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On convergence and performance certification of a continuous-time economic model predictive control scheme with time-varying performance index[☆]

Andrea Alessandretti[a,b], A. Pedro Aguiar[c,b], Colin N. Jones[a]

[a] École Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland
[b] Institute for Systems and Robotics (ISR), Instituto Superior Tecnico (IST), Lisbon, Portugal
[c] Faculty of Engineering, University of Porto (FEUP), Porto, Portugal

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ABSTRACT

This paper addresses the design of convergence and performance certified sampled-data model predictive control (MPC) laws with a time-dependent economic performance index. More precisely, using a dissipativity property of the system, we provide a set of sufficient conditions that guarantee convergence of the closed-loop state trajectory to a, possibly time-varying, average economically optimal state trajectory. Moreover, the average performance of the closed-loop system is shown to be no worse than the one obtained by operating the system at the average economically optimal state trajectory. Constructive methods to design an appropriate terminal set and terminal cost that satisfy the proposed sufficient conditions are presented and illustrated with numerical examples.

1. Introduction

A key factor that allowed Model Predictive Control (MPC) schemes to gain an important role in many practical applications lies in their ability to explicitly optimize over a desired performance index, while satisfying state and input constraints. Ideally one would wish to minimize the infinite horizon time integral of a predefined stage cost evaluated along the constrained state and input trajectories. Although, since this problem is generally intractable, MPC is often used to approximate the infinite horizon optimization with a sequence of easier finite horizon optimizations. Depending on the meaning of the chosen stage cost, we can distinguish between classic MPC schemes, also termed Tracking MPC, and Economic MPC schemes.

The main objective of a Tracking MPC controller is to steer the state of a system to a desired steady-state or state trajectory. Toward this goal, the stage cost is properly designed to penalize the distance from the current state to the desired one. This approach has been widely investigated in the literature and, depending on the methodology used to approximate the infinite horizon optimal control problem, we can identify two families of schemes: the ones that utilize the so-called terminal sets and terminal costs and the terminal-set-free schemes. For the first family we refer the reader to, e.g., Mayne, Rawlings, Rao, and Scokaert (2000), Morari and Lee (1999) and Rawlings and Mayne (2009) for the discrete-time case and Chen and Allgöwer (1998), Fontes (2001) and Jadbabaie, Yu, and Hauser (2001) for the continuous-time. Similarly, for the second family, we refer to Grüne and Pannek (2011) and Reble and Allgöwer (2012) for the discrete-time and continuous-time case, respectively.

In recent years, a growing attention has been devoted to Economic MPC schemes, where the main objective is the minimization of a performance index associated with a given economic stage cost. Here, the term economic is utilized to emphasize that such function is not designed to penalize the distance of the current state to the desired one, but it rather represents an index of interest to be minimized, e.g., an economic index. Such generality gives rise to many interesting applications. Similarly to the Tracking MPC case, also in this case the infinite horizon problem can

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E-mail addresses: andrea.alessandretti@epfl.ch (A. Alessandretti), pedro.aguiar@fe.up.pt (A.P. Aguiar), colin.jones@epfl.ch (C.N. Jones).

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be approximated using a terminal set and a terminal cost or, in a terminal-set-free approach, by properly selecting a long horizon. In the works Diehl, Amrit, and Rawlings (2011), Amrit, Rawlings, and Angeli (2011) and Angeli, Amrit, and Rawlings (2012), the economically optimal steady-state is precomputed and used to constrain the terminal state of the prediction with a terminal equality (Amrit et al., 2011; Diehl et al., 2011) or inequality (Angeli et al., 2012) constraint. Sufficient conditions for stability of the economically optimal steady-state were initially provided in Diehl et al. (2011), and later generalized by Amrit et al. (2011) and Angeli et al. (2012) using a dissipativity property of the system. In Grüne (2013), the author provides conditions on the horizon length and stage cost for closed-loop convergence to an arbitrarily small neighborhood of the optimal steady-state. Stability and recursive feasibility properties of the Economic MPC controller for the case of changes in the economic stage cost are addressed in Ferramosca, Rawlings, Limon, and Camacho (2010).

Although the use of dissipativity properties of a system provides an elegant approach for analysis and design of Economic MPC schemes, alternative methods have been proposed. The works Fagiano and Teel (2013)and Müller, Angeli, and Allgöwer (2013) employ a generalized terminal set, consisting of all the feasible steady-states, and a constraint on the increase of the terminal cost to guarantee convergence to *a* steady-state. In Heidarinejad, Liu, and Christofides (2012), the authors discuss the use of a given control Lyapunov function (CLF), defined over the whole desired region of attraction, to design a dual mode scheme. Here, initially the economic optimization is performed while enforcing the state within a level set of the CLF, and then, at a given point triggered at an arbitrary time, a Lyapunov decrease is enforced driving the state to the desired equilibrium point. In the works Maree and Imsland (2014), Alessandretti, Aguiar, and Jones (2013) and Alessandretti, Aguiar, and Jones (2015) a combination of a classic stage cost and an economic stage cost is adopted. In this case, assumptions on the magnitude of the economic stage cost are introduced in order to preserve stability (Maree & Imsland, 2014), convergence (Alessandretti et al., 2013), and ultimate boundedness (Alessandretti et al., 2015) guarantees.

