Digital Signal Processing ••• (••••) •••–•••

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Extended state space recursive least squares

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
EKF
Nonlinear systems
SSRLS
State estimation
UKF

ABSTRACT

A new extended state space recursive least squares (ESSRLS) algorithm is proposed for state estimation of nonlinear systems. It is based on state space recursive least squares (SSRLS) approach and uses first order linearization of the system. It inherits the capability of obtaining state estimate without knowledge of process and measurement noise covariance matrices (Q and R respectively). The proposed approach is considered to provide new design option for scenarios where noise statistics and system dynamics vary. ESSRLS is initialized using delayed recursion method and a forgetting factor λ is employed to optimize the performance. The selection of λ can be problem specific as shown through experimental validations. However a value closer to and less than unity is generally recommended. Theoretical bases are validated by applying this algorithm to problems of tracking a non-conservative oscillator, a damped system with amplitude death and a signal modeled by mixture of Gaussian kernels. Simulation results show an MSE performance gain of 20 dB and 23 dB over extended Kalman filter (EKF) and unscented Kalman filter (UKF) while tracking van der Pol oscillator without knowledge about noise variances. The computational complexity of ESSRLS falls within that of EKF and UKF.

1. Introduction

Signal processing and control have widespread applications ranging from small-scale industrial appliances to numerous research areas (e.g., biomedical, navigation, mining, speech processing, etc.). Their main goal remains to accomplish process control and provide a performance monitoring and optimization mechanism. State estimation plays a key role by estimating current state of the system, using current and past values of system input and output signals. Major application categories include system identification, parameter estimation, inverse modeling, denoising and interference cancellation. A few practical examples are target tracking, simultaneous localization and mapping of robots, power systems, weather forecasting, autopilot, satellite navigation, speech enhancement, traffic control and ECG processing.

Approximate mathematical models for practical time varying systems usually suffer because of modeling uncertainties, assumptions, measurement uncertainties, unknown external disturbances and non-stationary noise sources acting on the system. In this scenario, utilizing a state estimation algorithm becomes the preferred choice. Both Kalman Filter (KF) and state space recursive least-squares (SSRLS) assist in obtaining accurate state estimates for linear dynamic systems, whereby KF performs in an optimal fashion in minimum mean square error sense while encountering Gaussian noise and provides optimal least-squares estimate when noise distribution is unknown, whereas SSRLS does not claim optimality and obtains state estimates in a least-squares sense without requirement of estimating process and measurement noise covariance matrices (Q and R respectively) [1–3]. Performance of SSRLS largely depends on selection of forgetting factor (λ) which ranges from 0 to 1 [4].

Most physical systems are inherently nonlinear and pose a great challenge to state estimation. There is no optimal algorithm available in literature, however two popular variants of KF exist, known as EKF and UKF. In EKF, system state is propagated through a first order linear approximation at each time instant. Higher order EKFs and iterated EKF are used to reduce first order linearization error where situation permits [2]. UKF is based upon unscented transform and performs accurately up to second order in estimating mean and variance of system state distribution. Square root UKF is utilized to resolve divergence issues associated with UKF [5–7].

The accurate estimation of Q and R is vital in ensuring convergence and accuracy of estimates in EKF and UKF [2,8,9]. There is no state of the art mechanism available for their estimation, usually diagonal matrices are assumed and diagonal entries are estimated using available observations and known modeling inac-

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http://dx.doi.org/10.1016/j.dsp.2015.10.017

curacies. Practical scenarios may arise where estimation of diagonal entries is not possible. Capturing non-stationarity of process and measurement noise in Q and R is difficult. Inaccurate estimation leads to poor performance [5,8,10]. Different variants of these two filters have been proposed and compared in literature in this context [6,11–13]. Wu et al. presented a self-adaptive UKF specific for underwater navigation [14]. Hu et al. [15] applied the idea of adaptive memory on KF to adaptively adjust forgetting factors under condition of optimality. Further variants are developed utilizing maximum-likelihood estimation, fuzzy logic techniques for estimating noise statistics and updating process noise covariance matrices online [16–19]. Iterated UKF proposed for structural system identification is more robust but it still relies on similar techniques to evaluate Q and R [20]. Robust UKF tries to reduce estimation error in presence of model uncertainties [21]. Nevertheless all these variants in some way or the other require *a priori* estimation of noise distribution and considerable increase in computational complexity is also noted.

Another approach in this regard has been developed using kernel methodology [22]. Liu presented kernel version of extended recursive least squares (Ex-RLS) [2] algorithm known as Extended kernel recursive least squares (Ex-KRLS) which is suitable for nonlinear systems with slow fading and a small variation in state [23]. A better approach suggested in [24] combines Ex-KRLS with variant of KF to form Ex-KRLS-KF whereby, a state space model is used to obtain hidden state estimates using KF and measurement model is updated by Ex-KRLS. Better estimation is gained at the cost of added computational complexity. Ref. [25] presents a different approach which does not assume a system model and instead learns state of nonlinear dynamic system using available measurements. It enhances filtering capability for non-Gaussian noise environments.

