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A quadratic model for the far-field direction of arrival estimation

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A B S T R A C T

In this paper, we address the problem of estimating the directions of arrival of far-field source signals. For that, we propose a new nonuniform linear array that allows the analyst to model the array data snapshot as a second-order polynomial phase signal. This permits the use of polynomial phase parameter estimation and time-frequency based estimation techniques, in addition to existing direction of arrival estimation methods. Furthermore, for the purpose of simplifying the analysis, we propose here an estimation technique based on the Wigner–Ville distribution kernel. Examples and Monte-Carlo simulations are presented to show the validity, effectiveness, and statistical efficiency of this technique.

1. Introduction

Parameter estimation theory has been of great interest to many researchers and scientists for decades. Its importance stems from the fact that it underpins the design of many engineering systems for information extraction [1,2]. Array processing can be considered as a natural spin-off of parameter estimation theory, whereby, relevant temporal as well as spatial parameters are estimated. Array processing techniques have the ability to combine data collected at several sensors in order to perform a particular estimation task [3].

The use of an array of sensors to extract information from a propagating wave (or signal) is of importance in a variety of applications such as radar, sonar, communications, and seismology [4–12]. The information of interest could be the content of the signal itself, the signal source location, or the reflection that produces it (e.g., radar, sonar) [2]. In general, the array geometry can be in different shapes and sizes, depending on the problem investigated; however, the linear is the most popular and widely used in real-life applications [13].

A linear array can be constructed by using sensors that are uniformly or nonuniformly spaced. In a uniform linear array (ULA), the sensors are located on a straight line with the same inter-element spacing. For nonuniform linear arrays (NLAs), sensors are usually placed at integer multiples of a unit distance [14]. NLAs strive to cover a large array aperture with a limited number of sensors. NLAs outperform ULAs when the same number of sensors is considered [13–15]. NLAs are also referred to as sparse arrays, which have surged over the last decade as part of area of sparse sensing. Sparse arrays include minimum redundant arrays, minimum hole arrays, co-prime arrays and nested arrays [16–18]. The aforementioned arrays attempt to place the sensors such as the coarray have larger aperture than the ULA counterpart. Other sparse configurations construct sparse arrays for stable DOA estimation and increased signal-to-noise ratio [19,20]. In general, the analysis for this type of arrays invites subspace and singular value decomposition techniques [21,22].

In this paper, we consider the problem of estimating the directions of arrival (DOA) of far-field source signals. Unlike the above array design criteria, we propose a new NLA geometry that allows modeling the array signal in a quadratic form similar to a chirp signal. The progressive inter-element spacing with sensor number along the array, which is associated to a chirp quadratic phase, makes the proposed array structure applicable to situations where enforcing the ULA configuration is difficult or challenged. Also, in addition to existing DOA estimation techniques, the proposed quadratic signal model permits the possibility to apply mature and well established techniques such as polynomial phase signal (PPS) estimation techniques [23–25] and time-frequency based estimation techniques [26,27]. Furthermore, for the purpose of simplifying the analysis, we propose here to use the kernel of the Wigner–Ville distribution (WVD) [28–30] to convert the problem of DOA estimation to that of a simple frequency estimation of a sinusoid (or sum of sinusoids) corrupted by interference and noise. In addition to its simplicity, we show that the proposed estimation is computationally and statistically efficient in the estimation of the DOA of arbitrary source signals.

The paper is organized as follows. In Section 2, we consider the single source case. In this section, we formulate the problem under investigation, present the new array geometry, propose an

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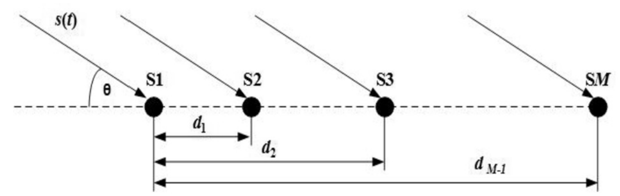

Fig. 1. A signal $s(t)$ impinging on a linear array from an angle θ.

estimation technique, and develop the Cramer–Rao lower bound (CRLB) for the new model. In Section 3, we extend the analysis to the multiple sources case. Various examples are presented in both cases to show the validity and accuracy of the proposed method. Section 4 concludes the paper.

2. Single source

2.1. Proposed nonlinear array configuration & model

The problem set up, shown in Fig. 1, represents a signal $s(t)$ propagating at a velocity $v(t)$ and impinging on a linear array at an angle θ. The array consists of M sensors, S_i, $i = 1, \ldots, M$, all assumed to be omnidirectional and located on the same elevation.

