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Brief paper

Distributed circular formation control of ring-networked nonholonomic vehicles☆

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ABSTRACT

This paper investigates the circular formation control problem of multiple nonholonomic vehicles of unicycle type. The measurement of each vehicle is based on its local coordinate frame and the communication network among vehicles is modeled by a directed cycle graph. A distributed dynamic control law is designed by only using the local measurement of each vehicle and information of its neighbors in the network. The proposed control law guarantees that all vehicles move with a prespecified angular velocity along a common circle with the given center and radius, and maintain evenly spaced along the circle. Furthermore, the velocity constraint including positive-minimum linear velocity of each vehicle is explicitly taken into account.

1. Introduction

Circular formation control aims at designing a distributed control law such that a group of vehicles travel along a common circle and maintain evenly spaced. In practice, circular formation can be applied in the scenario where vehicles are required to enclose, capture, secure or monitor a target, e.g., mobile sensors for ocean sampling (Leonard et al., 2007).

Many efforts have been devoted to the study of circular formation control. Sepulchre, Paley, and Leonard (2007, 2008) presented a comprehensive investigation on the circular formation of multiple vehicles with identical unit linear velocity. In Sepulchre et al. (2007), gradient control laws based on potential functions were proposed for vehicles with all-to-all communication, which was generalized to vehicles with limited communication in Sepulchre et al. (2008). Chen and Zhang (2011) studied the collective circular motion under a jointly connected condition. An average system was used to approximate the closed-loop system by ignoring the first order of smallness $O(1/\omega_0)$, where ω_0 is the steady-state angular velocity. Then, stability analysis was given on the approximated system. Later, Chen and Zhang (2013) further considered the case where each vehicle has a local coordinate frame. El-Hawwary and Maggiore (2013) formulated the circular formation control problem as a set stabilization problem and proposed a hierarchical design approach.

Another research direction on this topic is to consider the ring-networked vehicles, which needs minimum communication links. Marshall, Broucke, and Francis (2004, 2006) considered the formation of multiple vehicles in cyclic pursuit and gave stability analysis on the linearized system. It was shown that the equilibrium formations of multi-vehicle systems are generalized regular polygon formations. Sinha and Ghose (2007) considered the cyclic pursuit problem of vehicles with heterogenous constant linear velocities. Later, Zheng, Lin, and Yan (2009) proposed a projection-based cyclic pursuit control law such that the trajectories of vehicles will never diverge. It is also noted that several works studied the case where all vehicles know the center and radius of the common circle and the common steady-state velocity. In particular, Ceccarelli, Di Marco, Garulli, and Giannitrapani (2008) considered the vehicle of which the onboard sensor has limited visibility region. In Summers, Akella, and Mears (2009), even spacing along a given circle was achieved by vehicles with velocity constraint. Lan, Yan, and Lin (2010) proposed a hybrid control law using local measurements. In addition, some results on networked double-integrators (Pavone & Frazzoli, 2007; Ramirez-Riberos, Pavone, Frazzoli, & Miller, 2010; Ren, 2009; Sharma, Ramakrishnan, & Kumar, 2013) can be applied to nonholonomic

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vehicles after local feedback linearization as shown in Ren and Atkins (2007). Recently, Seyboth, Wu, Qin, Yu, and Allgower (2014) and Zheng, Lin, Fu, and Sun (2015) studied circular formations where the linear velocities of vehicles are nonidentical in the steady state.

In this paper, the circular formation is prespecified with the center and radius of the common circle and the steady-state velocity of vehicles. Each vehicle has its local coordinate frame and the communication network is modeled by a directed cycle graph. A distributed dynamic control law without using any global information is proposed to globally stabilize vehicles to the prespecified circular formation. Similar to Sepulchre et al. (2008), the proposed dynamic control law requires both local measurement and communication.

