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Nonlinear and adaptive undecimated hierarchical multiresolution analysis for real valued discrete time signals via empirical mode decomposition approach

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ABSTRACT

Hierarchical multiresolution analysis is an important tool for the analysis of signals. Since this multiresolution representation provides a pyramid like framework for representing signals, it can extract signal information effectively via levels by levels. On the other hand, a signal can be nonlinearly and adaptively represented as a sum of intrinsic mode functions (IMFs) via the empirical mode decomposition (EMD) algorithm. Nevertheless, as the IMFs are obtained only when the EMD algorithm converges, no further iterative sifting process will be performed directly when the EMD algorithm is applied to an IMF. As a result, the same IMF will be resulted and further level decompositions of the IMFs cannot be obtained directly by the EMD algorithm. In other words, the hierarchical multiresolution analysis cannot be performed via the EMD algorithm directly. This paper is to address this issue by performing a nonlinear and adaptive hierarchical multiresolution analysis based on the EMD algorithm via a frequency domain approach. In the beginning, an IMF is expressed in the frequency domain by applying discrete Fourier transform (DFT) to it. Next, zeros are inserted to the DFT sequence and a conjugate symmetric zero padded DFT sequence is obtained. Then, inverse discrete Fourier transform (IDFT) is applied to the zero padded DFT sequence and a new signal expressed in the time domain is obtained. Actually, the next level IMFs can be obtained by applying the EMD algorithm to this signal. However, the lengths of these next level IMFs are increased. To reduce these lengths, first DFT is applied to each next level IMF. Second, the DFT coefficients of each next level IMF at the positions where the zeros are inserted before are removed. Finally, by applying IDFT to the shorten DFT sequence of each next level IMF, the final set of next level IMFs are obtained. It is shown in this paper that the original IMF can be perfectly reconstructed. Moreover, computer numerical simulation results show that our proposed method can reach a component with less number of levels of decomposition compared to that of the conventional linear and nonadaptive wavelets and filter bank approaches. Also, as no filter is involved in our proposed method, there is no spectral leakage in various levels of decomposition introduced by our proposed method. Whereas there could be some significant leakage components in the various levels of decomposition introduced by the wavelets and filter bank approaches.

1. Introduction

Hierarchical multiresolution analysis provides a pyramid like framework for the analysis of signals [1]. The global information of the signals can be obtained from the components in the coarse level, where the bandwidths of the components in the coarse level are large. Then, more detail information of the signals can be obtained by zooming the signals from the components in the coarse level to the components in the detail level, where the components in the detail level are with narrow bandwidths. This technique of

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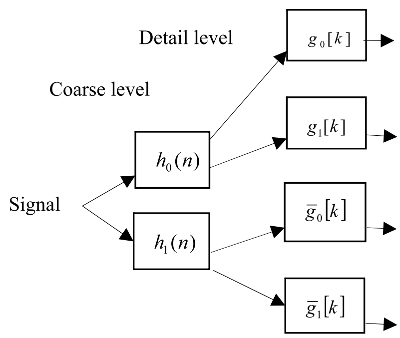

Fig. 1. Tree structure filter bank.

signal decompositions from the coarse level to the detail level is widely used in many engineering applications such as in the scalable image coding [11], edge detection of images [2], etc.

The most common hierarchical multiresolution analysis is based on the wavelets and tree structure filter bank approach [1]. The signal is linearly projected to a predefined wavelets kernel at the coarse level to obtain the coarse components. Then, the coarse components are further linearly projected to the same or different predefined wavelets kernel at a detail level to obtain the detail components. The tree structure filter bank is shown in Fig. 1. However, as the projection is linear and the wavelets kernel is predefined, this kind of linear and nonadaptive signal decompositions is not effective for some applications that required nonlinear and adaptive signal processing [8–10].

Recently, a new nonlinear and adaptive signal decomposition method based on the EMD approach is proposed [3,4]. The EMD algorithm is an iterative sifting process. First, the envelopes of the signal are obtained by interpolating its extrema via the cubic spline function. Then, the signal is subtracted from the average of its upper and lower envelopes. These procedures are iterated until the algorithm converges and the first IMF is obtained. Next, the signal is subtracted by the obtained IMFs. The above procedures are iterated until all the IMFs are obtained. From here, we can see that the decomposition does not depend on any predefined function except the cubic spline function. It mainly depends on the signal itself. Hence, the decomposition is adaptive. Moreover, the decomposition is nonlinear, so the EMD algorithm is useful for applications required nonlinear and adaptive signal processing [8–10]. However, as the sifting process is iterative and the IMFs are obtained only when the algorithm converges, no further iteration will be performed when the EMD algorithm is applied to these IMFs. This implies that the same set of IMFs will be resulted when the EMD algorithm is applied to these IMFs. Therefore, the components in the next level cannot be obtained directly via the EMD approach. This paper is to address this issue.

