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Stability of consensus extended Kalman filter for distributed state estimation☆

Giorgio Battistelli, Luigi Chisci

Università di Firenze, Dipartimento di Ingegneria dell'Informazione (DINFO), Via di Santa Marta 3, 50139 Firenze, Italy

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ABSTRACT

The paper addresses consensus-based networked estimation of the state of a nonlinear dynamical system. The focus is on a family of distributed state estimation algorithms which relies on the extended Kalman filter linearization paradigm. Consensus is exploited in order to fuse the information, both prior and novel, available in each network node. It is shown that the considered family of distributed Extended Kalman Filters enjoys local stability properties, under minimal requirements of network connectivity and system collective observability. A simulation case-study concerning target tracking with a network of nonlinear (angle and range) position sensors is worked out in order to show the effectiveness of the considered nonlinear consensus filter.

1. Introduction

Consensus is a widely exploited tool for distributing computations over networks in a scalable way. An especially important application of consensus, which has recently received great attention, is networked state estimation, i.e., distributed estimation of the state of a dynamical system given measurements provided by a wireless sensor network. The literature on the subject is quite vast and includes approaches based on consensus Kalman filtering (Cattivelli & Sayed, 2010; Kamgarpour & Tomlin, 2008; Kar & Moura, 2011; Li & Jia, 2012; Olfati-Saber, 2007; Wang, Ren, & Li, 2014; Zhou, Fang, & Hong, 2013), Luenberger-like consensus estimation (Matei & Baras, 2012; Millan, Orihuela, Jurado, Vivas, & Rubio, 2015; Millan, Orihuela, Vivas, Rubio, Dimarogonas, & Johansson, 2013; Stankovic, Stankovic, & Stipanovic, 2009), consensus H_∞ estimation (Ugrinovskii, 2011, 2013), distributed particle filtering (Hlinka, Hlawatsch, & Djuric, 2013; Mohammadi & Asif, 2013), and distributed moving-horizon estimation (Farina, Ferrari-Trecate, & Scattolini, 2010, 2012). The interested reader is referred to the above-cited papers as well so to the references therein for an overview of the different existing approaches. In the context of networked state estimation, the main challenge is to design distributed estimation algorithms that preserve as much as possible the stability, performance and robustness requirements of their centralized counterparts.

In this respect, significant advances have been made, in the last years, in the linear setting by developing distributed state estimation (DSE) algorithms able to guarantee stability under minimal requirements of network connectivity and system collective observability, i.e. observability from the whole network but not necessarily from individual sensors. Such algorithms include the *consensus on information* (CI) filter of Battistelli and Chisci (2014) and Battistelli, Chisci, Morrocchi, and Papi (2011) and the *information weighted consensus filter* (ICF) of Kamal, Farrell, and Roy-Chowdhury (2012, 2013). The CI, in which consensus is carried out on the posterior information of the network nodes, can be interpreted in terms of consensus to the average of the local posteriors according to the pseudo-metric induced by the Kullback–Leibler average (Battistelli & Chisci, 2014). The ICF algorithm performs a consensus with a suitable weighting of the prior state and measurement information so as to ensure convergence to the centralized estimate as the number of consensus steps goes to infinity. Recently in Battistelli, Chisci, Mugnai, Farina, and Graziano (2015), it was shown that both the CI and the ICF belong to a broader family of DSE algorithms, and a generalization to a nonlinear setting was

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E-mail addresses: giorgio.battistelli@unifi.it (G. Battistelli), luigi.chisci@unifi.it (L. Chisci).

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proposed by exploiting the Extended Kalman Filter (EKF) linearization argument. Hereafter, the family of DSE algorithms resulting from such a generalization will be referred to as Distributed EKFs (DEKFs).

The present paper provides a contribution by proving that the family of DSE algorithms of Battistelli et al. (2015) enjoy nice stability properties also in the more general nonlinear setting, provided that, similarly to the linear case, suitable connectivity and collective observability assumptions hold. In the lines of the classical results on stability of centralized EKF (La Scala, Bitmead, & James, 1995; Reif, Gunther, Yaz, & Unbehauen, 1999; Reif & Unbehauen, 1999), the stability analysis is based on the idea of writing the estimation error dynamics in a suitable way so that the linearized part is separated from the nonlinear (higher-order) terms. Then, the stability of the linear part of the estimation error dynamics can be analyzed via Lyapunov-like methods, and local stability results can be derived for the overall estimation error dynamics. As a further contribution, an explicit connection is established between the boundedness of the filter covariance matrix and the invertibility of the collective observability mapping.

