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A new music-empirical wavelet transform methodology for time–frequency analysis of noisy nonlinear and non-stationary signals

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ABSTRACT

The goal of signal processing is to estimate the contained frequencies and extract subtle changes in the signals. In this paper, a new adaptive multiple signal classification-empirical wavelet transform (MUSIC-EWT) methodology is presented for accurate time–frequency representation of noisy non-stationary and nonlinear signals. It uses the MUSIC algorithm to estimate the contained frequencies in the signal and build the appropriate boundaries to create the wavelet filter bank. Then, the EWT decomposes the time-series signal into a set of frequency bands according to the estimated boundaries. Finally, the Hilbert transform is applied to observe the evolution of calculated frequency bands over time. The usefulness and effectiveness of the proposed methodology are validated using two simulated signals and an ECG signal obtained experimentally. The results demonstrate clearly that the proposed methodology is immune to noise and capable of estimating the optimal boundaries to isolate the frequencies from noise and estimate the main frequencies with high accuracy, especially the closely-spaced frequencies.

1. Introduction

An important issue in signal processing is the development of effective data analysis methods capable of modeling the underlying phenomena in signals obtained from physical or biological events. Most data analysis methods use a pre-determined basis to process data and therefore are considered non-adaptive or *rigid*. Many physical signals include significant noise and closely-spaced frequencies that cannot be effectively analyzed using these methods [12]. Further, real-life time series signals such as biomedical signals [38,43,30,48,8,2,9–11,6,7], or vibration signals obtained from civil structure subjected to dynamic excitations [33,14,26,13,32,27] include nonlinear and non-stationary properties that cannot be adequately analyzed using these methods. In order to overcome these limitations, time–frequency representation (TFR) has become a good alternative to analyze nonlinear and non-stationary signals since a TFR can provide information about the frequencies contained in signal over time [40].

In recent years, different TFRs have been used for analyzing nonlinear and non-stationary signals, such as short-time Fourier transform (STFT) [17], Wigner–Ville distribution (WVD) [46], wavelet transform (WT) [16,3], and Hilbert–Huang transform (HHT) [22]. Despite providing useful results, the aforementioned methods present some unresolved difficulties. For instance, STFT cannot adequately describe instantaneous frequencies due to its fixed window size [45]. The WVD introduces *cross-term interference* between estimated signal components which impedes efficient estimation of the instantaneous frequencies [39]. To overcome the limitations of STFT, the WT provides an efficient TFR for nonlinear and non-stationary signals since it decomposes the signal into multiple time–frequency levels retaining the transient characteristics of the analyzed signal [4,5,42]. Unfortunately, the WT capabilities are degraded in highly noisy environments. Further, WT is a non-adaptive method that is based on the use of some basis independent of the processed signal. It does not decompose a time signal according to its contained information and consequently cannot estimate the instantaneous frequencies effectively. To introduce adaptability, the wavelet packet transform [24,34,41] was introduced, but, it is still based on the use of prescribed basis limiting its adaptability.

In contrast, empirical mode decomposition (EMD) combined with Hilbert transform (HT), known as Hilbert–Huang transform

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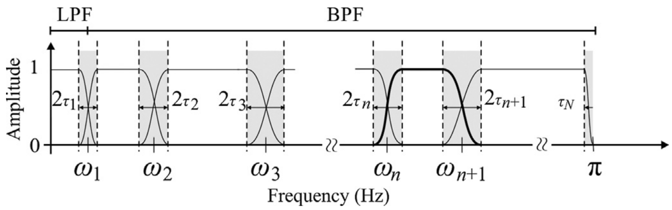

Fig. 1. EWT basis construction (LPF = low pass filter; BPF = band pass filter).

or HHT, is an adaptive signal processing method capable of analyzing time-varying or nonlinear signals according to information contained in the signal [22]. The HHT method also suffers from the mode mixing effect encountered in the EMD method which limits the accurate estimation of the instantaneous frequencies [47].

Recently, Gilles [19] introduced empirical wavelet transform (EWT) to lessen the problems found in the HHT method. EWT provides an efficient TFR of nonlinear and non-stationary time-series signals, since it can decompose the time signal according to frequency information contained in the signal. Unfortunately, the EWT capabilities are degraded under noisy signals because an unexpected segmentation of the signal may result limiting an accurate estimation of the instantaneous frequencies.

It is highly desirable to have an algorithm capable of estimating the fundamental frequencies of a signal with great accuracy, noise immunity, and without the need of significant additional computing processing resources. Jiang and Adeli [25] introduced a new vibration signal-based damage detection method based on a power density spectrum method, called *pseudospectrum*. They note that "*the pseudospectrum provides a reliable solution for eigenvalues of a non-normal matrix* (*whose eigenvectors are not orthogonal*)" and point out "*the non-normal matrices exist in the chaotic motion of transient nonlinear systems represented by measured data*." They employ the multiple signal classification (MUSIC) method to determine the pseudospectrum from the time series of the structural response. This MUSIC method produces a high-resolution spectral or frequency content estimation from a set of eigenvectors of an autocorrelation matrix generated by the input signals, even for data with high noise or a low signal-to-noise ratio. It provides an increased detectability of the fundamental frequencies contained in the signal, especially, the closed spaced frequencies compared to the conventional FFT. Jiang and Adeli [25] apply the MUSIC methodology in combination with their dynamic wavelet neural network (WNN) to detect damage in highrise buildings with only a small amount of sensed data successfully.

