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Global leader-following consensus of a group of general linear systems using bounded controls[☆]

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ABSTRACT

This paper studies the global leader-following consensus problem for a multi-agent system with bounded controls. The follower agents and the leader agent are all described by a general linear system. Both a bounded state feedback control law and a bounded output feedback control law are constructed for each follower agent in the group. The feedback law for each input of an agent uses a multi-hop relay protocol, in which the agent obtains the information of other agents through multi-hop paths in the communication network. The number of hops each agent uses to obtain its information about other agents for an input is less than or equal to the sum of the number of eigenvalues at the origin and the number of pairs of non-zero imaginary eigenvalues of the sub-system corresponding to the input, and the feedback gains are constructed from the adjacency matrix of the communication network. It is shown that global leader-following consensus is achieved under these feedback control laws when the communication topology among follower agents is a strongly connected and detailed balanced directed graph and the leader is a neighbor of at least one follower.

1. Introduction

As a central problem in coordinated control of multi-agent systems, consensus entails the states of all agents in the system to converge to an agreement state through the use of local information by each agent. The consensus problem has drawn vast attention in recent years. This was in part due to the superiority of a multi-agent system over individual systems performing solo tasks, and in part due to its many applications, such as unmanned air vehicles, autonomous underwater vehicles, distributed sensor networks and mobile robots (Alighanbari & How, 2005; Chen, Lu, Yu, & Hill, 2013; Cook & Hu, 2010; Cortés & Bullo, 2005; Klein, Bettale, Triplett, & Morgansen, 2008).

In the early literature on coordinated control of multi-agent systems, the dynamics of an agent is often simplified to the kinematics of a single-integrator (Jadbabaie, Lin, & Morse, 2003; Olfati-Saber & Murray, 2004; Ren & Beard, 2005) or the dynamics of a double-integrator (Ren, 2008; Ren, Beard, & Atkins, 2005). Various aspects of the consensus problem have been studied for single-integrator or double-integrator agents. For example, multi-hop relay protocols were proposed and shown to achieve fast consensus seeking (Jin & Murray, 2006). The dynamics of most practical agents are however much more complex than those of a single-integrator or a double-integrator. Consequently, many of the early results cannot be directly used in practical applications, which has motivated the study of coordinated control of more general multi-agent systems. It is proven in Ren, Moore, and Chen (2006) that a group of networked agents described by a chain of multiple integrators can achieve global leaderless consensus when the number of zero eigenvalue of a certain matrix constructed from the Laplacian matrix associated with the communication topology is the same as the number of the integrators in an agent, and the remaining eigenvalues all have negative real parts. On the other hand, the consensus problem for a multi-agent system whose agents are described by a general higher order linear system is considered in He and Cao (2011), Seo, Shim, and Back (2009) and Yu, Chen, Ren, Kurths, and Zheng (2011). In particular, it is proven in Seo et al. (2009) that a group of N networked agents

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can achieve consensus if $N-1$ systems constructed in a special form can be simultaneously stabilized by a stable compensator, which is constructed with the low gain feedback design technique (Lin, 1999). Coordinated control of more general agents has also been studied in the contexts of, for example, output regulation (Su, Hong, & Huang, 2013; Su & Huang, 2012; Yang, Stoorvogel, Grip, & Saberi, 2014), nonlinear agent dynamics (Liu & Jiang, 2016), and heterogeneous agent dynamics (Ding, 2013; Yang, Saberi, Stoorvogel, & Grip, 2014).

The ubiquity of actuator saturation in control systems has also motivated the study of the consensus problem with bounded controls. Global consensus in the presence of actuator saturation has only been studied for agents with simple dynamics. Global leaderless consensus was considered in Li, Xiang, and Wei (2011), where the agents are represented by single integrator systems and the communication topology among agents is assumed to contain a directed spanning tree. Global leader-following consensus for agents that are represented by double integrator dynamics or general higher order but neutrally stable linear systems was studied in Meng, Zhao, and Lin (2013) and Yang, Meng, Dimarogonas, and Johansson (2014). In particular, for double integrator agents, it was established that global leader-following consensus can be achieved by linear local feedback laws over a fixed communication topology or by non-linear local feedback laws over a switching communication topology. For neutrally stable agents, it is shown that global leader-following consensus can be achieved by linear local feedback laws over either a fixed communication topology or a switching communication topology. On the other hand, semi-global leader-following consensus was achieved in Su, Chen, Lam, and Lin (2013) for general higher order systems whose open loop poles are all in the closed left-half plane by using the low gain feedback design technique (Lin, 1999).

In contrast with the limited number of results on multi-agent consensus with bounded controls, global stabilization of individual linear systems with bounded controls has been systematically studied. It has been established (Sussmann, Sontag, & Yang, 1994; Sussmann & Yang, 1991) that a linear system subject to control input saturation can be globally asymptotically stabilized only when it is asymptotically null controllable with bounded controls (ANCBC), that is, it is stabilizable and all its open-loop poles are located in the closed left-half plane, and even for such systems, linear feedback is in general not capable of achieving global asymptotic stabilization except for special classes of systems such as double integrators and neutrally stable systems. Nonlinear feedback laws of nested saturation type and in the form of weighted sum of saturated linear feedbacks were proposed in Sussmann et al. (1994) and Teel (1992) to achieve global asymptotic stabilization of such ANCBC systems.