Thanks to this result, the considered family of DEKFs emerges as an effective tool for the solution of many practically relevant distributed nonlinear filtering problems like, e.g., distributed tracking of a moving object given measurements of angle, range and/or Doppler wireless communicating sensors spread over the area of interest; such sensors, in fact, are highly nonlinear and unable to individually guarantee observability.

The rest of the paper is organized as follows. Section 2 introduces the problem setting. Section 3 describes the considered family of DEKF algorithms for networked state estimation and Section 4 analyzes its stability properties. Section 5 demonstrates, via simulation experiments, the effectiveness of such a consensus filter in a nonlinear target tracking case-study. Section 6 ends the paper with concluding remarks. All mathematical proofs are reported in the Appendix.

2. Problem setting

This paper addresses Distributed State Estimation (DSE) over a sensor network consisting of two types of nodes: *communication* nodes have only *processing* and *communication* capabilities, i.e. they can process local data as well as exchange data with neighboring nodes, while *sensor* nodes have also *sensing* capabilities, i.e. they can sense data from the environment. Notice that communication nodes are introduced to act as "relays" of information whenever sensor nodes are too far away to communicate. For insights on the importance of considering the effect of communication nodes when studying the properties of a distributed state estimation algorithm we refer the reader to Kamal et al. (2013), Wang et al. (2014) (where this type of nodes is referred to as "naive nodes"). In the sequel, the network will be denoted by the triplet $(\mathcal{S}, \mathcal{C}, \mathcal{A})$ where: $\mathcal{S}$ is the set of sensor nodes, $\mathcal{C}$ the set of communication nodes, $\mathcal{N} = \mathcal{S} \bigcup \mathcal{C}$, $\mathcal{A} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of arcs (connections) such that $(i, j) \in \mathcal{A}$ if node j can receive data from node i (clearly $(i, i) \in \mathcal{A}$ for all $i \in \mathcal{N}$). Further, for each node $i \in \mathcal{N}$, $\mathcal{N}^i$ will denote the set of its in-neighbors (including i itself), i.e. $\mathcal{N}^i \triangleq \{j : (j, i) \in \mathcal{A}\}$.

The DSE problem over the sensor network $(\mathcal{S}, \mathcal{C}, \mathcal{A})$ can be formulated as follows. Consider a dynamical system

$$\mathbf{x}_{t+1} \quad = \quad \mathbf{f}(\mathbf{x}_t) + \mathbf{w}_t \tag{1}$$

and a set of sensors $\mathcal{S}$ with measurement equations

$$\mathbf{y}_t^i \quad = \quad \mathbf{h}^i(\mathbf{x}_t) + \mathbf{v}_t^i, \quad i \in \mathcal{S}. \tag{2}$$

Notice that the above measurement equation is defined only for sensor nodes, since no measurement is supposed to be collected by the communication nodes. Then the objective is to have, at each time $t \in \{1, 2, \ldots\}$ and in each node $i \in \mathcal{N}$, an estimate $\hat{\mathbf{x}}_{t|t}$ of the state $\mathbf{x}_t$ constructed only on the basis of the local measurements (when available) and of data received from all adjacent nodes $j \in \mathcal{N}^i \setminus \{i\}$.

Table 1
Information CEKF Algorithm, to be implemented at each sampling interval $t = 1, 2, \ldots$ starting from initial conditions $\hat{\mathbf{x}}_{1|0}$, $\boldsymbol{\Omega}_{1|0}$, $\mathbf{q}_{1|0} = \boldsymbol{\Omega}_{1|0}\,\hat{\mathbf{x}}_{1|0}$.