To perform the frequency domain analysis of the IMFs, the Hilbert–Huang transform method is applied [6]. In particular, a complex valued signal is defined in such a way that its real part is the IMF itself and its imaginary part is the Hilbert transform of the IMF. Then, the instantaneous frequency of the IMF is defined as the derivative of the phase of this complex valued signal. Also, the Hilbert spectrum is defined as a triple ordered pair of the frequency, the time and the amplitude of this complex valued signal. As the instantaneous frequency is a function of time, the Hilbert spectrum is a curve showing the relationship among the time, the frequency and the amplitude of the IMF in a three dimensional space. However, as the domain of the Hilbert spectrum of the IMF is not purely the time domain, the above procedures cannot be applied again on the obtained Hilbert spectrum of the IMF. In other words, the Hilbert–Huang transform method only provides the time–frequency information of the first level decomposition of the signal, but it does not provide the time–frequency information of the second level or further level decomposition of the signal. Hence, the hierarchical multiresolution decomposition is not supported by the Hilbert–Huang transform method. Besides, it is found that the IMFs obtained by applying the EMD algorithm to a fractional Gaussian noise are similar to the corresponding outputs of a dyadic nonuniform filter bank [5]. In other words, the IMFs of the fractional Gaussian noise are localized in the dyadic frequency bands. Nevertheless, there is no further nonuniform filter bank model on the obtained IMFs. From here, this is still a single level decomposition on the fractional Gaussian noise but this is not a tree structure decomposition on the fractional Gaussian noise. In other words, the hierarchical multiresolution decomposition is not supported by the dyadic nonuniform filter bank model.

The outline of this paper is as follows. A nonlinear and adaptive hierarchical multiresolution analysis based on the EMD algorithm via a frequency domain approach is proposed in Section 2. Computer numerical simulation results are presented in Section 3. Finally, a conclusion is drawn in Section 4.

2. Nonlinear and adaptive hierarchical multiresolution analysis based on the EMD algorithm via a frequency domain approach

For majority one dimensional signals, they can be decomposed into a finite number of IMFs [3]. Since the hierarchical multiresolution analysis cannot be performed directly via the EMD approach, this section is to address this issue by performing the hierarchical multiresolution analysis based on the EMD algorithm via a frequency domain approach.

Although most of existing results of the EMD are based on continuous time signals, the signals can be sampled to discrete time signals if the sampling frequency is higher than the Nyquist rate. The whole EMD process is actually implemented based on the discrete time signals because of the use of a computer.

The notation of our proposed algorithm is as follows. Denote the superscript "0" as the index referring to the first level of decomposition. Denote M_0 as the total number of IMFs in the first level of decomposition. Define m_0 as the index of the IMFs in the first level of decomposition. That is, $0 \le m_0 \le M_0 - 1$. Denote ξ as the index of the selected IMF in the first level for the further decomposition. That is, $\xi \in \{0, \ldots, M_0 - 1\}$. Denote N as the length of the ξth IMF. Let n be the time index of the ξth IMF. That is, $0 \le n \le N - 1$. After applying the DFT to the ξth IMF, we have a frequency domain representation of the ξth IMF. Denote k as the frequency index of the ξth IMF. That is, $0 \le k \le N - 1$. Denote a as the total number of zeros to be inserted to the ξth IMF in the frequency domain. Denote IS as the symbol referring to the inserted sequence. After inserting zeros to the DFT sequence, denote k' as the frequency index. That is, $0 \le k' \le N + a - 1$. After applying the IDFT to the inserted DFT sequence, denote n' as the time index. That is, $0 \le n' \le N + a - 1$. Then, perform the empirical mode decomposition to the inserted DFT sequence in the time domain. Denote the superscript "1" as the index referring to the second level of decomposition. Denote M_1 as the total number of IMFs in the second level of decomposition. Define m_1 as the index of the IMFs in the second level of decomposition. That is, $0 \le m_1 \le M_1 - 1$. In general, denote the lowercase letters c, c_{IS} and t as the symbols referring to the sequences in the time domain while the capital letters C, C_{IS} and T referring to the sequences in frequency domain.

The proposed algorithm (Algorithm 1) is summarized below and the flow chart of this algorithm is shown in Fig. 2.