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Minimum variance estimation for linear uncertain systems with one-step correlated noises and incomplete measurements

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ABSTRACT

This paper deals with state estimation problem for linear uncertain systems with correlated noises and incomplete measurements. Multiplicative noises enter into state and measurement equations to account for the stochastic uncertainties. And one-step autocorrelated and cross-correlated process noises and measurement noises are taken into consideration. Using the latest received measurement to compensate lost packets, the modified multi-step random delays and packet dropout model is adopted in the present paper. By augmenting system states, measurements and new defined variables, the original system is transformed into the stochastic parameter one. On this basis, the optimal linear estimators in the minimum variance sense are designed via projection theory. They depend on the variances of multiplicative noises, the one-step correlation coefficient matrices together with the probabilities of delays and packet losses. The sufficient condition on the existence of steady-state estimators is then given. Finally, simulation results illustrate the performance of the developed algorithms.

1. Introduction

As is well known, if the system model under consideration is exactly known, Kalman filter is an optimal filter in the minimum variance sense for linear systems [1]. Under different hypotheses on the processes involved in the observation equation, corresponding algorithms have been proposed, such as Unscented Kalman filter, Extended Kalman filter, Particle filter and so on [2]. However, in the network environment, the standard observation model becomes inappropriate due to the existence of network-induced uncertainties like packet dropouts, transmission delays, missing measurements, and/or disorder. Accordingly, modeling the observation process is vitally important to the filtering or state estimation problem for the networked control systems (NCSs).

Studies on filtering or state estimation with one or two, even three network-induced uncertainties have attracted considerable attention in the past few decades. For instance, using an innovation analysis approach, [3] presented the optimal linear estimators for the systems with multiple packet dropouts. Given measurements transmitted by different sensors subject to random packet dropouts, [4] discussed least-squares linear estimation problem using covariance information. For random delayed systems, the optimal and suboptimal estimators [5] and recursive filtering and smoothing algorithms [6] were proposed. Taking packet dropouts, missing measurements as well as random measurement delays into account, [7] designed adaptive filtering schemes. Differently from the aforementioned observation models, [8] developed a model to describe multi-step transmission delays and packet dropouts by introducing some Bernoulli distributed random variables. Then, the optimal linear estimators for single sensor were derived. Based on the novel model in [8], [9,10] developed corresponding estimation algorithms for different systems. To be specific, [9] extended the results in [8] to the multi-sensor distributed case, and derived a distributed fusion filter. By means of the reorganized innovation approach, [10] investigated the optimal estimator for the linear systems with and without time-stamped data packets. However, multi-step random delays and packet losses model used in [8–10] can result in complete loss of packets at times, which affect the performance of proposed estimator. To overcome this deficiency, the latest measurement transmitted successfully can be used for the estimation. The resultant model is described in the problem formulation section, called as the compensation multi-step random delays and packet losses model, or compensation model for short.

In many engineering applications, the process noise and measurement noise are assumed to be correlated. As pointed out in [11], the radar system is a typical example of this aspect. On this account, a great number of estimation results concerning systems

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with correlated noises have been obtained. To mention a few, [12] designed the optimal robust non-fragile Kalman-type recursive filter for a class of uncertain systems with finite-step autocorrelated measurement noises and multiple packet dropouts. Compared with the results in [12], a globally optimal filtering was proposed in [13] by exploiting sufficiently the statistical properties of correlated noises. Considering the correlation between the signal and the observation noise, the least-squares linear smoothing problem was investigated in [14]. Recently, [15] has coped with the optimal least-squares linear estimation problem, in which one-step correlated and cross-correlated parameter matrices other than correlated noises were considered. For the distributed filtering problem subject to correlated noises, a distributed Kalman filtering fusion algorithm [16], the distributed weighted robust Kalman filter [17], and optimal sequential and distributed fusion schemes [18] were developed, respectively.

On the other hand, parameter uncertainties inevitably exist in the system model because of model reduction, varying parameters and so on. In general, parameter uncertainties include deterministic uncertainties and stochastic uncertainties. Multiplicative noises, as we all known, are regarded as the stochastic uncertainties. Hence, so far, many valuable results with respect to state estimation for uncertain systems have been reported. Specifically, an exact, closed-form minimum variance filter [19], the optimal linear estimators [20] and a robust distributed state fusion Kalman filter [21] were proposed, respectively. Merging the stochastic uncertain terms into the process and observation noises of the original system, a robust Kalman filter was presented in [22]. Considering the parameter uncertainties may occur in a probabilistic way, [23] addressed the distributed filtering problem in terms of linear matrix inequalities. Both deterministic uncertainties and stochastic uncertainties being considered, [24] designed a robust finite-horizon Kalman filter for discrete time-varying uncertain systems. Recently, [25] designed optimal linear estimators for NCSs with stochastic uncertainties, multiple sensors and packet losses of both sides from sensors to an estimator and from a controller to an actuator. Although [20] and [25] derived the optimal estimators for uncertain system with compensation model, but multi-step random delay and correlated noises were not involved.

