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Brief paper

Robust synthesis for linear parameter varying systems using integral quadratic constraints☆

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ABSTRACT

A robust synthesis algorithm is developed for a class of uncertain, linear parameter varying (LPV) systems. The uncertain system is described as an interconnection of a nominal LPV system and a block structured uncertainty. The nominal part is a "gridded" LPV system with state matrices that are arbitrary functions of the parameter. The input/output behavior of the uncertainty is described by integral quadratic constraints (IQCs). The robust synthesis problem leads to a non-convex optimization. The proposed algorithm is a coordinate-wise descent similar to the well-known DK iteration for μ synthesis. It alternates between an LPV synthesis step and an IQC analysis step. Both steps can be efficiently solved as semidefinite programs. The derivation of the synthesis algorithm is less obvious for LPV systems as compared to its LTI counterpart due to the lack of a valid frequency response interpretation. The main contribution is the construction of the iterative synthesis algorithm using time domain dissipation inequalities and a scaled system analogous to that appearing in μ synthesis. It is shown that the proposed algorithm ensures that the robust performance is non-increasing at each iteration step. The effectiveness of the proposed method is demonstrated on a simple numerical example.

1. Introduction

This paper considers the robust synthesis problem for a class of uncertain linear parameter varying (LPV) systems. The uncertain system is described as an interconnection of a nominal (not-uncertain) LPV system and a block structured uncertainty. The state matrices of the nominal system have an arbitrary dependence on parameters, i.e. it is a "gridded" LPV system. Such models arise naturally in many applications via linearization of a nonlinear model around parameterized operating (trim) points (Bobanac, Jelavić, & Perić, 2010; Moreno, Seiler, & Balas, 2012). The input/output behavior of the uncertainty is described by integral quadratic constraints (IQCs) (Megretski & Rantzer, 1997). The use of IQCs is sufficiently general to describe "uncertain" components that include nonlinearities, in addition to (parametric or dynamic) uncertainty.

The robust synthesis problem, formulated in Section 3.1, is to synthesize a controller that minimizes a closed-loop robust performance metric. This leads to a non-convex optimization that involves a search for both the controller state matrices and the IQC analysis variables. The proposed algorithm, given in Section 3.2, consists of a coordinate-wise descent similar to the well-known DK-iteration (Balas, Chiang, Packard, & Safonov, 2007; Zhou, Doyle, & Glover, 1996) for μ synthesis. Specifically, the proposed algorithm alternates between an LPV synthesis step and an IQC analysis step. The synthesis step essentially relies on existing results for nominal LPV systems in Wu, Yang, Packard, and Becker (1996). The analysis step is performed using a matrix inequality condition to bound the robust performance of the closed-loop uncertain LPV system (Section 4.1). Both steps can be efficiently solved as semidefinite programs (SDPs). The effectiveness of the proposed method is demonstrated on a numerical example in Section 5.

There are two main technical challenges. First, the nominal LPV system does not have a valid frequency response interpretation and hence the analysis requires a time domain approach. Section 4.1 develops a matrix inequality robustness analysis

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condition (Theorem 2) using (time domain) dissipation inequality techniques. This analysis condition is an extension of the worst-case gain condition in Pfifer and Seiler (2015). An alternative to the dissipation inequality based approach for IQCs in the time domain is given in Cantoni, Jönsson, and Khong (2013). It is purely based on operator theory and uses homotopy arguments to proof stability. This alternative approach can potentially be used to develop synthesis algorithms complementary to the one developed here or provide an alternative proof for the presented algorithm. The second technical challenge is that an appropriate scaled system must be constructed to link the analysis and synthesis steps. This construction, described in Section 4.2, is such that the next synthesis step on the scaled plant yields a controller that improves the closed-loop robust performance. These technical results are used to show the following main result in Section 4.3: the robust performance metric is non-increasing at each iteration step and hence the algorithm converges.

This paper builds on known results for both LPV systems and IQCs. A brief review is provided in Section 2. In addition, there are several related robust synthesis results for LPV systems whose state matrices have a rational dependence on the parameters (Apkarian & Adams, 1998; Scherer & Kose, 2012; Veenman & Scherer, 2010, 2014). This rational dependence leads to finite-dimensional matrix inequalities in the algorithm. In contrast, the algorithm in this paper is developed for the case where the state matrices have an arbitrary dependence on the parameters. This leads to parameter-dependent matrix inequalities for both the synthesis and analysis steps. As a result, parameter gridding is required to obtained finite-dimensional matrix inequality conditions. Finally, this paper builds on Wang, Pfifer, and Seiler (2014) which only considered LTI uncertainty. This paper extends the algorithm to uncertainties described by a general class of IQCs.

2. Background

2.1. Linear Parameter Varying (LPV) Systems

LPV systems are a class of systems whose state-space matrices are continuous functions of a time-varying parameter $\rho : \mathbb{R}^+ \to \mathbb{R}^{n_\rho}$. The set of admissible parameter trajectories is defined as $\mathcal{T} := \{\rho : \mathbb{R}_+ \to \mathbb{R}^{n_\rho} : \rho(t) \in \mathcal{P}\ \forall t \geq 0$ and $\rho(t)$ is continuously differentiable$\}$ where $\mathcal{P} \subset \mathbb{R}^{n_\rho}$ is a known compact set. In some applications, the parameter varying rate $\dot{\rho}$ are assumed to be bounded. However, only the rate unbounded case is considered here for simplicity. All results in this paper generalize, but with extensive notations, to the rate bounded case using existing results in Pfifer and Seiler (2015) and Wu et al. (1996). An n_Gth order LPV system, G_ρ, is defined by

$$\begin{bmatrix} \dot{x} \\ e \end{bmatrix} = \begin{bmatrix} A(\rho) & B(\rho) \\ C(\rho) & D(\rho) \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} \tag{1}$$

