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Brief paper

Optimal control over multiple erasure channels using a data dropout compensation scheme☆

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ABSTRACT

In this paper we focus on networked control systems subject to data dropouts. We consider that the controller communicates with the plant through multiple independent erasure channels, and that each actuator sends acknowledgement packages to the controller. We restrict our attention to the optimal design of a class of controllers that embeds a data dropout compensator and where, besides this compensation mechanism, all processing is an affine function of the available data. Our first result shows that the optimal design consists in solving, separately, an estimation problem and a control problem. The optimal controller is a linear function of the estimated state, which is obtained exploiting the information given by the acknowledgement packages and the data dropout compensator. Since we focus on a specific class of controllers, our proposal is suboptimal, however, unlike the optimal LQG controller found in the literature, our proposal is easier to implement, and its steady state behaviour can be simply determined. We also show that, under standard conditions, the affine part of the optimal controller converges to an LTI filter, which can be used as a simple alternative to the time varying controller.

1. Introduction

Networked control systems (NCSs) are control systems closed over non-transparent communication links. Such systems have received much interest due to their many practical benefits. However, in NCSs there are also several practical limitations imposed by networks such as random delays, packet dropouts, and quantization errors (see, e.g. Hespanha, Naghshtabrizi, & Xu, 2007 and the references therein). In this paper we focus on packet dropouts, which can be modelled by Bernoulli stochastic processes (Elia & Eisenbeis, 2011; Schenato, Sinopoli, Franceschetti, Poolla, & Sastry, 2007). Several works have been focused on determining conditions in which the NCS can be stabilized (see e.g. Elia & Eisenbeis, 2011; Zhang & Yu, 2007), and have shown that there exists a threshold for the probabilities of successful transmission below which stabilization is not possible. Provided that these conditions are met, one can deal with the performance problem where, in general, three main setups can be found: control packets can be dropped, measurement packets can be dropped, or both control and measurement packets can be dropped.

In the first setup, the control signal may not reach the actuator and, hence, the focus is on setting the plant input properly. Some works consider a simple compensation scheme in which the actuator receives *zero-input* action if the control packet is lost during transmission (Imer, Yüksel, & Başar, 2006). Another popular compensation scheme corresponds to the *hold-input* action (Sun, Xie, Xiao, & Soh, 2008), in which the actuator receives the last available control signal if an outage occurs. A generalized hold-input strategy was presented in Moayedi, Foo, and Soh (2010) to replace the lost control signal, which covers the *zero-input* and *hold-input* strategies as special cases.

On the other hand, when only measurement packets may be dropped, the question that arises is how to set the controller input to replace the lost data. In that context, optimal estimation is studied in Sinopoli et al. (2004), where the standard Kalman filter is adapted to deal with missing data assuming instantaneous knowledge of the sensor-to-controller channel state. The optimal estimator gain is time-varying and stochastic. As a consequence, the optimal estimator does not converge to a steady state and, furthermore, the estimation error covariance matrix becomes a random variable. The above limitations have motivated the design of alternative filters that, beside being suboptimal, are

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computational inexpensive and simpler to implement. With that aim in mind, a constant-gain Kalman filter for a steady state framework is proposed in Schenato (2008). An extension of the results in Schenato (2008) that deals with multiple sensors subject to independent packet loss is given in Schenato (2007). A suboptimal time-varying filter is given in Zhang, Song, and Shi (2012), where has been shown that, under some standard conditions, it converges to the constant gain Kalman filter in Schenato (2008). More basic strategies consider that the missing data is replaced by zeros (zero-input strategy) (Hadjicostis & Touri, 2002), by the latest measurement received at the controller (hold-input strategy) (Sun et al., 2008) and by a weighted value of the last available measurement (generalized hold-input strategy) (Zhang, Yu, & Feng, 2011). In Ling and Lemmon (2003), a data-dropout compensator that minimizes the overall performance among the classical *zero-input* and *hold-input* schemes is given. However, this approach is limited to SISO systems, has only one channel in the loop, and the estimator is an LTI filter, thus, suboptimal. Inspired by Ling and Lemmon (2003), a linear time-varying estimator that embeds a data-dropout compensator is proposed in Silva and Solis (2013) which encompasses the above suboptimal strategies and, additionally, shows that the minimal steady state estimation error-covariance for such class of estimators is achieved with the constant gain Kalman filter obtained in Schenato (2008). The estimator presented in Silva and Solis (2013) was successfully exploited in Silva, Vargas, and Maass (2013) to design optimal controllers assuming no control signal loss.

In the third setup, when both links are subject to data loss, a mixture of the previously discussed strategies are employed for control design (Moayedi et al., 2010; Zhang & Yu, 2007). One of the most relevant results is reported in Schenato et al. (2007), in which the optimal LQG problem is solved, and where the use of acknowledgement packages sent from the actuator to the controller plays a crucial role. If such packets are not available (UDP-like protocol), then the optimal controller is in general non-linear. However if such acknowledgements are available (TCP-like protocols), then separation principle holds, and the optimal LQG controller is a linear function of the estimated state, which can be obtained exploiting the results in Sinopoli et al. (2004). Now, since the optimal estimator gain is a random variable, the optimal cost cannot be computed analytically and its steady state value cannot be explicitly determined, however it can be bounded. An extension of these results to multiple channels is given in Garone, Sinopoli, Goldsmith, and Casavola (2012) assuming a TCP-like protocol for both finite and infinite horizon control problems. A simpler but non trivial alternative approach is presented in Silva, Maass, and Vargas (2014), where the standard LQG cost function is minimized based on the class of estimators with the data-dropout compensator presented in Silva and Solis (2013). They showed that the separation principle also holds but, unlike (Schenato et al., 2007), the optimal cost can be easily characterized and, indeed, it coincides with the upper bound of the minimum cost given in Schenato et al. (2007).

