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State estimation for invariant systems on Lie groups with delayed output measurements[☆]

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ABSTRACT

This paper proposes a state estimation methodology for invariant systems on Lie groups where outputs of the system are measured with delay. The proposed method is based on cascading an observer and a predictor. The observer uses delayed measurements and provides estimates of delayed states. The predictor uses those estimates together with the current inputs of the system to compensate for the delay and to provide a prediction of the current state of the system. We consider three classes of left-invariant, right-invariant, and mixed-invariant systems and propose predictors tailored to each class. The key contribution of the paper is to exploit the underlying symmetries of systems to design novel predictors that are computationally simple and generic, in the sense that they can be combined with any stable observer or filter. We provide a rigorous stability analysis demonstrating that the prediction of the current state converges to the current system trajectory if the observer state converges to the delayed system trajectory. The good performance of the proposed approach is demonstrated using a sophisticated Software-In-The-Loop simulator indicating the robustness of the observer–predictor methodology even when large measurement delays are present.

1. Introduction

Recent work on applications in navigation and control of mechanical systems has demonstrated advantages of modeling such systems as kinematics on appropriate Lie groups (Bullo, 2005; Jurdjevic, 1997; Markley & Crassidis, 2014). In particular, state estimation for systems on Lie groups has been motivated by real world applications such as; attitude estimation on $\mathsf{SO}(3)$ (Bonnabel, Martin, & Rouchon, 2008; Grip, Fossen, & Johansen, 2015; Izadi & Sanyal, 2014; Khosravian & Namvar, 2010; Mahony, Hamel, & Pflimlin, 2008; Markley & Crassidis, 2014; Roberts & Tayebi, 2011; Sanyal, Lee, Leok, & McClamroch, 2008; Vasconcelos, Silvestre, & Oliveira, 2008a; Zamani, Trumpf, & Mahony, 2013), pose estimation on $\mathsf{SE}(3)$ (Hua, Zamani, Trumpf, Mahony, & Hamel, 2011; Rehbinder & Ghosh, 2003; Rodrigues, Crasta, Aguiar, & Leite, 2010; Vasconcelos, Cunha, Silvestre, & Oliveira, 2010), homography estimation on $\mathsf{SL}(3)$ (Hamel, Mahony, Trumpf, Morin, & Hua, 2011), and motion estimation of chained systems on nilpotent Lie groups (Leonard & Krishnaprasad, 1995) (e.g. front-wheel drive cars or kinematic cars with k trailers).

State observers rely on fusing the measurements of inputs and outputs of systems to estimate their internal states. For mechanical systems, output measurements usually provide low frequency information about the state of the system while input measurements provide high frequency information about the rate of change of the state. For invariant systems on Lie groups, systematic observer design methodologies have been proposed that lead to strong stability and robustness properties of the resulting observers. Specifically, Bonnabel et al. (2008) propose a symmetry-preserving observer that utilizes the invariance of the system and the moving frame method, leading to local

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convergence properties of the observers. The authors propose methods in Khosravian, Trumpf, Mahony, and Lageman (2015), Lageman, Trumpf, and Mahony (2010) and Mahony, Trumpf, and Hamel (2013) to achieve almost globally convergent observers using globally well defined errors and error dynamics. Other work in this area is the consideration of multiple output maps formulated by actions of the Lie group on homogeneous output spaces and also adaptive estimation of an unknown input bias (Barrau & Bonnabel, 2014; Bonnabel et al., 2008; Khosravian, Trumpf, Mahony, & Lageman, 2015; Mahony et al., 2013). All of the above mentioned observer design methodologies require delay free measurement of the current outputs and inputs of the system. In many practical scenarios, however, measurement of the outputs of the system is inherently delayed (comparing to the ideal outputs of the system) while the inputs are measured without significant delays. For example, in the velocity aided attitude estimation scenario, measurements of linear velocity (output) provided by commercial GPS units are usually delayed with respect to the actual velocity of the vehicle. The delay can be several hundred milliseconds (and even up to half a second) due to various environmental effects and in-sensor processing delays (Kingston & Beard, 2004). In contrast, measurements of the vehicle's angular velocity and linear acceleration provided by an onboard IMU are almost instantaneous. Another example is attitude estimation for satellites using star trackers and gyros. The image processing inside a star-tracker sensor can cause significant delays in the order of tens of milliseconds while gyroscope measurements on the satellite are obtained without significant delays. Similar delay problems occur in attitude estimation for aerial robots when vision based sensors compute landmarks. Even in instrumented indoor flight environments, the attitude data from devices such as VICON or OptiTrack are delayed by the communication channel from these sensors to the onboard attitude estimation system of the vehicle. Finally, heavy filtering of noisy sensor measurements before data fusion can introduce significant delays in the filtered data. This is particularly important, for example, for obtaining air velocity measurements from noisy air pressure readings (Riseborough, 2015). It is well understood that measurement delays can negatively affect the stability and robustness of observers or filters and degrade their performance (Aggoune, Boutayeb, & Darouach, 1999; Ahmed-Ali, Karafyllis, & Lamnabhi-Lagarrigue, 2013; Battilotti, 2014; Cacace, Germani, & Manes, 2010; Germani, Manes, & Pepe, 2002; Karafyllis & Kravaris, 2009; Kazantzis & Wright, 2005; Van Assche, Ahmed-Ali, Hann, & Lamnabhi-Lagarrigue, 2011).