This paper presents a new algorithm for state estimation of nonlinear systems which is based upon SSRLS approach and uses first order linear approximation. It assumes an unforced system model with zero process noise (process noise is being dealt with separately in a different paper as an extension of SSRLS with adaptive memory (SSRLSWAM) in [26] for nonlinear systems). The inherited dependency of forgetting factor remains an issue and needs to be optimally adjusted to attain the best possible estimates. A few experiments have been carried out to validate the algorithm, its dependence upon initial conditions, forgetting factor and comparison with EKF and UKF. Three different types of nonlinear systems have been considered for this purpose. Computational complexity is also discussed in terms of time consumed by each function call.

Section 2 presents a review of SSRLS algorithm, derivation of new algorithm is carried out in Section 3; Section 4 explores the ability of proposed algorithm by applying it on relevant problems; Section 5 reviews computational complexity and in the end, findings are concluded with a view to provide a new option to designers for nonlinear state estimation.

2. Review of SSRLS

The state-space recursive least-squares (SSRLS) provides an important tool to estimate a wide class of deterministic signals modeled by linear state-space models and corrupted by observation noise [4]. This algorithm assumes an unforced system model. Batch processed least squares state estimation forms the basis for its derivation. The concept of exponential forgetting factor is introduced to achieve a recursive algorithm. Unlike KF, SSRLS does not claim optimality and attempts to obtain state estimates in least-squares sense. It does not require Q and R. SSRLS achieves better tracking performance and faster convergence rate as compared to RLS by virtue of using an appropriate system model. Additionally, it can track multiple observations simultaneously while RLS can only track scalar random processes. System model is given as:

$$\begin{aligned} x[k+1] &= A[k]x[k] \\ y[k] &= C[k]x[k] + v[k] \end{aligned} \tag{1}$$

where $x \in \mathscr{R}^n$ is state vector, $y \in \mathscr{R}^m$ is output vector and $v[k]$ is observation noise corrupting the measurements. A, C pair is assumed to be l-step observable with invertible A. State estimate $\hat{x}[k]$ given by SSRLS is:

$$\hat{x}[k] = \bar{x}[k] + K[k]\varepsilon[k] \tag{2}$$

where $\bar{x}[k] = A[k]\hat{x}[k-1]$ is the predicted state estimate. Gain $K[k]$ is determined by SSRLS. $\varepsilon[k]$ is innovation signal defined as difference between observed output $y[k]$ and predicted output $\bar{y}[k] = C[k]\bar{x}[k]$.

2.1. SSRLS observer gain

Observer gain can be computed in two different ways. One method involves the inversion of $n \times n$ matrix and is known as SSRLS form I. Gain computation is done as follows:

$$\phi[k] = \lambda A^{-T}[k]\phi[k-1]A^{-1}[k] + C^T[k]C[k] \tag{3}$$

$$K[k] = \phi^{-1}[k]C^T[k] \tag{4}$$

Second method, known as SSRLS form II requires the inversion of $m \times m$ matrix and is computationally less complex. The Riccati equation of SSRLS is evaluated as follows:

$$\begin{aligned} P[k] = {} & \lambda^{-1}A[k]P[k-1]A^T[k] \\ & - \lambda^{-2}A[k]P[k-1]A^T[k]C^T[k] \\ & \times [I + \lambda^{-1}C[k]A[k]P[k-1]A^T[k]C^T[k]]^{-1} \\ & \times C[k]A[k]P[k-1]A^T[k] \end{aligned} \tag{5}$$

Here $P[k] = \phi^{-1}[k]$. Observer gain is computed using (4). λ represents forgetting factor and inherits its characteristics from standard RLS. Although its optimal value is experimentally judged, for linear systems values closer to but less than unity provide better results. It is also termed as memory length whereby closer to unity means more memory [3,2]. The algorithm can be initialized using regularization term or delayed recursion method [3].

3. Extended state-space recursive least-squares (ESSRLS) algorithm

SSRLS described above addresses the general problem of trying to estimate state of a discrete-time controlled process that is governed by a linear stochastic difference equation. The idea is to extend it to nonlinear systems which will enable its application to a vast majority of practical problems. In something akin to a Taylor series, we can linearize the system around current estimate using partial derivatives of process and measurement functions.

3.1. Non-linear system model

We consider nonlinear discrete time system represented by

$$x[k] = f(x[k-1])$$

$$y[k] = h(x[k], v[k]) \tag{6}$$

where $x \in \mathscr{R}^n$, $y \in \mathscr{R}^m$ and $v[k]$ represents measurement noise with $p(v) \sim \mathscr{N}(0, R)$, non-linear function f relates the state at