The main contribution of this paper lies in the following aspects. First, the proposed control law for a prespecified circular formation does not rely on any global information, including the center and radius of the common circle, the steady-state velocity, the total number of vehicles, as well as a global inertial frame and a common reference direction. Second, global asymptotical stability of the closed-loop system instead of a linearized system (Marshall et al., 2004; Marshall, Broucke et al., 2006) or an approximated system (Chen & Zhang, 2011, 2013) is guaranteed. Third, the proposed control law explicitly takes into account the velocity constraint including positive-minimum linear velocity, and thus can be applied to vehicles subject to stall conditions, such as fixed-wing unmanned aerial vehicles (Ren & Beard, 2004).

This paper is organized as follows. In Section 2, the problem formulation and a technical lemma are given. In Section 3, the design procedure of the proposed control law is presented. Section 4 presents the main results and Section 5 shows the simulation results of an illustrative example. Finally, the conclusion is drawn in Section 6.

Notations: Throughout the paper, the norm $\|x\|$ of a vector $x \in \mathbb{R}^n$ is defined as $\|x\| = (\sum_{i=1}^{n} |x_i|^2)^{\frac{1}{2}}$.

2. Preliminaries

2.1. Problem formulation

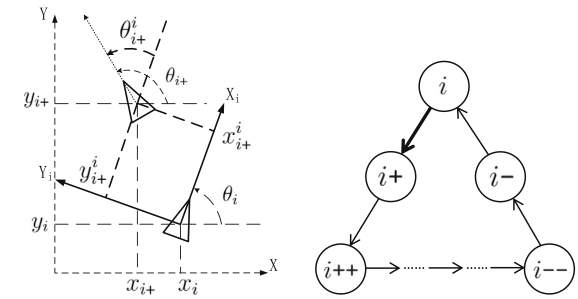
Consider N nonholonomic vehicles of unicycle type. The kinematic model of each vehicle is described by:

$$\dot{x}_i = v_i \cos\theta_i, \qquad \dot{y}_i = v_i \sin\theta_i, \qquad \dot{\theta}_i = \omega_i, \quad i = 1, \ldots, N, \quad (1)$$

where $p_i := [x_i\ y_i]^{\mathrm{T}} \in \mathbb{R}^2$ is the coordinate of the center of mass (position) and $\theta_i \in \mathbb{R}$ is the heading angle (orientation) of each vehicle in the inertial frame (see Fig. 1(a)). The linear velocity $v_i \in \mathbb{R}$ and the angular velocity $\omega_i \in \mathbb{R}$ are control inputs.

Based on the pursuit graph (Marshall, Broucke et al., 2006, Definition 2), the topology of the communication network among vehicles is modeled by a directed cycle graph $\mathcal{G} = \{\mathcal{O}, \mathcal{E}\}$ which consists of a finite set of nodes $\mathcal{O} = \{1, \ldots, N\}$ representing N vehicles, and a set of edges $\mathcal{E} = \{(j, i) : i \neq j,\ i, j \in \mathcal{O}\}$ containing directed edges from node j to node i. For each node i, there exists one incoming edge $(i-, i)$ and one outgoing edge $(i, i+)$, where node $i-$ is called the *pre-neighbor* of node i and node $i+$ is called the *next-neighbor* of node i (see Fig. 1(b)).

A directed edge $(i, i+)$ means that vehicle i can send its information to vehicle $i+$ and can measure the states of vehicle $i+$ in its local coordinate frame. The coordinate frame of vehicle i has the origin at its position p_i and the x-axis coincident with its orientation θ_i. The states $p_{i+} = [x_{i+}\ y_{i+}]^{\mathrm{T}}$ and θ_{i+} measured in the coordinate frame of vehicle i is denoted by p^i_{i+} and θ^i_{i+} respectively (see Fig. 1(a)). It is noted that the information flow of this network is not strictly one-way since the relative states between vehicle



(a) Local measurement. (b) Directed cycle graph $\mathcal{G}$.

Fig. 1. Illustration of measurement in the coordinate frame of vehicle i when there is a directed edge $(i, i+)$ in $\mathcal{G}$.

i and $i+$ are measured by vehicle i. Each vehicle i can measure neither its state $[p_i^{\mathrm{T}}\ \theta_i]^{\mathrm{T}}$ nor the relative position $p_i - p_j$ due to a lack of a global inertial frame and a common reference direction respectively.