To the best of our knowledge, almost all the research devoted to Economic MPC is developed for discrete-time systems, even if the applications, addressed via discretization, are often stemming from continuous-time dynamical models. One exception is the work of Heidarinejad et al. (2012) that has the restriction of requiring a CLF defined over the whole desired region of attraction. Moreover, while a vast effort was dedicated to the analysis and synthesis of MPC schemes leading to the convergence of the closed-loop state trajectory to a steady-state, few results address the interesting scenario of time-varying stage cost and convergence to potentially time-varying state trajectories. See, for instance, the works Angeli et al. (2012), Zanon, Gros, and Diehl (2013) and Limon, Pereira, Muñoz De La Peña, Alamo, and Grosso (2014), for the case of closed-loop convergence to periodic orbits in the discrete-time setting. Furthermore, only Amrit et al. (2011) and Müller, Angeli, Allgöwer, Amrit, and Rawlings (2014) address the relaxation from terminal equality to terminal inequality in the dissipativity-based approach.

Inspired by these observations, this work addresses the design of a continuous-time Economic MPC with terminal constraint for time-varying continuous-time systems with time-dependent stage cost and with convergence and performance guarantees. The main dissipativity-based results introduced in the time-invariant discrete-time case are extended to the time-varying setting for continuous-time systems. Moreover, an average performance analysis of the MPC controller is performed and constructive methods for the computation of a suitable terminal set and a terminal cost are presented. We build on our previous result in Alessandretti, Aguiar, and Jones (2014), extending it to the case of time-varying systems, constraints, performance indexes, and dissipative functions. The differentiability assumption on dissipativity function and terminal cost is dropped. Moreover, convergence to a time-varying state trajectory, rather than a steady-state, and the associated extension on the performance analysis is presented.

The remainder of this paper is organized as follows: Section 2 contains the problem definition. Sections 3 and 4, similarly to Amrit et al. (2011) but for the continuous-time case, address the convergence properties and the performance analysis of the closed-loop system, respectively. Design methodologies for a suitable terminal set and terminal cost are presented in Section 5, followed by Section 6 with some numerical examples. Section 7 closes the paper with some conclusions. All the proofs are reported in Appendix.

Notation. For a generic continuous-time trajectory $\boldsymbol{x}$, the term $\boldsymbol{x}([t_1, t_2])$ denotes the trajectory considered in the time interval $[t_1, t_2]$ and $x(t)$ the trajectory evaluated at a specific time t. The notation $x(\tau; t, z)$ is used whenever we want to make explicit the dependence of $x(\tau)$ on the optimization problem parameters t and z. For a generic scalar function $g : \mathbb{R}_{\geq t_0} \to \mathbb{R}$ and time t_0 we denote by $\mathrm{Av}[g(t), t_0]$ the set

$$\mathrm{Av}[g(t), t_0] := \left[\liminf_{\delta \to +\infty} \frac{\int_{t_0}^{t_0+\delta} g(t)dt}{\delta}, \; \limsup_{\delta \to +\infty} \frac{\int_{t_0}^{t_0+\delta} g(t)dt}{\delta} \right]$$

where if the limit exists, then we have $\mathrm{Av}[g(t), t_0] = \{\lim_{\delta \to +\infty} \frac{1}{\delta} \int_{t_0}^{t_0+\delta} g(t)dt\}$. Moreover, for a generic function $g : \mathbb{R}^n \to \mathbb{R}$, with n being a positive integer, the terms $g_x(\hat{x})$ and $g_{xx}(\hat{x})$ denote the Jacobian and the Hessian, respectively, of $g(\cdot)$ with respect to the vector $x \in \mathbb{R}^n$ evaluated at $\hat{x} \in \mathbb{R}^n$. For a given matrix A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and the maximum real valued eigenvalue of A. The notation $A \succ 0$ is used to denote that A is a positive definite matrix. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to belong to class-$\mathcal{K}_\infty$, or to be a class-$\mathcal{K}_\infty$ function, if it is zero at zero, strictly increasing and radially unbounded, i.e., $\alpha(x) \to \infty$ as $x \to \infty$. For a given function $g : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ and scalars r and t we denote by $\mathcal{L}(t; g, r)$ the r-sub-level-set $\mathcal{L}(t; g, r) := \{x : g(t, x) \leq r\}$ parametrized with t. For a generic set $\mathcal{A} \subseteq \mathbb{R}^n$ we denote by $\mathrm{int}\,\mathcal{A}$ the interior of $\mathcal{A}$. The generic closed ball of radius r is denoted by $\mathcal{B}(r) := \{x : \|x\| \leq r\}$. Given two generic continuous-time trajectories $\boldsymbol{x}$ and $\boldsymbol{y}$ we say that $\boldsymbol{x}$ asymptotically converges to $\boldsymbol{y}$ if $\|x(t) - y(t)\| \to 0$ as $t \to +\infty$. The term $\mathcal{PC}(a, b)$ denotes the space of piecewise continuous trajectories defined over $[a, b]$. For the sake of simplicity, the dependence on time and parameters is dropped whenever it is clear from the context.

2. Problem definition

Consider the continuous-time time-varying dynamical system

$$\dot{x}(t) = f(t, x(t), u(t)), \quad x(t_0) = x_0, \; t \geq t_0 \tag{1}$$

and let the state and input vectors $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ be constrained as

$$(x(t), u(t)) \in \mathcal{X}(t) \times \mathcal{U}(t), \quad t \geq t_0, \tag{2}$$

where the set-valued maps $\mathcal{X} : \mathbb{R} \rightrightarrows \mathbb{R}^n$ and $\mathcal{U} : \mathbb{R} \rightrightarrows \mathbb{R}^m$ denote the time-varying state and input constraint sets, and t_0 and $x_0 = x(t_0)$ are the initial time and state, respectively.

Definition 1 (*Open-loop MPC problem*)**.** Given a pair $(t, z) \in \mathbb{R}_{\geq t_0} \times \mathcal{X}(t)$ and a horizon length $T > 0$, the open-loop MPC