider almost cyclostationary processes with jitter effect. We propose a bootstrap approach based on the Moving Block Bootstrap method to construct pointwise and simultaneous confidence intervals for the Fourier coefficients of the autocovariance function of such processes. In the simulation study we showed how our results can be used to detect the significant frequencies of the autocovariance function. We compared the behavior of our approach for jitter effects caused by perturbations from two distributions, namely uniform and truncated normal. Moreover, we present a real data application of our methodology.

1. Introduction

In this paper we consider almost cyclostationary (ACS) processes. They are generalizations of cyclostationary (CS) processes. A process $X(t)$ with finite second moments is called CS with period d, if it has periodic mean and covariance function, i.e.,

$$\mathrm{E}\big(X(t+d)\big)=\mathrm{E}\big(X(t)\big) \quad \text{and}$$
$$\mathrm{Cov}\big(X(t),X(s)\big)=\mathrm{Cov}\big(X(t+d),X(s+d)\big).$$

For more details we refer the reader to [1].

Moreover, a process $X(t)$ with finite second moments is called ACS, if its mean and autocovariance functions are almost periodic. Let us recall that almost periodic functions were introduced by Besicovitch in [2]. A function $f:\mathcal{R}\to\mathcal{R}$ is called *almost periodic* if for every $\varepsilon>0$ there exists a number l_ε such that for any interval of length greater than l_ε, there is a number p_ε in this interval such that

$$\sup_{t\in\mathcal{R}}\big|f(t+p_\varepsilon)-f(t)\big|<\varepsilon. \tag{1}$$

Equivalently, the almost periodic functions can be defined as the uniform limits of trigonometric polynomials (see [2]). For more information on ACS processes we refer the reader to [3].

To analyze ACS processes, Fourier analysis is often applied. Fourier expansions of the mean and the autocovariance function are used to detect significant frequencies. Although, results establishing the estimators of the Fourier coefficients and their properties are well known (see [4]), in practical applications one needs also a method to obtain the range of possible values of the considered parameters. Unfortunately, the asymptotic confidence intervals cannot be constructed because the asymptotic variances of the estimators depend on the unknown parameters. Thus, to compute confidence intervals resampling methods are used. They allow us to approximate the distribution of the statistics of interest.

One of the most popular resampling techniques is the bootstrap method. It was introduced by Efron in [5]. The method was initially designed for independent and identically distributed data, but in the late 1980s and beginning of the 1990s, there appeared modifications dedicated for stationary time series (see [6] and [7]). Finally, techniques for nonstationary processes have been developed in the last 10 years. Methods dedicated to stationary or nonstationary time series are designed to preserve the dependence structure contained in the data. The idea is to randomly sample blocks of observations and hence to keep inside of each block the dependence structure contained in the original data. Currently there exist three block bootstrap methods that can be applied to CS/ACS processes. These are the Moving Block Bootstrap (MBB) introduced independently in [6] and [7], the Generalized Seasonal Block Bootstrap (GSBB) proposed in [8], and the Generalized Seasonal Tapered Block Bootstrap (GSTBB) proposed in [9]. All can be used for CS processes, but since the GSBB and the GSTBB require knowledge of the period length, they cannot be applied to the ACS case. The first bootstrap consistency result for CS/ACS processes was obtained in 2007 by Synowiecki in [10]. The author showed validity of the MBB for the overall mean of the ACS time series. Dudek et al. proved the

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GSBB consistency for the overall mean and the seasonal means in [8]. The corresponding results for the GSTBB can be found in [9]. The applicability of the modified MBB for the Fourier coefficient of the mean and the autocovariance functions of ACS time series was proved in [11]. Fourier coefficient of the mean and the autocovariance functions of CS time series were considered in [12] (GSBB) and [9] (GSTBB). Moreover, Dehay and Dudek showed the MBB validity for Fourier coefficient of the mean and the autocovariance functions of ACS continues time process that is not fully observed (see [13] and [14]).

In the following we extend applicability of the bootstrap method to the ACS processes, which are observed in instants that are randomly disturbed. This effect is called jitter. It appears in many signal analysis problems [15–17], e.g., receiver design in telecommunication, audio applications, optical encoders, etc. Jitters in clock signals are typically caused by noise or other disturbances in the system. Contributing factors include thermal noise, power supply variations, loading conditions, device noise, and interference coupled from nearby circuits.

When acquiring a signal, the jitter of the sampling clock can be voluntary or involuntary. For example, in the case of compressed sensing it is essential to achieve a random signal acquisition. In practice this operation can be performed by adding a random jitter on the clock signal. In other applications, the jitter can be undergoing. This is the case for the angular acquisitions of vibratory signals issued from rotating machinery. In this situation, angular sampling is sensitive to hardware imperfections (optical encoder precision, electrical perturbation, etc.) (see [18]). The angular sampled signal quantification step or sampling frequency determination is not identical to the time domain [19,20]. Angular sampling is also not adapted to study time domain signals like impulse response. Some of the imperfections can be viewed as non-uniform sampling or random jitter. In these voluntary or involuntary circumstances, it appears appropriate to develop CS signal analysis tools sampled in the presence of jitter.