In this paper, we consider the problem global leader-following consensus for a group of networked agents using bounded controls. The dynamics of each agent is represented by a general linear system. For each follower agent, we construct both a bounded state feedback control law and a bounded output feedback control law. The feedback law for each input of an agent uses a multi-hop relay protocol, in which the agent obtains the information of other agents through multi-hop paths in the communication network. The number of hops each agent uses to obtain its information about other agents for an input is less than or equal to the sum of the number of eigenvalues at the origin and the number of pairs of non-zero imaginary eigenvalues of the sub-system corresponding to the input, and the feedback gains are constructed from the adjacency matrix of the communication network. We show that global leader-following consensus is achieved under these feedback control laws when the communication topology among the follower agents is a strongly connected and detailed balanced directed graph and the leader is a neighbor of at least one follower.

An outline of this paper is as follows. Section 2 recalls some basic definitions and notations in graph theory. Section 3 contains problem statement. Section 4 focuses on the case that the follower agents have a single input. Section 5 extends the results in Section 4 to the multiple input case. Section 6 concludes the paper.

2. Graph theory

A directed graph is denoted as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_N\}$ is a finite, nonempty set of nodes (each node denotes a follower agent) and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is a set of edges (each edge denotes an ordered pair of nodes). An edge (ν_i, ν_j) in a directed graph denotes that agent j has access to the information of agent i. A directed path in a directed graph is a sequence of edges of the form $(\nu_{i1}, \nu_{i2}), (\nu_{i2}, \nu_{i3}), \ldots$. A directed path $(\nu_i, \nu_{i1}), (\nu_{i1}, \nu_{i2}), \ldots, (\nu_{ik-1}, \nu_j)$ between ν_i and ν_j is called a k-hop, and ν_i is called a kth neighbor of ν_j. A directed graph is strongly connected if there exists a directed path between any pair of distinct nodes.

Let $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ be the adjacency matrix associated with $\mathcal{G}$, where $a_{ij} > 0$ if $(\nu_j, \nu_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Here we assume that $a_{ii} = 0$ for all $i = 1, 2, \ldots, N$. Let $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ be the Laplacian matrix associated with $\mathcal{A}$, where $l_{ii} = \sum_{i=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$. A directed graph is detailed balanced if there exist some real numbers $\upsilon_i > 0,\ i = 1, 2, \ldots, N$, such that $\upsilon_i a_{ij} = \upsilon_j a_{ji}$, for all $i = 1, 2, \ldots, N$ (Jiang & Wang, 2009).

Besides the N follower agents, the leader agent is labeled as ν_0. The communication between follower agent i and the leader agent is denoted as a_{i0}, where $a_{i0} > 0$ if agent i has access to the information of the leader agent and $a_{i0} = 0$ otherwise. The communication topology $\mathcal{G}$ we consider in this paper satisfies the following assumption.

Assumption 1. The directed graph $\mathcal{G}$ is strongly connected and detailed balanced and $a_{i0} > 0$ for at least one $i,\ i = 1, 2, \ldots, N$.

Denote $M = \mathcal{L} + \mathrm{diag}\{a_{10}, a_{20}, \ldots, a_{N0}\}$. Let $\upsilon = [\upsilon_1,\ \upsilon_2, \ldots, \upsilon_N]^{\mathrm{T}}$, $\mathrm{diag}\{\upsilon\} = \mathrm{diag}\{\upsilon_1, \upsilon_2, \ldots, \upsilon_N\}$, $\upsilon_{\min} = \min\{\upsilon_1, \upsilon_2, \ldots, \upsilon_N\}$ and $\upsilon_{\max} = \max\{\upsilon_1, \upsilon_2, \ldots, \upsilon_N\}$.

Lemma 1. *Under Assumption 1, all eigenvalues of M are on the open right-half plane, and the matrix $\mathrm{diag}\{\upsilon\}M + M^{\mathrm{T}}\mathrm{diag}\{\upsilon\} = 2M^{\mathrm{T}}\mathrm{diag}\{\upsilon\}$ is positive definite.*

In the above lemma, the fact that all eigenvalues of M are on the open right-half plane is established in Ren and Cao (2011) and the fact that $\mathrm{diag}\{\upsilon\}M + M^{\mathrm{T}}\mathrm{diag}\{\upsilon\} = 2M^{\mathrm{T}}\mathrm{diag}\{\upsilon\}$ is positive definite can be established based on the analysis given in the proof of Lemma 4 in Hu and Hong (2007).

3. Problem statement

Consider a group of N networked follower agents, each described by a linear system,

$$\dot{x}_i = Ax_i + Bu_i, \qquad y_i = Cx_i, \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^{\mathrm{T}} \in \mathbb{R}^n$, $u_i = [u_{i1}, u_{i2}, \ldots, u_{im}]^{\mathrm{T}} \in \mathbb{R}^m$ and $y_i = [y_{i1}, y_{i2}, \ldots, y_{ir}]^{\mathrm{T}} \in \mathbb{R}^r$ are respectively the states, control inputs and outputs of agent i. Let the leader be also described by a linear system,

$$\dot{x}_0 = Ax_0, \qquad y_0 = Cx_0, \tag{2}$$

where $x_0 = [x_{01}, x_{02}, \ldots, x_{0n}]^{\mathrm{T}} \in \mathbb{R}^n$ and $y_0 = [y_{01},\ y_{02}, \ldots, y_{0r}]^{\mathrm{T}} \in \mathbb{R}^r$ are respectively the states and outputs of the leader agent.

Assumption 2. All eigenvalues of A are on the closed left-half plane and the pair (A, B) is stabilizable.

Assumption 3. The pair (A, C) is detectable.

The global leader-following consensus problems we are to study are stated as follows.