If we assume the velocity $v(t)$ to be constant during the time the wave crosses the array [8], and consider sensor S_1 to be the reference sensor, then, the sensor outputs can be expressed as

$$\mathbf{x}(t) = \mathbf{v}(\theta)s(t) + \mathbf{w}(t), \tag{1}$$

where

$$\mathbf{x}(t) = [x_1(t)\ \ x_2(t)\ \ \ldots\ x_M(t)]^T \tag{2}$$

$$\mathbf{v}(\theta) = \left[e^{j2\pi F_0 d_0 \cos\theta/v}\ \ e^{j2\pi F_0 d_1 \cos\theta/v}\ \ \ldots\ e^{j2\pi F_0 d_{M-1} \cos\theta/v}\right]^T \tag{3}$$

$$\mathbf{w}(t) = [w_1(t)\ \ w_2(t)\ \ \ldots\ w_M(t)]^T. \tag{4}$$

In these expressions, $\mathbf{v}(\theta)$ represents the data steering vector, $\mathbf{w}(t)$ is a zero-mean Gaussian noise with covariance matrix $C_{\mathbf{w}} = \sigma^2\mathbf{I}$, $s(t)$ is the source signal, and $d_{i-1}, i = 1, 2, \ldots, M$ (with $d_0 = 0$), is the distance between sensor S_i and the reference one. If we select these distances such that

$$d_1 = d, \quad d_2 = 4d, \quad d_3 = 9d, \quad \ldots \quad d_{M-1} = (M-1)^2 d, \tag{5}$$

where d is a pre-defined arbitrary distance, then, the steering vector becomes

$$\mathbf{v}(a) = \left[e^{j2\pi\cdot a\cdot(0)^2}\ \ e^{j2\pi\cdot a\cdot(1)^2}\ \ e^{j2\pi\cdot a\cdot(2)^2}\ \ e^{j2\pi\cdot a\cdot(3)^2}\ \ldots \right.$$
$$\left. e^{j2\pi\cdot a\cdot(M-1)^2}\right]^T \tag{6}$$
$$= \left[e^{j2\pi a m^2}\right], \quad m = 0, 1, \ldots, M-1,$$

where $a = F_0 d\cos\theta/v$. We observe that $\mathbf{v}(a)$ is an M-sample, unit-amplitude, second-order polynomial phase signal, uniformly sampled at a sampling period equal to unity. Thus, for a fixed snapshot t, the column vector $\mathbf{x}(t)$ is just a noisy linear FM signal and, consequently, estimating θ is equivalent to estimating the phase coefficient of $\mathbf{x}(t)$.

2.2. Proposed estimation method

The WVD is defined as [26]

$$W(t', f) = \int_{-\infty}^{+\infty} z(t' + \frac{\tau}{2}) \cdot z^*(t' - \frac{\tau}{2})\, e^{-j2\pi f\tau} d\tau$$
$$= \int_{-\infty}^{+\infty} K_z(t', \tau)\, e^{-j2\pi f\tau} d\tau, \tag{7}$$

where $z(t')$ is the analytic signal associated with the signal under consideration, and $K_z(t', \tau)$ is called the WVD kernel.

In our present case, for a fixed snapshot t, the noiseless quantity of the data model in (1) can be written in the following form

$$z(t') = s(t)e^{j2\pi a t'^2}. \tag{8}$$

Thus, if we apply the WVD kernel on this quantity, we obtain

$$K(t', \tau) = z(t' + \frac{\tau}{2}) \cdot z^*(t' - \frac{\tau}{2}) = |s(t)|^2 e^{j2\pi(2at')\tau}. \tag{9}$$

The above result indicates that, for a fixed time instant t', the WVD kernel, as a function of the variable τ, is just a complex sinusoid with frequency equal to $2at'$. Consequently, its Fourier transform can be used as an estimator of the frequency $2at'$, or equivalently the parameter a. Also, we know that the best estimation performance occurs when the implementation window length for τ is the largest possible [31]. In our current model, this happens at $t' = (M-1)/2$ (with M selected to be an odd integer). Therefore, we use this particular time instant to estimate the parameter a. That is, we compute the FFT of the WVD kernel for $t'_0 = (M-1)/2$, i.e.,

$$W_z(t'_0, f) = \underset{\tau \to f}{\mathcal{F}} \left[K_z(t'_0, \tau)\right]$$
$$= \underset{\tau \to f}{\mathcal{F}} \left[z(t'_0 + \frac{\tau}{2}) \cdot z^*(t'_0 - \frac{\tau}{2})\right], \tag{10}$$

and search for the maximizer of this spectrum to estimate the desirable parameter a.

It is worth noting that by an alternative choice of the distances in (5), we can model the array signal by a higher-order PPS, for which we apply the kernel of the corresponding optimal polynomial WVD [29] to convert it into a complex sinusoid. An advantage of this new distance distribution is the reduced number of sensors needed to cover the same array aperture, but at the price of a slight estimation performance degradation [31]. This point is out of the scope of this presentation and will be considered in a future work.

2.3. Example

In this example, we consider a seismic wave, $s(t)$, impinging on a linear array consisting of $M = 21$ sensors. These sensors, with respect to the first one, are located according to (5) with $d = 10$ [m]. The seismic wave at the first sensor (selected as the reference sensor) is expressed as

$$s(n) = \exp\left[j2\pi(2nT_s + 0.6(nT_s)^3)\right],$$
$$n = -500, -499, \ldots, 500,$$

where T_s, the sampling period, is set equal to 1/100 [sec]. The WVD of this signal, computed according to Equation (7), is displayed in Fig. 2.

The wave is assumed to be propagating at an apparent velocity $v = 1000$ m/s, with an angle $\theta = 85°$, and frequency $F_0 = 5$ [Hz].

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