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Distributions of multivariate equally-correlated fading employing selection combining

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A R T I C L E I N F O

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A B S T R A C T

Selection combiner output distributions of multivariate correlated fading have been very scarce in the literature. Distributions of multivariate for popular correlated fading environments employing selection combining such as Rayleigh, Rician and Nakagami-m have been obtained and thoroughly studied. However, selection combiner output distributions of multivariate correlated generalised-Rayleigh, central chi-square, non-central chi-square fading and performance of popular modulation schemes under correlated generalised-Rician and correlated generalised-Rayleigh fading have not been rigorously studied. In this paper, single-integral and infinite-summation selection combiner output distributions of multivariate correlated fading environments including generalised-Rayleigh, central chi-square, generalised-Rician and non-central chi-square are examined. Using the newly-derived distributions, average bit error rates of Binary Phase-Shift Keying, Differential Binary Phase-Shift Keying, $\frac{\pi}{4}$-Shift Differential Quadrature Phase Shift Keying, Minimum-Shift Keying and M-ary Frequency-Shift Keying under correlated and identically distributed (c.i.d.) generalised-Rician and c.i.d. generalised-Rayleigh are derived and plotted against common branch normalised SNR, diversity order, fading parameter, number of degrees of freedom and correlation coefficient. Simulation results exactly match the proposed findings which validate the new findings. Detailed discussions are given. Mathematical proofs are given in Appendices A–D.

1. Introduction

Correlated fading has been an important topic in wireless communications [11,18,20] from which statistical distributions of multivariate correlated fading environments have been obtained using selection combining (SC). The application of SC is mainly to combat fading severity and also it is the simplest combiner whose compact mathematical cumulative distribution function (cdf) expressions can be readily obtained. In the current open literature, without incorporating Millimetre Wave (mmWave) technologies, selection combiner output (SCO) cdfs of multivariate exponentially-correlated Rician fading have been specifically difficult to obtain and so far only the trivariate cdf is available [13] which currently is insufficient for massive-multiple-input-multiple-output (MIMO) Fifth-Generation (5G) networks,[1] suggesting that sophisticated multivariate cdfs are required for theoretical performance prediction of massive-MIMO 5G networks and beyond. This evidently shows that there are knowledge gaps to be filled in the current literature for correlated fading.

It should be reminded that the cdf of trivariate exponentially-correlated Rician fading has seven nested infinite summations which are computationally intensive and its corresponding integral form is not yet available in the open literature. It is noted that unsolved integrals such as single-integral cdfs via [11, (15), (21), (22)] can be efficiently computed using modern packages such as MATLAB. In contrast, the SCO cdfs of multivariate cor-

[1] The SCO exponentially-correlated Rician multivariate cdf for MIMO systems must be obtained for massive-MIMO 5G networks. However, this cdf is not yet available in the open literature. The SCO exponentially-correlated Nakagami-m multivariate cdf for single-input-multiple-output (SIMO) systems is available via [20, (6)] which is still insufficient for fronthaul and backhaul communications. It is noted that [20, (6)] is based on [9, (2.1)].

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related and identically distributed (c.i.d.) Rician and Nakagami-m fading are available in [5,11,18,20,22] in two forms: (i) single-integral, and (ii) infinite-summation from which the former is typically more computation efficient than the latter. This shows that (i) SCO cdfs of multivariate c.i.d. fading have been well-studied than their SCO exponentially-correlated multivariate counterparts and (ii) worst-case-scenario or upper-bound studies on performance indicators such as average bit error rates (ABER) under exponentially-correlated fading environments will likely be studied because of the difficulty in obtaining exact compact expressions. If exact and compact expressions are obtained, a powerful supercomputer will be required to compute infinite-summation cdf expressions for exponentially-correlated environments. In the last few years, with the rapid introduction of 5G networks employing massive-MIMO and mmWave technologies utilising a large number of amplify-powered and directional antennas, it is inevitable that antenna correlation is one of the challenges that must be theoretically and practically addressed by the research community, especially in ultra-dense 5G networks [3,15,16]. This places further burden on theoretical developments for cdfs of multivariate correlated fading, which are currently scattered with mistakes [22–25],[2] and have not yet been fully developed for sophisticated systems such as massive-MIMO and mmWave 5G.

Theoretical developments on cdfs of multivariate correlated fading started in the 1960's via the pioneer works of Miller [28] from which infinite-summation multivariate cdfs have been employed. Since the 1990's, the topic was revisited and advanced, notably, Karagiannidis and others obtained the cdf of multivariate exponentially-correlated Nakagami-m fading [20]. Distributions of trivariate correlated Nakagami-m fading have also been obtained in [1,29] under a generalised correlation model. Chen and Tellambura derived SCO distributions in both single-integral and infinite-summation forms of c.i.d. Rayleigh, Rician and Nakagami-m fading. Mallik [27] introduced a multi-integral multivariate cdf scheme, however, the computation becomes intensive as the cdf order is increased. SCO cdf of multivariate correlated generalised-Rician was studied in [24] from which it was mathematically shown that Rayleigh, Rician and Nakagami-m fading are the former's special cases. The cdf of multivariate correlated and non-identically distributed (c.n.i.d.) generalised-Rician fading was given in [5] but its SCO version was not derived. Even though these generalised fading environments are more advanced than the popular Rayleigh, Rician and Nakagami-m fading, they have not been thoroughly studied in the literature. In addition, they can be employed to obtain their corresponding SCO cdfs of multivariate c.i.d. central chi-square and non-central chi-square fading, forming a useful quartet of SCO distributions which can be mathematically generalised to obtain other popular SCO distributions. In 2012, Li and Cheng [26] approximated the Marcum Q-function for small arguments, then re-obtained [11, (22)] for the SCO cdf of multivariate c.n.i.d. Nakagami-m fading under an unequal-channel-gain scenario. Careful studies of [26] show that it is a special case of [18]. Even though efforts were spent in [2,11,18,26,30], the SCO cdfs of multivariate c.i.d. Nakagami-m fading are only valid for integer fading parameter m.

