Automatica 68 (2016) 1–8

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Smooth dynamic output feedback control for multiple time-delay systems with nonlinear uncertainties[☆]

Chang-Chun Hua[a], Xin-Ping Guan[a,b]

[a] Institute of Electrical Engineering, Yanshan University, Qinhuangdao, 066004, China

[b] Department of Automation, Shanghai Jiao Tong University, Shanghai, China

ARTICLE INFO

Article history:
Received 13 November 2014
Received in revised form
25 August 2015
Accepted 13 December 2015
Available online 22 February 2016

Keywords:
Dynamic output feedback
Lyapunov–Krasovskii method
Multiple time delays
Adaptive method

ABSTRACT

The dynamic output feedback control problem is considered for a class of systems with multiple time delays and nonlinear uncertainties. Based on the control input matrix and output matrix, we decompose the system into two subsystems. The dynamic compensator is designed for the first subsystem, and then the output feedback controller is constructed based on the compensator and the second subsystem. By using the introduced new Lyapunov–Krasovskii functional, we show that the solution of the resultant closed-loop system converges exponentially to an adjustable bounded region. Compared with the previous works, the developed controller in this paper is memoryless and smooth, which only uses the system output. The control design conditions are relaxed because of the developed dynamic compensator. The result is further extended to the general nonlinear case. The corresponding dynamic nonlinear output feedback control method is proposed. Finally, simulations are performed to show the potential of the proposed methods.

1. Introduction

Time delay is a common phenomenon for many industrial process systems. The stability analysis and control for the time delay dynamic systems have attracted considerable attentions (see Gu, Kharitonov, & Chen, 2003 and the references therein). The Lyapunov functional method and Razuminkin lemma are two common tools for the analysis and design of time delay system. The obtained results are often given in the form of linear matrix inequality (LMI). For a practical system, it may be a difficult task to obtain all the state variables, and then the output feedback control method is developed. Compared with the state feedback control, the output feedback control problem is more challenging because of the limited information of state variables. The output feedback can be classified into two categories: static output feedback and dynamic output feedback.

During the past decades, the static output feedback and dynamic output feedback have been received lots of attention. Xu and Chen (2004) considered the dynamic output feedback control for linear time delay system. Mirkin and Gutman (2005) proposed the output feedback model adaptive control approach for the linear time delay systems with unknown coefficients. In He, Liu, Rees, and Wu (2007); He, Wu, Liu, and She (2008), the static output feedback control problem and the dynamic output feedback control problem were considered, and the new corresponding stability criteria were proposed. For the system with uncertainties, the sliding mode control method was developed for solving the output feedback control problem. Xu, Lam, Shi, Boukas, and Zou (2009) considered the dynamic output feedback control problem for linear time delay systems. Du, Lam, and Shu (2010) investigated the static and dynamic output feedback control problem for linear systems with time delays. The static output feedback control problem was considered in Mahmoud and Qureshi (2012) for a class of large-scale time delay systems and a novel sliding mode control method was proposed. The time delay systems considered in the above references are in the linear form. The LMI based control design conditions were proposed.

For the time delay systems with nonlinear uncertainties, some works have been done. In Hua, Wang, and Guan (2008), a delay independent state feedback control method was proposed for dealing with this problem. With the use of sliding mode control technique, Han, Fridman, Surgeon, and Edwards (2009) investigated the discontinuous static output feedback control problem. Mirkin and Gutman (2008) constructed the memoryless controllers.

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Emilia Fridman under the direction of Editor Ian R. Petersen.

E-mail addresses: cch@ysu.edu.cn (C.-C. Hua), xpguan@sjtu.edu.cn (X.-P. Guan).

http://dx.doi.org/10.1016/j.automatica.2016.01.007
0005-1098/

In Hua, Ding, and Guan (2012), the actuator failure problem was considered and the delay dependent smooth state feedback controller was designed. For the output feedback control problem, in Yan, Spurgeon, and Edwards (2009), the dynamic observer was designed to estimate the unmeasurable system state and then the sliding mode controller was constructed based on the observer. In Yan, Spurgeon, Zhu, and Zhang (2013b), the delay independent static output feedback controller was constructed. In Yan, Spurgeon, and Edwards (2013a), the decentralized static output feedback controller was constructed for interconnected system with time delays. Zhang, Liu, Feng, and Zhang (2013) investigated the dynamic output feedback control for large-scale time delay systems with triangular structure and proposed a novel observer-based memoryless control design method. In Yan, Spurgeon, and Edwards (2014), a new memoryless static output feedback sliding mode controller was designed for nonlinear systems with time delays.

It is well known that the static output feedback is easy for implementation, but some strict conditions should be imposed on the system. The dynamic output feedback technique is more flexible and the required conditions on the considered systems are less conservative. In addition, the sliding mode control method is very efficient for the control design of the time delay systems with uncertain disturbances, but the designed controller is discontinuous and the precise time delays are often required for the control implementation. With the above observations, we consider the dynamic output feedback control problem for a class of nonlinear time delay systems and aim to design the smooth output feedback controller.

In this paper we focus on the controller design for the multiple time-delay system with nonlinear uncertainties. The uncertain functions are bounded by nonlinear functions with unknown gains. Then the dynamic compensator is designed to relax the control design conditions. The adaptive output feedback controller is designed based on the compensator. New Lyapunov functional is proposed to prove the stability of the whole system. Furthermore, the obtained result is further extended to the general nonlinear case and the corresponding dynamic output feedback control problem is discussed. Finally, simulations are performed and the results show the effectiveness of the proposed control method.

