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An edge-weighted second order variational model for image decomposition

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ABSTRACT

Decomposing an image into structure and texture is an important procedure for image understanding and analysis. Structure retains object hues and sharp edges whilst texture contains oscillating patterns of an observed image. The classical Vese–Osher model has been used for image decomposition, but its resulting structure image tends to show the undesirable staircase effect. Second order variational models that use a bounded Hessian regulariser have been proposed to remedy this side effect, but they tend to blur edges of objects in structure components. In this paper, we propose an edge-weighted second order variational model for image decomposition, which is able to eliminate staircase effects and preserve object edges. To avoid directly calculating the high order nonlinear partial differential equations of the proposed model, a fast split Bregman algorithm is developed, which uses the fast Fourier transform and analytical generalised soft thresholding equations. Extensive experiments demonstrate that the proposed variational image decomposition model outperforms state-of-the-art first and second order image decomposition models. By removing the texture component from the original noisy image, the effectiveness of the proposed model for image denoising has also been validated.

1. Introduction

Image decomposition into structure and texture has been used, for instance, for image similarity analysis [1], texture synthesis [2], texture image segmentation [3], texture decomposition and feature selection [4–6], and structure and texture image inpainting [7], etc.

A popular method for image decomposition is the variational approach. Vese and Osher proposed a first order variational model (FOVO) [8] for image decomposition. However, the FOVO model suffers from the undesirable staircase effect, that is, the resulting structure image has a jagged appearance. This is because the energy minimisation in FOVO is in the bounded variation (BV) space [9], which results in a piecewise constant function leading to the staircase side effect. Recently, a second order regulariser defined in bounded Hessian (BH) space, the BH regulariser, has been employed to remedy this side effect [10–14]. Compared with other non-convex high order regularisers, such as the mean curvature [15–17] and Euler-elastica [18,19] etc., the BH regulariser is a convex high order extension of the total variation (TV) regulariser [9] which is less dependent on initialisation. Compared with the convex total generalised variation (TGV) regulariser [20–22], the BH regulariser is also more efficient to implement. However, the edges of objects in the resulting structure image are often blurred due to the fact that the model imposes too much regularity on the image. In addition, the Euler-equations of the BH regulariser are fourth order nonlinear partial differential equations (PDEs), which are very difficult to discretise to solve computationally.

In this paper, we propose an edge-weighted second order (EWSO) variational model for image decomposition to overcome the problems with the existing first and second order models mentioned above. Preliminary results of this work have been presented in a conference [24]. Instead of using the TV regulariser as in the FOVO model, the proposed EWSO model uses the BH regulariser together with an edge diffusivity function. The former aims to remove staircase effects, whilst the latter aims to preserve object edges. The split Bregman algorithm [23] is adapted to improve the computational speed by transforming the energy minimisation problem of the proposed EWSO model into four subproblems, which are then efficiently solved using the fast Fourier transform (FFT) and analytical soft thresholding equations without any iterations. The new model is validated through extensive experiments. Experimental results show that the proposed new model outperforms the state-of-the-art variational methods for both image de-

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composition and image denoising. The contributions of the paper are threefold: 1) A new second order variational image decomposition model is proposed; 2) A fast split Bregman algorithm is developed for image decomposition based on a finite difference scheme; 3) The new image decomposition model is used for image denoising applications.

The paper is organised as follows: Section 2 sets the background for the paper by introducing some existing variational models and their drawbacks for image decomposition. Section 3 introduces the proposed second order variational model for image decomposition. Section 4 describes the proposed split Bregman algorithm for solving the variational model efficiently. Section 5 gives details of the experiments using the proposed model for image decomposition and denoising. Section 6 concludes the paper. Implementation details of the proposed split Bregman algorithm are given in Appendix.

2. Background

In image decomposition, an image f is split into two components $f = u + v$ where u represents structure containing object edges and hues, and v represents texture (noise or oscillating patterns).

2.1. First order variational models

In order to extract structure u and texture v from an image f, Rudin, Osher and Fatemi [9] proposed the following TV model

$$\min_{u\in BV(\Omega)}\left\{E(u)=\frac{1}{2}\|v\|^2_{L^2(\Omega)}+\alpha\int_\Omega|\nabla u|,\ f=u+v\right\},\tag{2.1}$$

where $BV(\Omega)=\left\{u\in L^1(\Omega)\mid \mathrm{TV}(u)<+\infty\right\}$ is a bounded variation functional space on Ω, an open subset of $\mathbb{R}^n$ ($n=2$ in this paper) with Lipschitz boundary; α is a smoothness parameter; $\int_\Omega|\nabla u|$ is the TV regulariser; and $\mathrm{TV}(u)$ is the total variation semi-norm of u, defined as

$$\mathrm{TV}(u):=\sup\left\{\int_\Omega u\,div(\varphi)\,dx:\varphi\in C^1_0\left(\Omega,\mathbb{R}^n\right),\|\varphi\|_{L^\infty}\le 1\right\},\tag{2.2}$$

where the first order divergence operator $div(\varphi)$ is defined as $div(\varphi)=\sum_{i=1}^n\frac{\partial\varphi_i}{\partial x_k}(x)$.

