Automatica 68 (2016) 369–377

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Backstepping-based boundary observer for a class of time-varying linear hyperbolic PIDEs☆

Andreas Deutschmann[a,1], Lukas Jadachowski[b], Andreas Kugi[a,b]

[a] *Automation and Control Institute, Complex Dynamical Systems Group, Vienna University of Technology, Vienna, Austria*

[b] *Christian Doppler Laboratory for Model-Based Process Control in the Steel Industry, Vienna University of Technology, Vienna, Austria*

ARTICLE INFO

Article history:
Received 8 May 2015
Received in revised form
16 October 2015
Accepted 25 January 2016
Available online 15 March 2016

Keywords:
Distributed-parameter system
Hyperbolic PIDE
Luenberger-type observer
Boundary observer
Backstepping

ABSTRACT

In this paper, a Luenberger-type boundary observer is presented for a class of distributed-parameter systems described by time-varying linear hyperbolic partial integro-differential equations. First, known limitations due to the minimum observation time for simple transport equations are restated for the considered class of systems. Then, the backstepping method is applied to determine the unknown observer gain term. By avoiding the framework of Gevrey-functions, which is typically used for the time-varying case, it is shown that the backstepping method can be employed without severe limitations on the regularity of the time-varying terms. A modification of the underlying Volterra transformation ensures that the observer error dynamics is equivalent to the behavior of a predefined exponentially stable target system. The magnitude of the observer gain term can be traded for lower decay rates of the observer error. After the theoretic results have been proven, the effectiveness of the proposed design is demonstrated by simulation examples.

1. Introduction

While the topic of boundary control for first-order hyperbolic partial differential equations (PDEs) has been thoroughly investigated over the last twenty years, see, e.g., Christofides and Daoutidis (1996), Coron, d'Andrea Novel, and Bastin (2007), Diagne, Bastin, and Coron (2012), Krstic and Smyshlyaev (2008a), Prieur, Winkin, and Bastin (2005), Prieur, Winkin, and Bastin (2008), the problem of state observation has been addressed only recently Castillo, Witrant, Prieur, and Dugard (2013), Di Meglio, Bresch-Pietri, and Aarsnes (2014), Di Meglio, Vazquez, and Krstic (2013) and Vazquez, Krstic, and Coron (2011). Most of these contributions apply the backstepping method introduced by Smyshlyaev and Krstic (2004), which maps the observer error dynamics onto a desired (exponentially stable) target system using Volterra integral transformations. The strength of this approach is its structural simplicity, the broad range of possible time-invariant and time-varying plants (Krstic & Smyshlyaev, 2008b; Smyshlyaev & Krstic, 2005) and the possibility to combine it with other concepts, as for instance flatness-based feedforward control (Meurer & Kugi, 2009).

A time-invariant version of the class of linear first-order hyperbolic PIDEs considered in this paper was introduced in Krstic and Smyshlyaev (2008a) which is closely related to the parabolic type treated in Smyshlyaev and Krstic (2004). Such PIDEs usually arise from two coupled PDEs where one can be perturbed suitably. Very recently, the boundary control concept presented in Krstic and Smyshlyaev (2008a) was extended to an adaptive output-feedback design able to deal with unknown parameters (Bernard & Krstic, 2014) and systems with Fredholm operators that do not exhibit a strict-feedback structure (Bribiesca-Argomedo & Krstic, 2015).

While a filter-based state observer with non-adjustable error dynamics is used in Bernard and Krstic (2014) for time-invariant plants in the course of designing an output-feedback law using a backstepping pre-transformation, this paper is concerned with a Luenberger-type observer for time-varying hyperbolic PIDEs. Time-varying backstepping designs are usually treated by

☆ The financial support in part by the Austrian Federal Ministry of Science, Research and Economy, the National Foundation for Research, Technology and Development, and voestalpine Stahl GmbH is gratefully acknowledged. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Rafael Vazquez under the direction of Editor Miroslav Krstic.

E-mail addresses: deutschmann@acin.tuwien.ac.at (A. Deutschmann), jadachowski@acin.tuwien.ac.at (L. Jadachowski), kugi@acin.tuwien.ac.at (A. Kugi).

[1] Tel.: +43 1 58801 376263.

http://dx.doi.org/10.1016/j.automatica.2016.02.007
0005-1098/

employing the framework of Gevrey-functions (Jadachowski, Meurer, & Kugi, 2014; Meurer, 2013; Meurer & Kugi, 2009; Vazquez, Trelat, & Coron, 2008) which imposes strict conditions on the regularity of the time-varying terms. As this paper shows, this can be avoided for linear hyperbolic equations. By using a modified backstepping transformation, a general PIDE can serve as target system and thus a desired observer error dynamics can be chosen. Therefore, it enables to trade slower error dynamics for reduced sensitivity to noise.

The paper is structured as follows: First, the problem under consideration is introduced in Section 2. Minimum observation times known from simple transport systems give a lower bound of what can be achieved theoretically. Thus, it is shown in Section 3 that hyperbolic PIDEs are subject to the same limitation. Section 4 follows the backstepping approach to calculate the desired observer gain term. Finally, Section 5 applies the design to specific examples and analyzes the influence of the design parameters, introduced by the general target system in Section 4, on the error dynamics.