As stated above, several key results given for single erasure channels have been extended to the multiple channel scenario. Similarly, in this paper we complete our previous set of works (Silva et al., 2014, 2013), dealing with a more general setup. That is, here we consider NCSs where both the communication from sensor-to-controller and controller-to-actuator are made through multiple independent erasure channels. We focus our attention in a class of controllers that embeds a data-dropout compensator mechanism and where almost all processing is an affine function of the collected data. The optimal controller is obtained in such setup for both finite and infinite horizon and we show that separation principle holds provided that TCP like protocol is used. Our optimal design is computationally inexpensive, easy to implement, and its steady state behaviour can be easily characterized. Thus, our proposal is an alternative to the optimal LQG controller given in Garone et al. (2012). Also, we show that, under certain conditions, the affine part of the optimal controller converges to an LTI filter. This LTI filter is the optimal among all the LTI filters that use the proposed compensation scheme, and it can be used as a simpler alternative to the optimal time-varying controller. In order to obtain our results we derived an extension of the class of optimal estimators with the data-dropout compensator presented in Silva and Solis (2013) to a multiple sensor scenario. These estimators include the well-known zero-input, hold-input and generalized hold-input strategies as special cases.

The paper is organized as follows. Section 2 states the problem formulation. Optimal estimation is given in Section 3, while the optimal controller designs are derived in Section 4. Section 5 focuses on the steady state behaviour. Numerical examples are given in Section 6, followed by conclusions in Section 7.

Notation. $\mathbb{N}$ and $\mathbb{R}$ denotes the natural and the real numbers respectively. I_n denotes the $n \times n$ identity matrix and $\odot$ the standard Hadamard product (Bernstein, 2005), while $\text{diag}\{e_i\}$ defines a diagonal matrix with elements e_i for $i \in \{1, \dots, n\}$. Given a matrix A, A^T denotes its transpose, while $A > 0$ (resp. $A \geq 0$) points out that A is positive definite (resp. semi-definite). For any $x \in \mathbb{R}^n$ and $M \geq 0$, $\|x\|_M^2 \triangleq x^T M x$. For any sequence x, x_k denotes its kth sample, while x^k is used as shorthand for $x_0, \dots, x_k$. $\Pr\{\cdot\}$ and $\mathcal{E}\{\cdot\}$ denote the probability and expectation operators respectively. If a and b are random variables, then $\mu_a \triangleq \mathcal{E}\{a\}$, $P_a \triangleq \mathcal{E}\{(a-\mu_a)(a-\mu_a)^T\}$ and $P_{ab} \triangleq \mathcal{E}\{(a-\mu_a)(b-\mu_b)^T\}$. If x is an stochastic process, then μ_x and P_x denote its steady-state covariance matrix (if exists). $\hat{\mathcal{E}}\{a|b\} \triangleq \mu_a + P_{ab}P_b^{-1}(b-\mu_b)$ denotes the best linear least squares estimator of a, given b (Doob, 1953).

2. Setup and problem definition

Consider the NCS architecture of Fig. 1, where P is a MIMO discrete-time LTI plant modelled by

$$x_{k+1} = Ax_k + Bu_k + v_k, \quad k \in \mathbb{N},\ x_0 = x_o, \tag{1a}$$

$$y_{i,k} = C_i x_k + e_{i,k}, \quad i = 1, \dots, m, \tag{1b}$$

where m is the total number of sensors, $x_k \in \mathbb{R}^{n_x}$ is the state, x_o is the initial state, $y_{i,k} \in \mathbb{R}^{n_i}$ is the *i-th* sensor output and $n_y \triangleq \sum_{i=1}^m n_i$, $u_k \in \mathbb{R}^{n_u}$ is the control input, v_k models process noise, $e_{i,k}$ corresponds to the *i-th* measurement noise and (A, B, C_i) are known matrices of appropriate dimensions. We assume that x_o is a second-order random variable with mean μ_o and covariance matrix $P_o \geq 0$, independent of $(v_k, e_{1,k}, \dots, e_{m,k})$, and that $(v_k, e_{1,k}, \dots, e_{m,k})$ are zero-mean mutually independent i.i.d. sequences, having constant covariance matrices $P_v \geq 0$ and $P_{e_i} > 0\ \forall i$. The plant P has to be controlled by a remote controller which communicates with the plant over multiple unreliable channels in both the sensor-to-controller and controller-to-actuator links (see Fig. 1). Due to the channels unreliability, packets may be randomly lost during communication. We model this scenario as follows. We assume that each sensor $i = 1, \dots, m$ sends a packet to the controller and it can be lost according to a Bernoulli random binary process $\theta_{i,k}$ $(i = 1, \dots, m)$, such that, $\theta_{i,k} = 1$ if the packet is successfully transmitted to the controller and $\theta_{i,k} = 0$ if it is lost. Similarly, the controller sends to each actuator $j = 1, \dots, n$ a control packet $u_{j,k}^c \in \mathbb{R}^{\ell_j}$, where $\sum_{j=1}^n \ell_j = n_u$, and it can be randomly lost according to another Bernoulli random binary process $\lambda_{j,k}$ $(j = 1, \dots, n)$. We can describe this situation for all the channels involved in the network, by the following multiple parallel erasure channels model:

$$y_k^c = \Theta_k y_k, \qquad u_k = \Lambda_k u_k^c, \tag{2}$$