2. Problem formulation

Let us consider a nonlinear time delay system

$$\begin{cases} \dot{x}(t) = \sum_{j=0}^{r} A_j x(t-\tau_j(t)) + B(u(t) + f(x(t), \\ \quad x(t-\tau_1(t)), x(t-\tau_2(t)), \dots, x(t-\tau_r(t)))) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$ and $y(t) \in \Re^p$ are the state, control input and output of the system, respectively; $A_j \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{p\times p}$ are the known matrices. Without loss of generality, we assume $n \ge p > m$ and $Rank\,(CB) = m$; $\tau_j(t)$ are time-varying delays for $j \in [1, r]$, and $\tau_0(t) = 0$. The time-varying delays satisfy $\dot{\tau}_j(t) \le \tau_j^* < 1$ and $\tau_j(t) \le \overline{\tau}_j$, where τ_j^* and $\overline{\tau}_j$ are positive scalars; The nonlinear function vector $f(\cdot)$ is uncertain and contains the multiple delayed states with $f(0, 0, \dots, 0) = 0$.

For the uncertain nonlinear function vector $f(\cdot)$ of system (1), we impose the following assumption:

Assumption 1. The nonlinear function vector f satisfies

$$\begin{aligned} &\|f(x(t), x(t-\tau_1(t)), x(t-\tau_2(t)), \dots, x(t-\tau_r(t)))\| \\ &\quad \le \sum_{j=0}^{r} \theta_j^T \beta_j \left(\left\|x\left(t-\tau_j(t)\right)\right\|\right), \end{aligned} \tag{2}$$

where $\theta_j \in \Re^{p_j}$ are unknown constant vectors, $\beta_j(\cdot) = \left[\beta_{j1}(\cdot), \beta_{j2}(\cdot), \dots, \beta_{jp_j}(\cdot)\right]^T$, β_{ji} are known smooth class-$\Bbbk$ functions (strictly increasing and $\beta_{ji}(0) = 0$), and there exist continuous functions $\overline{\beta}_{ji}(\cdot)$ such that $\beta_{ji}(\chi) \le \chi \overline{\beta}_{ji}(\chi)$.

In this paper, we consider the following control problem: Design a delay dependent smooth dynamic output feedback controller for system (1) with Assumption 1 such that the solution of the closed-loop system converges to an adjustable region with an exponential decay rate.

Remark 1. System (1) contains the matched uncertain nonlinearities and mismatched functions with multiple time delays. The state feedback sliding mode control method was proposed in Xia and Jia (2003) and the stability of the closed-loop system was proven. The state feedback control problem was discussed for system (1) in Hua et al. (2008) and the memoryless controller was constructed. Han et al. (2009) designed a novel static output feedback controller. For system (1), Yan et al. (2009) constructed the full-order observer and further designed the observer based sliding mode output feedback controller. It is well known that the static output feedback control will impose some strict conditions on the considered system. In addition, the sliding mode control method requires the precise time delay values of system state for control implementation. In Yan et al. (2014), the memoryless static output feedback sliding mode control design problem was discussed. In this paper our objective is to design a smooth and memoryless dynamic output feedback controller for system (1).

Remark 2. With the parameters unknown, the adaptive method was developed to estimate system parameters or the bound parameters of uncertainties. For the time delay systems with all time delay functions matched with the control input, Mirkin and Gutman (2005), Wu (2004) and Zheng, Wang, and Lee (2005) introduced the memoryless adaptive state controller design methods and the recent work (Mirkin & Gutman, 2010) proposed a novel output feedback control design method. In Mirkin, Gutman, and Shtessel (2014), the decentralized adaptive control problem was considered with the unknown interconnections. By observing that there contains the nonlinear function in the system, we see that the former methods are not applicable to system (1).

Remark 3. For the problem formulated, there are three challenging issues as follows. The first one is how to design a dynamic compensator for the control design with the output signal y. The second one is how to deal with the unknown parameters of the uncertain nonlinear functions. The third one is how to design the smooth and memoryless controller for the multiple time-delay system. If the above three problems are solved, the controller will be designed with easy implementation in practical engineering systems.

Without the loss of generality, we assume that $B = \begin{bmatrix} 0_{(n-m)\times m} \\ \overline{B}_{m\times m} \end{bmatrix}$ and $C = \begin{bmatrix} 0_{(n-p)\times p} & D_{p\times p} \end{bmatrix}$ where $\overline{B}_{m\times m}$ is a non-singular matrix, D is an orthogonal matrix. If it is not the case, we may choose the coordination matrices to transform it into the standard form, see Edwards and Spurgeon (1998).

Let $x = \begin{bmatrix} x_1^T & x_2^T \end{bmatrix}^T$ where $x_1 \in \Re^{n-m}$ and $x_2 \in \Re^m$. System (1) is further written as the following form

$$\begin{cases} \dot{x}_1(t) = \sum_{j=0}^{r} \left(A_{j11} x_1\left(t-\tau_j(t)\right) + A_{j12} x_2\left(t-\tau_j(t)\right)\right) \\ \dot{x}_2(t) = \sum_{j=0}^{r} \left(A_{j21} x_1\left(t-\tau_j(t)\right) + A_{j22} x_2\left(t-\tau_j(t)\right)\right) \\ \quad + \overline{B}u(t) + \overline{B}f(x(t), x(t-\tau_1(t)), \dots, x(t-\tau_r(t))), \end{cases} \tag{3}$$