Correction (measurement-update):

$$\mathbf{C}_t^i = \frac{\partial \mathbf{h}^i}{\partial \mathbf{x}}\left(\hat{\mathbf{x}}_{t|t-1}\right), \qquad i \in \mathcal{S}$$

$$\boldsymbol{\Omega}_{t|t} = \boldsymbol{\Omega}_{t|t-1} + \sum_{i \in \mathcal{S}} \left(\mathbf{C}_t^i\right)^\top \mathbf{V}^i\,\mathbf{C}_t^i$$

$$\overline{\mathbf{y}}_t^i = \mathbf{y}_t^i - \mathbf{h}^i\left(\hat{\mathbf{x}}_{t|t-1}\right) + \mathbf{C}_t^i \hat{\mathbf{x}}_{t|t-1}, \quad i \in \mathcal{S}$$

$$\mathbf{q}_{t|t} = \mathbf{q}_{t|t-1} + \sum_{i \in \mathcal{S}} \left(\mathbf{C}_t^i\right)^\top \mathbf{V}^i\,\overline{\mathbf{y}}_t^i$$

Prediction (time-update):

$$\hat{\mathbf{x}}_{t|t} = \boldsymbol{\Omega}_{t|t}^{-1}\mathbf{q}_{t|t}, \quad \text{and} \quad \mathbf{A}_t = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\left(\hat{\mathbf{x}}_{t|t}\right)$$

$$\boldsymbol{\Omega}_{t+1|t} = \mathbf{W} - \mathbf{W}\mathbf{A}_t\left(\boldsymbol{\Omega}_{t|t} + \mathbf{A}_t^\top \mathbf{W}\mathbf{A}_t\right)^{-1}\mathbf{A}_t^\top \mathbf{W}$$

$$\hat{\mathbf{x}}_{t+1|t} = \mathbf{f}\left(\hat{\mathbf{x}}_{t|t}\right), \quad \text{and} \quad \mathbf{q}_{t+1|t} = \boldsymbol{\Omega}_{t+1|t}\,\hat{\mathbf{x}}_{t+1|t}$$

2.1. Centralized extended Kalman Filter

Before describing the family of DEKF algorithms under consideration, it is convenient to briefly recall the equations of the centralized Extended Kalman Filter, which is assumed to simultaneously process all measurements $\{y_k^i, i \in \mathcal{S}\}$. Hereafter, for convenience, the *information filter* form will be adopted. The information filter propagates, instead of the estimate $\hat{\mathbf{x}}_{t|t-1}$ and covariance $\mathbf{P}_{t|t-1}$, the *information* (inverse covariance) matrices

$$\boldsymbol{\Omega}_{t|t-1} \triangleq \mathbf{P}_{t|t-1}^{-1}, \qquad \boldsymbol{\Omega}_{t|t} \triangleq \mathbf{P}_{t|t}^{-1}$$

and the vectors

$$\mathbf{q}_{t|t-1} \triangleq \mathbf{P}_{t|t-1}^{-1}\hat{\mathbf{x}}_{t|t-1}, \qquad \mathbf{q}_{t|t} \triangleq \mathbf{P}_{t|t}^{-1}\hat{\mathbf{x}}_{t|t}$$

that will be referred to as *information vectors*. Then, the recursive information filter of Table 1 can be derived (Battistelli et al., 2015), where $\mathbf{W}$ and $\mathbf{V}^i$, $i \in \mathcal{N}$, are given positive definite matrices. A typical choice for such matrices is to take $\mathbf{W}$ as an estimate of the inverse covariance of the process disturbance $\mathbf{w}_t$, and each $\mathbf{V}^i$ as an estimate of the inverse covariance of the measurement noise $\mathbf{v}_t^i$ affecting the ith sensor. Notice, however, that a specific choice of such matrices is immaterial for the subsequent developments.

The algorithm of Table 1 generalizes the *Information Kalman Filter* algorithm, corresponding to $\mathbf{f}(\mathbf{x}) = \mathbf{A}_t\mathbf{x}$ and $\mathbf{h}^i(\mathbf{x}) = \mathbf{C}_t^i\mathbf{x}$, to nonlinear systems (1) and/or sensors (2) via the Extended Kalman Filter paradigm of linearizing the state and measurement equations around the current estimate. With this respect, the following assumption is needed.

A1. The functions $\mathbf{f}$ and $\mathbf{h}^i$, $i \in \mathcal{S}$, are twice continuously differentiable on $\mathbb{R}^n$, where $n = \dim(\mathbf{x})$.

Notice that, in order to streamline the presentation, here and in the following it is supposed that the functions $\mathbf{f}$ and $\mathbf{h}^i$, $i \in \mathcal{S}$, are defined over the whole $\mathbb{R}^n$. However, all the results presented hereafter could be suitably modified to account for the case when the system trajectories are confined to a given set $\mathcal{X} \subset \mathbb{R}^n$.

3. Distributed extended Kalman Filter

The focus of this paper is on a family of DSE algorithms proposed in Battistelli et al. (2015) wherein each network node runs a local