The paper is organized as follows. In Section 2 the problem is formulated and the considered assumptions are presented. In addition, the estimators of the Fourier coefficients are introduced and their asymptotic properties are discussed. Section 3 is dedicated to the bootstrap method. The MBB approach adapted to our problem is presented and its consistency for the Fourier coefficients of the autocovariance function is shown. Finally, the construction of the bootstrap pointwise and the simultaneous confidence intervals are provided. Section 4 is devoted to the alternative bootstrap technique that can be used in the considered problem. In Section 5, a simulation study is presented in which the performance of the proposed bootstrap method is verified. Finally, in Section 6 the real data vibratory gear vibration signal is analyzed and the obtained results are discussed in Section 7.

2. Problem formulation

Let $X = \{X(t), t \in \mathbb{R}\}$ be a zero-mean real-valued process that is uniformly almost cyclostationary (UACS), i.e., for any $s \in \mathbb{R}$ $\mathrm{E}(X^2(s)) < \infty$ and autocovariance function $B(t,\tau) = \mathrm{Cov}(X_t, X_{t+\tau}) = \mathrm{E}(X(t)X(t+\tau))$ is almost periodic in t uniformly in τ.

In the following, we use notation introduced in [21].

The process $X(t)$ is not observed continuously but only in instants $t_k = kh + U_k, h > 0$. Hence one observes the discrete time process $X_k = \{X(kh + U_k), k \in \mathcal{Z}\}$. Random variables U_k are independent and identically distributed (iid) and are independent of X. They can be considered as random errors. In fact we assume that each instant in which the process is observed is disturbed. In the sequel we assume that $h > 0$ is fixed and small enough to avoid aliasing. In [18] a similar model was considered with U_k being iid random variables from the standard normal distribution.

In this paper we focus on the Fourier analysis of X. Generally, for a fully observed process X, the Fourier coefficients of the autocovariance function are of the form

$$a(\lambda,\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \mathrm{E}\big(X(t)X(t+\tau)\big)\exp(-i\lambda s)ds \tag{2}$$

For a continuous time observation of a UACS process X the estimator

$$\widehat{a}_T(\lambda,\tau) = \frac{1}{T}\int_0^T X(t)X(t+\tau)\exp(-i\lambda t)dt \tag{3}$$

is consistent and asymptotically normal (see, e.g., [4,22]).

To estimate the Fourier coefficients of the autocovariance function of X in our case, we need to use some approximation. The problem is caused by the fact that we need to know values $X(t)X(t+\tau)$. Since τ is not always a multiple of h, we approximate τ by the nearest multiple of h. Let k_τ be the nearest integer to τ/h. We have

$$\frac{\tau}{h} - \frac{1}{2} < k_\tau < \frac{\tau}{h} + \frac{1}{2}.$$

Finally, $U_{k,\tau} = U_{k+k_\tau} + k_\tau h - \tau$ is a time perturbation for the time moment $kh + \tau$.

We observe a sample $\{X(kh + U_k) : 1 \le k \le n\}$. The estimator of $a(\lambda,\tau)$ is defined as follows

$$\widetilde{a}(\lambda,\tau) = \frac{1}{n}\sum_{k=1}^{n} \widetilde{b}_k(\lambda,\tau), \tag{4}$$

where

$$\widetilde{b}_k(\lambda,\tau) = X(kh+U_k)X\big((k+k_\tau)h + U_{k+k_\tau}\big)\exp(-i\lambda kh) \tag{5}$$

and $0 \le k \le n$ and $0 \le k + k_\tau \le n$. Note that for fixed $h > 0$, the time series $\widetilde{b}_k(\lambda,\tau)$ is ACS.

In the next subsection we discuss the assumptions that we used to derive our results.

2.1. Assumptions

In the sequel the following conditions are used:

(i) X is sampled at a constant rate greater than the Nyquist rate with time step $h > 0$ (h is small enough to avoid aliasing);
(ii) the random perturbations U_k are iid from some distribution on $(-h/2, h/2)$;
(iii) set $\Lambda = \{\lambda \in \mathbb{R} : a(\lambda,\tau) \neq 0$ for some $\tau \in \mathbb{R}\}$ is finite;
(iv) $\sup_t \mathrm{E}\{|X(t)|^{8+2\eta}\} < \infty$ for some $\eta > 0$;
(v) X has almost periodic fourth moments, i.e. for each $t \in \mathbb{R}$, $\mathrm{E}\{X(t)^4\} < \infty$; the function $(t,\tau_1,\tau_2,\tau_3) \mapsto \mathrm{E}\{X(t)X(t+\tau_1) \times X(t+\tau_2)X(t+\tau_3)\}$ is almost periodic in t uniformly with respect to τ_1, τ_2, τ_3 varying in $\mathbb{R}$;
(vi) $X(t)$ is α-mixing and $\sum_{k=1}^{\infty} k\alpha_X^{\frac{\eta}{\eta+4}}(k) < \infty$.

Condition (ii) prevents permutation of observations caused by the jitter effect, i.e., we assume that the instants of the observations are perturbed but the order of the observations is unchanged. Assumption (iii) denotes that for each τ there is a finite number of non-zero coefficients $a(\lambda,\tau)$ or equivalently a finite number of significant frequencies λ. This condition is not necessary but allows us to simplify the presentation of the results. Finally, to obtain the asymptotic normality of $\widetilde{a}(\lambda,\tau)$, a mixing condition (vi) is needed.

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