The work of Jiang and Adeli [25] leads us to conclude that the MUSIC algorithm can be an effective tool for estimating the appropriate boundaries of the wavelet filter bank. In this paper, a new MUSIC-EWT methodology is presented to obtain accurate time–frequency representation of a noisy signal. To demonstrate the effectiveness and usefulness of the proposed methodology, it is applied to a) a simulated free vibration problem with two closely-spaced frequencies and a high-level of noise, b) a non-stationary simulated signal with a high-level of noise, and c) a real-life example of the electrocardiogram (ECG) signal. The goal of the first two examples is to validate the accuracy and noise immunity of the proposed methodology for estimating the appropriate boundaries to create the wavelet filter bank. The efficacy of the proposed methodology to estimate the boundaries of the EWT and frequencies of time-series signal is compared with the original EWT proposed by Gilles [19]. Further, in order to demonstrate the advantages of the proposed methodology, it is compared with three advanced and widely-used signal processing techniques: Hilbert-Huang transform (HHT), the discrete wavelet transform (WT), and Extended Compact Kernel (ECK) [15].

2. Empirical wavelet transform

EWT is an adaptive wavelet transform capable of extracting individual instantaneous frequencies of a time series signal [19]. To provide signal processing adaptability, the segmentation of the Fourier spectrum is the most important step in the EWT. First, the local maxima of the Fourier spectrum $x(\omega)$ are estimated. Next, the boundaries of various frequencies ω_i are defined as the center between two consecutive maxima. Thus, the Fourier support $[0, \pi]$ is segmented into N contiguous segments. Each segment or frequency band is indicated by $S_n = [\omega_{n-1}, \omega_n]$ and $U_{n=1}^{N} S_n = [0, \pi]$ where U represents the union operator. A transition phase of width $2\tau_n$ is defined around each ω_n (Fig. 1). A more detailed explanation of how to select τ_n can be found in Gilles [19]. The empirical wavelets are defined as one low-pass $\phi_n(\omega)$ and $N-1$ band-pass $\psi_n(\omega)$ filters corresponding to the approximation and details components, respectively, on each S_n.

Fig. 1 displays an example of low-pass and band-pass wavelet filters identified as LPF, and BPF on the figure, where the vertical axis corresponds with the amplitude of the filters. The low-pass filter has a cut-off frequency of ω_1, which corresponds to the first estimated boundary. The first band-pass filter has a frequency band $[\omega_1, \omega_2]$, the second band-pass filter has a frequency band $[\omega_2, \omega_3]$, and the last band-pass filter has a frequency band $[\omega_{n+1}, \omega_\pi]$.

Following the idea used in deriving the Meyer's wavelet, Gilles [19] defines the empirical scaling function as [16]:

$$\phi_n(\omega) = \begin{cases} 1 \quad \text{if } |\omega| \le \omega_n - \tau_n \\ \cos[\frac{\pi}{2}\beta(\frac{1}{2\tau_n}(|\omega| - \omega_n + \tau_n))] \\ \quad \text{if } \omega_n - \tau_n \le |\omega| \le \omega_n + \tau_n \\ 0 \quad \text{otherwise} \end{cases} \tag{1}$$

and the empirical wavelet function as

$$\psi_n(\omega) = \begin{cases} 1 \quad \text{if } \omega_n + \tau_n \le |\omega| \le \omega_{n+1} - \tau_{n+1} \\ \cos[\frac{\pi}{2}\beta(\frac{1}{2\tau_{n+1}}(|\omega| - \omega_{n+1} + \tau_{n+1}))] \\ \quad \text{if } \omega_{n+1} - \tau_{n+1} \le |\omega| \le \omega_{n+1} + \tau_{n+1} \\ \sin[\frac{\pi}{2}\beta(\frac{1}{2\tau_n}(|\omega| - \omega_n + \tau_n))] \\ \quad \text{if } \omega_n + \tau_n \le |\omega| \le \omega_n + \tau_n \\ 0 \quad \text{otherwise} \end{cases} \tag{2}$$

where $\beta(x)$ is any arbitrary polynomial function with values in the range $[0, 1]$ with the following properties:

$$\beta(x) = \begin{cases} 0 \quad \text{if } x \le 0 \\ \quad \text{and } \beta(x) + \beta(1-x) = 1 \quad x \in [0, 1] \\ 1 \quad \text{if } x \ge 1 \end{cases} \tag{3}$$

Many polynomial functions satisfy these properties. The following polynomial first suggested by Daubechies [16] and used by Gilles [19] is also used in this research:

$$\beta(x) = x^4(35 - 85x + 70x^2 - 20x^3) \tag{4}$$

After the wavelet filters have been built (Eqs. (1) and (2)), the time series signal $x(t)$ is decomposed into different frequency bands through empirical wavelet transform defined by

$$W_f^\varepsilon(n, t) = F^{-1}(x(\omega)\psi_n(\omega)) \tag{5}$$

$$W_f^\varepsilon(0, t) = F^{-1}(x(\omega)\varphi_n(\omega)) \tag{6}$$

where the details $W_f^\varepsilon(n, t)$ and approximation $W_f^\varepsilon(0, t)$ coefficients are obtained by the inner products of the signal with the empirical wavelets low-pass and band-pass filters, respectively, and F^{-1} denotes the inverse Fourier transform.

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