A circular formation requires vehicles to move with a constant angular velocity ω_c along a common circle with the center $q_c := [x_c\ y_c]^{\mathrm{T}}$ and radius r_c, and to maintain evenly spaced along the circle. The counterclockwise ($\omega_c > 0$) *circular formation* is formally defined as:

Definition 2.1. A set $\bar{\mathcal{C}}_\rho(t) = \{[\bar{p}_i^{\mathrm{T}}(t)\ \bar{\theta}_i(t)\ \bar{\omega}_i(t)\ \bar{v}_i(t)]^{\mathrm{T}} \in \mathbb{R}^5,\ i = 1, \ldots, N\}$ is a *circular formation* with the *formation parameter* $\rho = [q_c^{\mathrm{T}}\ r_c\ \omega_c]^{\mathrm{T}} \in \mathbb{R}^2 \times \mathbb{R}^+ \times \mathbb{R}^+$, if

$$\bar{p}_i(t) - q_c = r_c[\sin\bar{\theta}_i(t)\ \ -\cos\bar{\theta}_i(t)]^{\mathrm{T}}, \quad (2)$$

$$\|\bar{p}_{i(i+)}(t)\| = \|\bar{p}_{i+(i++)}(t)\| = \cdots = \|\bar{p}_{(i-)i}(t)\|, \quad (3)$$

$$\bar{\omega}_i(t) = \omega_c,\ \bar{v}_i(t) = \omega_c r_c, \quad (4)$$

for all $t \geq 0$, where $\bar{p}_{i(i+)} = \bar{p}_i - \bar{p}_{i+}$. ■

Now, the *circular formation control problem* considered in this paper is formally defined as follows.

Definition 2.2. Consider N vehicles (1) and the communication digraph $\mathcal{G}$, and define $\mathcal{C}(t) = \{[p_i^{\mathrm{T}}(t)\ \theta_i(t)\ \omega_i(t)\ v_i(t)]^{\mathrm{T}} \in \mathbb{R}^5,\ i = 1, \ldots, N\}$. Given a formation parameter $\rho = [q_c^{\mathrm{T}}\ r_c\ \omega_c]^{\mathrm{T}} \in \mathbb{R}^2 \times \mathbb{R}^+ \times \mathbb{R}^+$, for vehicle i, $i = 1, \ldots, N$, with any initial states $[p_i^{\mathrm{T}}(t_0)\ \theta_i(t_0)]^{\mathrm{T}} \in \mathbb{R}^3$, $\forall t_0 \geq 0$, find a dynamic control law in the form of

$$[v_i\ \omega_i]^{\mathrm{T}} = \sigma(\hat{\rho}^i_i,\ m_i,\ \varrho^{i-}_{i-}), \qquad \dot{\hat{\rho}}^i_i = \kappa(\hat{\rho}^i_i,\ \varrho^{i-}_{i-}), \quad (5)$$

such that $\mathcal{C}(t)$ converges to a *circular formation* $\bar{\mathcal{C}}_\rho(t)$ as $t \to \infty$, where $\hat{\rho}^i_i$, to be designed later, is the estimate of ρ in the local coordinate frame, ϱ^{i-}_{i-} is the information of its *pre-neighbor*, m_i is the local measurement, and functions $\sigma(\cdot)$ and $\kappa(\cdot)$ are both sufficiently smooth. ■

In this paper, we consider the *circular formation control problem* under the following assumption:

[A1] Only one vehicle l knows the formation parameter in its local coordinate frame at the initial time t_0, i.e., $[q_c^{l\mathrm{T}}(t_0)\ r_c\ \omega_c]^{\mathrm{T}}$.

In general, vehicle l is anonymous and other vehicles cannot identify it.

Remark 2.1. To make vehicles maintain a cyclic pursuit manner, each vehicle is only required to connect to its neighbors by onboard point-to-point communication device and sensor. In Marshall, Fung, Broucke, D'Eleuterio, and Francis (2006), the practicality of cyclic pursuit as a distributed control strategy for multiple mobile robots was demonstrated by experiment. Since devices and sensors are usually distance-constrained in practice, vehicles need to locate within a bounded space such that the network is connected. ■