The mathematical relationships of these four distributions are clearly shown in Fig. 5 from which the non-central chi-square (with underlying non-zero-mean equal-variance Gaussian random variables (RVs)) and central chi-square (with underlying zero-mean equal-variance Gaussian RVs) RVs are equivalently the squares of generalised-Rician and generalised-Rayleigh RVs respectively. It should be noted that this quartet of fading environments go beyond the typical fading environments and it is important to obtain their SCO multivariate cdfs so that a systematic approach to correlated fading can be achieved. Performance of wireless systems for correlated fading environments has also been assessed in [7,8,19,21,36].

Having learnt theoretical developments for correlated fading, there are two major points to note. First, with respect to theoretical developments on SCO multivariate cdfs, it is clear that even though significant research has been performed, limited findings have been available for correlated generalised-Rayleigh and generalised-Rician fading which involve more than two Gaussian RVs. The cdf of multivariate c.n.i.d. generalised-Rician fading, not employing SC, has been derived in [5, (22)] which can mathematically be transformed into SCO cdf of multivariate c.i.d. generalised-Rician fading under $\lambda_k^2 = \rho$ where ρ is the channel correlation coefficient. It is undeniable that results on SCO cdfs of multivariate c.i.d. generalised-Rayleigh, central chi-square, non-central chi-square and generalised-Rician fading are scarce, leading to currently-limited performance studies for typical modulation schemes in these environments. Even though SCO cdfs of multivariate exponentially-correlated generalised-Rayleigh and central chi-square fading have been studied [20, (6)], cdfs of multivariate c.i.d. non-central chi-square and generalised-Rician fading however have not been available in the literature and they remain challenges to the research community.

Second, with respect to performance evaluation in the form of average bit error rate (ABER), the ABERs for Binary Phase Shift Keying (BPSK), Binary Frequency Shift Keying (BFSK), and M-ary Phase Shift Keying (MPSK) under c.i.d. Rayleigh fading were plotted in [11,21,35]. However, findings for ABERs of other modulation schemes under c.i.d. generalised-Rayleigh and generalised-Rician fading environments have also been very limited. Thus it has become clear that there are significant knowledge gaps in these two areas which should be addressed by the wireless research community. As the number of antennas is increased under massive-MIMO 5G systems, thoroughly understanding correlated fading is crucial to accurately analyse their performance. The SCO cdfs of multivariate c.i.d. generalised-Rayleigh and generalised-Rician fading aim to maximise the even n degrees of freedom, i.e. this idea is essentially the idea of massive-MIMO and mmWave 5G networks. Till now, performance of typical modulation schemes under correlated fading as a function of n has not yet been performed in the literature. This paper therefore aims to systematically and mathematically obtain SCO cdfs of multivariate c.i.d. generalised-Rayleigh, central chi-square and non-central chi-square fading which can be considered as generalised cases of Rayleigh, Rician, generalised-Rician and Nakagami-m fading. Incorporating the recent findings on SCO cdf of multivariate c.i.d. generalised-Rician fading [24], these cdfs are employed to assess performance of wireless networks under c.i.d. generalised fading which has not been thoroughly studied in the literature. The ABERs of five typical modulation schemes: BPSK, Differential Binary Phase Shift Keying (DBPSK), $\frac{\pi}{4}$-DQPSK ($\frac{\pi}{4}$-shifted Differential Quadrature Phase Shift Keying), Minimum Shift Keying (MSK) and M-ary Frequency Shift Keying (MFSK) are plotted against $\rho, \overline{\gamma}, L, K, n$, where L is the number of available fading branches, K is the fading parameter of the Rician environment and n is the number of underlying Gaussian RVs.

The contributions of this paper thus are: (i) To obtain new single-integral SCO cdf of multivariate c.i.d. generalised-Rayleigh, central chi-square and non-central chi-square fading for even n, (ii) To obtain new infinite-summation SCO cdf of multivariate c.i.d. generalised-Rayleigh, central chi-square and non-central chi-square fading for even n, findings for odd n will be reported in a separate publication, (iii) To verify the new SCO cdf against existing

[2] The current literature for correlated fading has been plagued with mathematical mistakes which (i) deter researchers from applying theoretical results into practice because of unsimplified hence complicated cdf expressions [10,12,13,25], and (ii) confuse new researchers to the field because of inconsistent and non-unified findings, leading to unmatched mathematical cross-verification.

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