The classical approach to tackle sensor delay in estimation problem is to take an estimator that has the desired performance for delay free measurements, and modify its innovation term such that it compares each delayed measurement with its corresponding backward time-shifted estimate. If the delay-free estimator has a Lyapunov stability proof, the stability analysis for the modified estimator is approached by using Lyapunov–Krasovskii or Lyapunov–Razumikhin functions (Aggoune et al., 1999; Ahmed-Ali, Cherrier, & Lamnabhi-Lagarrigue, 2012; Bahrami & Namvar, 2014; Cacace et al., 2010; Van Assche et al., 2011). Although commonly used in practice, such modified estimators require complicated stability analyses and careful and conservative gain tuning which may lead to poor transient responses. Combined observer–predictor design is an alternative method that has received recent interests, see e.g. Ahmed-Ali et al. (2013), Battilotti (2014), Germani et al. (2002), Karafyllis and Kravaris (2009), Kazantzis and Wright (2005) and Khosravian, Trumpf, and Mahony (2015) and the references therein. These methods take observers that use delayed measurements to provide estimates of the delayed state and combine them with appropriate predictors to compensate for the delay. An observer–predictor combination is proposed in Hansen, Fossen, and Johansen (2015) for attitude, velocity, and position estimation of flying vehicles and its good performance is verified in practice. The predictor of Hansen et al. (2015) (which is called a fast simulator in the context of that paper) relies on buffering the IMU measurements and employing a nonlinear observer to approximate the velocity and position kinematics by a double integrator system. This approximation enables forward integrating the buffered IMU data each time a new GPS measurement is received to obtain predictions of the current position and velocity. The authors of this paper have recently proposed a cascade observer–predictor methodology to handle sensor delay in the particular case of attitude estimation on the Lie group $\mathrm{SO}(3)$ (Khosravian, Trumpf, Mahony, & Hamel, 2014). Exponential convergence of the attitude estimate to its current value was shown using the specific algebraic structure of the Lie group $\mathrm{SO}(3)$. To the authors' knowledge, there is currently no state estimation methodology (with stability proof) for systems on general Lie groups when output measurements are affected by delay.

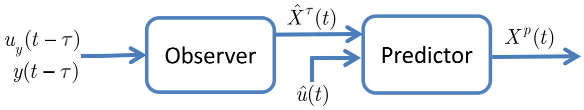

Fig. 1. Proposed observer–predictor methodology.

In this paper, we tackle the problem of state estimation for systems on general matrix Lie groups when measurements of system outputs are delayed. We propose an observer–predictor methodology for three classes of systems on Lie groups; left-invariant, right-invariant, and mixed-invariant. Our proposed methodology employs observers that use the delayed output measurements and estimate the delayed state of the system. We propose dynamic predictors that use the delayed estimates from the observers together with the current estimates of the inputs in order to predict the current state of the system (see Fig. 1). The key contribution of the paper is effective use of the underlying symmetry of systems in order to design the predictors such that the observer–predictor pairs are co-stable meaning that the observer–predictor combination provides (asymptotically/exponentially) stable estimates of the current state if the observer itself provides (asymptotically/exponentially) stable estimates of the delayed state. It turns out that the convergence rate of the combined predictor–observer depends only on the convergence rate of the observer independent of the magnitude of the delay (in contrast to what is common for the predictors for general systems on $\mathbb{R}^n$ Ahmed-Ali et al., 2013; Battilotti, 2014; Germani et al., 2002; Karafyllis & Kravaris, 2009; Kazantzis & Wright, 2005; Khosravian et al., 2015). We ensure separation of the observer design problem from the predictor design problem which gives us the freedom of employing any stable observer or filter without affecting the proposed co-stability analysis. An advantage of this approach is that the gain tuning process of the observer is independent of the magnitude of the delay, yielding robust performance of the observer–predictor pairs even with large delays. Rather than assuming the availability of current input measurements, we consider a more general case where only estimates of the current inputs of the system are available. This generalization is particularly useful when the input measurements have unknown biases and scaling factors that are adaptively estimated at the observer stage. The proposed predictors are recursive and hence computationally cheap making them ideal for embedded implementation in real-world applications. As an example, we consider the velocity aided attitude estimation problem and we provide realistic numerical simulations using a sophisticated Software-In-The-Loop (SITL) system designed for ArduPilot/APM, an open-source autopilot system that is widely used among the UAV enthusiasts community (ArduPilot/APM, 2014; ArduPilot Project, 2015; CanberraUAV, 2015; MAVProxy,