2. Problem statement

In the following, the observer design is considered for systems of the form

$$x_t(z,t) = x_z(z,t) + a(z,t)\,x(z,t) + g(z,t)\,x(0,t) + \int_0^z f(z,\xi,t)\,x(\xi,t)\,\mathrm{d}\xi, \tag{1a}$$

with boundary and initial conditions

$$x(z,0) = x_0(z) \tag{1b}$$

$$x(1,t) = u(t) \tag{1c}$$

and the system output

$$y(t) = x(0,t) \tag{1d}$$

defined on the domain $(z,t) \in \Omega = (0,1) \times \mathbb{R}^+$. Here, $u(t)$ represents an external input. The functions $a(z,t)$, $g(z,t)$ and $f(z,\xi,t)$ with $z, \xi \in [0,1]$ and $t \in \mathbb{R}^+$ are assumed to be continuous in z, t and ξ, respectively and bounded in time. A distributed-parameter Luenberger-type observer with the observer state $\hat{x}(z,t)$ is formulated in the form

$$\hat{x}_t(z,t) = \hat{x}_z(z,t) + a(z,t)\,\hat{x}(z,t) + g(z,t)\,\hat{x}(0,t) + p(z,t)\,(y(t) - \hat{y}(t)) + \int_0^z f(z,\xi,t)\,\hat{x}(\xi,t)\,\mathrm{d}\xi, \tag{2a}$$

with the observer's boundary and initial conditions

$$\hat{x}(z,0) = \hat{x}_0(z) \tag{2b}$$

$$\hat{x}(1,t) = u(t) \tag{2c}$$

and the corresponding observer output

$$\hat{y}(t) = \hat{x}(0,t). \tag{2d}$$

In view of (1) and (2) the dynamics of the observer error $e(z,t) = x(z,t) - \hat{x}(z,t)$ follows as

$$e_t(z,t) = e_z(z,t) + a(z,t)\,e(z,t) + p_1(z,t)\,e(0,t) + \int_0^z f(z,\xi,t)\,e(\xi,t)\,\mathrm{d}\xi, \tag{3a}$$

with the associated boundary and initial conditions

$$e(z,0) = e_0(z) \tag{3b}$$

$$e(1,t) = 0 \tag{3c}$$

using

$$p_1(z,t) = g(z,t) - p(z,t). \tag{4}$$

The unknown observer gain $p(z,t)$ has to be determined such that the observer state $\hat{x}(z,t)$ converges to the system state $x(z,t)$ in the sense of the L^2-norm, i.e., that the error dynamics (3) is exponentially stable in the L^2-norm.

Remark 1. If instability is introduced to (1) through the output feedback $g(z,t)x(0,t)$ only, the observer error dynamics can be stabilized by choosing $p(z,t) = g(z,t)$.

3. Minimum observation time

It is well known that the observability of simple transport systems (i.e. with $f \equiv g \equiv 0$) requires a minimum observation time $T_m = 1$. Analyzing observability of distributed-parameter systems is usually done by using operator semigroup theory, see, e.g., Pazy (1992) and Tucsnak and Weiss (2009). Since these methods require a closed-form or series solution, an alternative approach is chosen to show that the same minimum observation time also serves as a necessary condition for the considered class of PIDEs (1).

Lemma 2. *The system (1) can only be observable for*

$$t \geq T_m = 1. \tag{5}$$

Proof. Without loss of generality, (1) is restricted to $g \equiv 0$ since the term $g(z,t)x(0,t)$ is perfectly known. Applying the method of characteristics yields the implicit integral equation

$$x(z,t) = u\,(t+z-1) + \int_z^1 a(\sigma, t+z-\sigma)x(\sigma, t+z-\sigma)\mathrm{d}\sigma + \int_z^1\int_0^\sigma f\,(\sigma,\xi,t+z-\sigma)\,x\,(\xi,t+z-\sigma)\;\mathrm{d}\xi\;\mathrm{d}\sigma. \tag{6}$$

This equation shows that the solution at a single point (z^*, t^*) depends on the solution of a whole subset of Ω, i.e. the domain of dependence

$$\Omega_{D^*} = \{(z,t) \in \Omega \mid z^* \leq \sigma \leq 1 \text{ and } 0 \leq z \leq \sigma\} \subset \Omega \tag{7}$$

with $\sigma = z^* + t^* - t$. Hence, Ω_{D^*} is determined by the inequalities

$$z \geq 0, \tag{8a}$$

$$z - z^* \leq -(t - t^*), \tag{8b}$$

$$t \leq t^*, \tag{8c}$$

$$t - t^* \geq z^* - 1. \tag{8d}$$

However, the solution for points $(z,t) \in \Omega_{D^*}$ will depend on points outside of Ω_{D^*}. For example, a point $(z^1, t^1) \in \Omega_{D^*}$ has its own domain of dependence $\Omega_{D^1} \not\subset \Omega_{D^*}$ as shown in Fig. 1. Therefore, the question is if any inequality (8) will hold for iterative application of this dependence relation. It is easy to see that (8a)–(8c) indeed hold for this iterative relation. When considering the solution at the left boundary, e.g., $(z,t) = (0,t^*)$, points that comply with

$$t \geq t^* - z \tag{9}$$

do not influence the solution and thus the output $y(t^*)$. As a consequence, this part of the domain Ω cannot be observable at time t^* (see Fig. 1 on the right). □

Condition (5) is – without further investigation – just a necessary one for hyperbolic PIDEs of type (1).