Meyer [25] proposed an alternative image decomposition model (2.3) and introduced the concept of oscillating functions. He used the Banach space G (2.4) for texture v and replaced the L^2-norm in the data fitting term of the TV model by the G-norm defined in (2.5).

$$\min_{u\in BV(\Omega)}\left\{E(u)=\|v\|_G+\alpha\int_\Omega|\nabla u|,\ f=u+v\right\},\tag{2.3}$$

where

$$G=\left\{v\,\middle|\,v=div(g),\ g=(g_1\ g_2)\in L^\infty(\Omega,\mathbb{R}^2)\right\},\tag{2.4}$$

$$\|v\|_G=\inf_{g=(g_1\ g_2)}\left\{\left\|\sqrt{g_1^2+g_2^2}\right\|_{L^\infty}\middle|\,v=div(g),\ g\in L^\infty(\Omega,\mathbb{R}^2),\ |g|=\sqrt{g_1^2+g_2^2}\right\}.\tag{2.5}$$

Compared with the L^2-norm of the TV model, the weaker G-norm can capture the oscillations of a function better in energy minimisation. However, it is not possible to derive Euler–Lagrange equations for the G-norm such that a straightforward PDE method can be utilised to solve it.

Vese and Osher [8] then approximated Meyer's G-norm by the $div(L^p)$-norm and proposed the FOVO model that combines Meyer's oscillating function with the TV model [9]:

$$\min_{u,g}\left\{E(u,g)=\frac{1}{2}\int_\Omega(f-u-div(g))^2+\alpha\int_\Omega|\nabla u|+\beta\left[\int_\Omega|g|^p\right]^{1/p}\right\},\tag{2.6}$$

where α and β are two positive tuning parameters balancing the three energy terms. Vese and Osher confirmed that there are no obvious numerical differences using different values of p, with $1\le p\le 10$, but $p=1$ yields faster calculations.

Fig. 1 shows the decomposition results by the TV and FOVO models of the fingerprint image Fig. 1(a). The structure image Fig. 1(b) by the TV model contains some texture, and the texture image Fig. 1(c) contains some structure of the original image. The texture image Fig. 1(e) by the FOVO model however does not contain any structure, demonstrating the advantage of the G-norm (i.e. the L^1-norm of $|g|$ in the FOVO model) over the L^2-norm in the TV model in capturing texture. Unlike the structure image by the TV model, the structure image Fig. 1(d) by the FOVO model does not contain any texture of the original image. However, as the function u in the FOVO model is defined in the $BV(\Omega)$ space, the staircase effect is present in the structure image.

2.2. High order variational models

To remedy the staircase effect suffered by the decomposition model, high order variational models have been proposed for various applications [10–22,24,26–29]. Several representative models [20,26,28] are reviewed here for comparison with the proposed model later on in this paper. Chambolle et al. [26] proposed a high order model, named the INFCON model in this paper, by means of an infimal-convolution of the first order and second order derivative regularisers. The former regulariser preserves edge sharpness and the latter maintains image smoothness. Specifically, their model can be used to decompose an image f into three components $f=u_1+u_2+v$ as follows:

$$\min_{u_1,u_2\in BV(\Omega)\times BH(\Omega)}\left\{\frac{1}{2}\|f-u_1-u_2\|^2_{L^2(\Omega)}+\alpha\int_\Omega|\nabla u_1|+\beta\int_\Omega\left|\nabla^2u_2\right|\right\},\tag{2.7}$$

where u_1 and u_2 are the piecewise constant and piecewise affine parts of the input image f, and v is the noise/texture.

A model that directly combines the first and second order regularisers, e.g. the TV and BH regularisers, named the combined first and second order (CFS) model in this paper, was proposed in [28]:

$$\min_{u\in BV(\Omega)\times BH(\Omega)}\left\{E(u)=\frac{1}{2}\|f-u\|^2_{L^2(\Omega)}+\alpha\int_\Omega|\nabla u|+\beta\int_\Omega\left|\nabla^2u\right|\right\},\tag{2.8}$$