where $A : \mathcal{P} \to \mathbb{R}^{n_G \times n_G}$, $B : \mathcal{P} \to \mathbb{R}^{n_G \times n_d}$, $C : \mathcal{P} \to \mathbb{R}^{n_e \times n_G}$ and $D : \mathcal{P} \to \mathbb{R}^{n_e \times n_d}$. The performance of an LPV system G_ρ is specified by its induced L_2 gain $\|G_\rho\| := \sup_{0 \neq d \in L_2,\ \rho \in \mathcal{T}} \frac{\|e\|}{\|d\|}$. A generalization of the Bounded Real Lemma (Wu et al., 1996) provides a sufficient condition to upper bound $\|G_\rho\|$. The next theorem states the condition but simplified for the special case of rate unbounded LPV systems.

Theorem 1 (*Wu et al., 1996*). *G_ρ is exponentially stable and $\|G_\rho\| \leq \gamma$ if there exists $P = P^T \geq 0$ such that $\forall \rho \in \mathcal{P}$*

$$\begin{bmatrix} PA(\rho) + A(\rho)^T P & PB(\rho) \\ B^T(\rho)P & -I \end{bmatrix} + \frac{1}{\gamma^2} \begin{bmatrix} C(\rho)^T \\ D(\rho)^T \end{bmatrix} \begin{bmatrix} C(\rho) & D(\rho) \end{bmatrix} < 0. \tag{2}$$

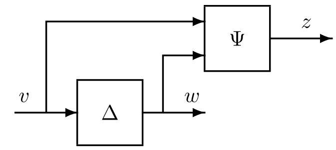

Fig. 1. Graphical interpretation of the IQC.

This theorem forms the basis for the induced L_2 norm controller synthesis in Wu et al. (1996). Consider an open loop LPV system G_ρ with inputs $[d^T, u^T]^T$ and outputs $[e^T, y^T]^T$. The objective is to synthesize a controller K_ρ:

$$\begin{bmatrix} \dot{x}_K \\ u \end{bmatrix} = \begin{bmatrix} A_K(\rho) & B_K(\rho) \\ C_K(\rho) & D_K(\rho) \end{bmatrix} \begin{bmatrix} x_K \\ y \end{bmatrix} \tag{3}$$

such that the closed-loop interconnection of G_ρ and K_ρ, which is given by the lower linear fractional transformation (LFT), denoted $\mathcal{F}_l(G_\rho, K_\rho)$, has the minimal induced L_2 gain: $\min_{K_\rho} \|\mathcal{F}_l(G_\rho, K_\rho)\|$. This LPV synthesis problem can be solved via parameterized LMI conditions. Details on the solution can be found in Wu et al. (1996). It should be noted that both the analysis and synthesis problems involve an infinite collection of LMI constraints parameterized by $\rho \in \mathcal{P}$. A remedy to this issue, which works in many practical examples, is to approximate the set $\mathcal{P}$ by a finite gridding set $\mathcal{P}_{grid} \in \mathcal{P}$.

2.2. Integral Quadratic Constraints (IQCs)

IQCs (Megretski & Rantzer, 1997) provide a framework for robustness analysis building on work by Yakubovich (1971). The IQC specifies a constraint on the input/output signals of the perturbation. The following definitions characterize the constraint in the frequency and time domain.

Definition 1. Let $\Pi \in \mathbb{RL}_\infty^{(n_v+n_w)\times(n_v+n_w)}$ be given. Two signals $v \in L_2^{n_v}[0,\infty)$ and $w \in L_2^{n_w}[0,\infty)$ satisfy the frequency domain IQC defined by the multiplier Π if

$$\int_{-\infty}^{\infty} \begin{bmatrix} \hat{V}(j\omega) \\ \hat{W}(j\omega) \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \hat{V}(j\omega) \\ \hat{W}(j\omega) \end{bmatrix} d\omega \geq 0 \tag{4}$$

where $\hat{V}$ and $\hat{W}$ are Fourier transforms of v and w. A bounded, causal operator $\Delta : L_{2e}^{n_v}[0,\infty) \to L_{2e}^{n_w}[0,\infty)$ satisfies the frequency domain IQC defined by Π if Eq. (4) holds for all $v \in L_2^{n_v}[0,\infty)$ and $w = \Delta(v)$.

Definition 2. Let Ψ be a stable LTI system, i.e. $\Psi \in \mathbb{RH}_\infty^{n_z \times (n_v+n_w)}$, and $M = M^T \in \mathbb{R}^{n_z \times n_z}$. Two signals $v \in L_{2e}^{n_v}[0,\infty)$ and $w \in L_{2e}^{n_w}[0,\infty)$ satisfy the time domain IQC defined by the multiplier Ψ and matrix M if the following inequality holds for all $T \geq 0$

$$\int_0^T z^T(t) M z(t)\, dt \geq 0 \tag{5}$$

where z is the output of Ψ driven by inputs (v, w) with zero initial conditions. A bounded, causal operator $\Delta : L_{2e}^{n_v}[0,\infty) \to L_{2e}^{n_w}[0,\infty)$ satisfies the time domain IQC defined by (Ψ, M) if Eq. (5) holds for all $v \in L_{2e}^{n_v}[0,\infty)$, $w = \Delta(v)$ and $T \geq 0$.

IQCs can be used to model a variety of nonlinearities and uncertainties, e.g. saturation and norm bounded uncertainty (Megretski & Rantzer, 1997). Fig. 1 provides a graphical interpretation for the time domain IQC. If Δ satisfies the time domain IQC defined by Ψ then the filtered signal z satisfies the constraint in Eq. (5) for any finite-horizon $T \geq 0$.