Up to now, to the best of the authors' knowledge, the minimum variance estimation problem for the uncertain systems with compensation multi-step random delays and packet losses, one-step autocorrelated and cross-correlated noises has not been considered yet, which motivates the present study. It should be note that the stochastic uncertainties together with correlated noises can bring many difficulties in designing the optimal estimators, not to mention the challenges brought from compensation multi-step random delays and packet losses model. Compared with the observation model adopted in [8], the compensation model in the present paper can use the latest measurement data transmitted successfully to compensate lost packets at some time. To transform the original system into the stochastic parameterized one, some new variables are defined. In particular, random variable ρ_{l+1} is introduced, which brings some difficulties in dealing with the relationships among this variable, other random variables and resultant stochastic coefficient matrices. Based on the stochastic parameter system, the filter, multi-step predictor and smoother are proposed via the innovation analysis approach. The designed estimators are optimal in the minimum variance sense, and consider the effect from multiplicative noises, one-step correlated noises together with multi-step delays and packet losses. In this sense, our work isn't a simple generalization of some existing results. In addition, estimation algorithms developed in the current paper can be applied to close loop control, target tracking, communications, fault diagnosis and so on.

The organization of the paper is as follows. Section 2 gives the problem under consideration. In Section 3, two lemmas and the main theorems on the design of minimum-variance estimators are provided. Furthermore, the sufficient condition on the existence of steady-state estimators is discussed. The performance of the proposed estimators is illustrated in Section 4 by a numerical example and some conclusions are drawn in Section 5. Proofs of the results in Section 3 are given in Appendices A–C.

Notations. Throughout the paper, the notations used are standard. R^m represents the m-dimensional Euclidean space. I_m and 0 mean the identity matrix and zero matrix with appropriate dimensions, respectively. $\delta_{t,l}$ denotes the Kronecker delta function, which is equal to zero if $t \neq l$, and one if $t = l$. $Prob(*)$ stands for the occurrence probability of the event $*$. $E(x)$ is the expectation of x. $\rho(A)$ represents the spectral radius of matrix A. $sym\{*\}$ denotes $* + *^T$. If not explicitly stated, all matrices are assumed to be of compatible dimensions.

2. Problem formulation

Consider the following discrete time-varying linear system with multiplicative noises:

$$x(t+1) = \Big(A(t) + \sum_{i=1}^{r} \alpha_i(t) A_{\mu,i}(t)\Big)x(t) + B(t)w(t) \tag{1}$$

$$z(t) = \Big(C(t) + \sum_{k=1}^{s} \beta_k(t) C_{\mu,k}(t)\Big)x(t) + v(t) \tag{2}$$

where $x(t) \in R^n$ is the system state, and the initial state $x(0)$ has mean $\bar{x}_0$ and variance P_0. $z(t) \in R^m$ is the measurement. $\alpha_i(t) \in R$ and $\beta_k(t) \in R$ represent mutually uncorrelated zero-mean multiplicative noises with variance Q_{α_i} and Q_{β_k}, respectively. r and s are known positive integers. $w(t) \in R^h$ and $v(t) \in R^m$ are, respectively, the one-step autocorrelated and cross-correlated process noise and measurement noise. $A(t)$, $A_{\mu,i}(t)$, $B(t)$, $C(t)$, and $C_{\mu,k}(t)$ are known time-varying matrices with appropriate dimensions. Also, $x(0)$ is assumed to be uncorrelated with $\alpha_i(t)$, $\beta_k(t)$, $w(t)$ and $v(t)$.

The network-induced uncertainties, such as random delays and packet losses, always occur during the measurement z(t) being sent to the estimator. For this reason, in our present work, $y(t)$ received by the estimator is modeled as follows:

$$\begin{aligned} y(t) &= \rho_0(t)z(t) + (1-\rho_0(t))\rho_1(t-1)z(t-1) + \cdots \\ &\quad + \Pi_{i=0}^{l-1}(1-\rho_i(t-i))\rho_l(t-l)z(t-l) + \rho_{l+1}(t)y(t-1) \end{aligned} \tag{3}$$

where l is the largest transmission delay, $\rho_i(t)$ satisfy $\rho_0(t) = \eta_0(t)$, $\rho_i(t) = \Pi_{k=0}^{i-1}(1-\eta_k(t+k))\eta_i(t+i)(i = 1, 2 \ldots l)$ and $\rho_{l+1}(t) = (1-\rho_0(t))(1-\rho_1(t-1))\ldots(1-\rho_l(t-l))$. $\eta_i(t)$ are mutually uncorrelated Bernoulli distributed random variables satisfying that $Prob\{\eta_i(t) = 1\} = \nu_i$ and $Prob\{\eta_i(t) = 0\} = 1-\nu_i (0 \leq \nu_i \leq 1)$. In addition, we assume that $\eta_i(t)$ are independent of $x(0)$, $w(t)$, $v(t)$, $\alpha_i(t)$ and $\beta_k(t)$.

To make the compensation model (3) more understandable, a simple demonstration of data transmission under $l = 2$ is given in Table 1, from which the main difference between the model in [8] and model (3) can be observed.

Table 1 shows that $z(1)$, $z(2)$, $z(4)$, $z(9)$ and $z(10)$ are received on-time, $z(5)$ is delayed one step, $z(3)$ is delayed two steps, $z(6)$, $z(7)$ and $z(8)$ are lost in [8], while the lost data are respectively compensated for by $y(2)$, $y(6)$ and $y(7)$ in model (3).

Assumption 1. The process noise $w(t)$ and the measurement noise $v(t)$ are one-step autocorrelated and cross-correlated noises, which