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A tutorial – game theory-based extended H infinity filtering approach to nonlinear problems in signal processing

Jaechan Lim

Department of Future IT Innovation Laboratory and Creative IT Engineering, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea

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ABSTRACT

In this paper, we provide a tutorial for the applications of "game-theory-based extended H infinity filtering (EHIF)" approach to various problems in disciplines of signal processing. The algorithm of this filtering approach is similar to that of the extended Kalman filtering (EKF). Since its invention, the Kalman filtering approach has been successfully and widely employed for many problems in scientific and engineering fields, e.g. target tracking, satellite systems, control, communications, etc. Therefore, the H infinity filtering approach also can be applied to all these problems. One big difference of EHIF from the EKF approach is that we apply it with unknown noise statistics of the state and measurement. In this tutorial, we introduce this non-well-known approach in spite of its practical usefulness, by providing the step by step algorithm with example problems of a number of signal processing disciplines. We also show that EHIF can outperform other approaches including the EKF that need to know the noise statistics in their applications, in some scenarios. By the contribution of this tutorial, we look forward to easy, and disseminative applications of EHIF to problems where, particularly, the EKF or particle filter could have been applied if noise statistics were known.

1. Introduction

Many problems in the disciplines of signal processing areas, a parameter (state) of interest is estimated based on some measurement. Particularly, if the system model can be described by the discrete-time-varying states with corresponding measurement, we can estimate the states sequentially by applying dynamic filters. The dynamic state system that describes the hidden state $\boldsymbol{x}$ and observed measurement $\boldsymbol{y}$ with zero mean and additive noise processes of $\boldsymbol{u}$ and $\boldsymbol{w}$ at time step k can be described as follows:

$$\boldsymbol{x}_k = g(\boldsymbol{x}_{k-1}) + \boldsymbol{u}_k, \tag{1}$$

$$\boldsymbol{y}_k = h(\boldsymbol{x}_k) + \boldsymbol{w}_k, \tag{2}$$

where bold face denotes a vector representation, $g(\cdot)$ and $h(\cdot)$ are the given state transition and the observation function, respectively, and are possibly nonlinear with respect to "$\boldsymbol{x}$." And, that "whether the problem is linear or nonlinear" is determined by if these functions are linear or nonlinear with respect to "$\boldsymbol{x}$." According to this system model, we can estimate the time-varying state $\boldsymbol{x}_k$ sequentially based on the corresponding measurement $\boldsymbol{y}_k$ at each time step by the dynamic filters such as the Kalman filter. The minimum mean squared error (MMSE) estimator will obtain the following:

$$\hat{\boldsymbol{x}}_k^{\mathrm{MMSE}} = \int \boldsymbol{x}_k p(\boldsymbol{x}_{0:k}|\boldsymbol{y}_{1:k}) d\boldsymbol{x}_k, \tag{3}$$

where "$1:k$" indicates the time indices from 1 to k, "ˆ" denotes an estimated version, and $p(\boldsymbol{x}_{0:k}|\boldsymbol{y}_{1:k})$ is the posterior density. In a particular case of: $g(\cdot)$ and $h(\cdot)$ are linear functions; and $\boldsymbol{u}_k$ and $\boldsymbol{w}_k$ are Gaussian distributed, the Kalman filter (KF) is the optimal MMSE estimator [1]. The KF estimates the state sequentially in a closed form as follows:

$$\hat{\boldsymbol{x}}_k^{\mathrm{KF}} = \bar{\boldsymbol{x}}_k + \mathcal{K}_k\big[\boldsymbol{y}_k - h(\bar{\boldsymbol{x}}_k)\big], \tag{4}$$

where $\mathcal{K}_k$ is the Kalman gain that is computed by the algorithm at each time step, and $\bar{\boldsymbol{x}}_k = g(\hat{\boldsymbol{x}}_{k-1}^{\mathrm{KF}})$. The KF is successfully applied to many nonlinear problems as well, by the Taylor expansion approximation. Since its invention [1], the KF has been successfully employed in many problems of scientific and engineering fields [2–12] e.g. target tracking, time-series analysis, satellite systems, control, communications, bio-medical image classification, speech recognition, etc.

The Kalman filter requires the knowledge of noise statistics in its application. Therefore, it is assumed that the mean and the variance of $\boldsymbol{u}$ and $\boldsymbol{w}$ are assumed to be known/obtained in their applications. However, we may encounter the situation when we may not able to know nor estimate the information of the noise statistics in practice. Therefore, in this paper, we introduce the

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approach, i.e. H infinity filter (HIF) that we can apply in this situation. Although we need a preliminary tuning process for weighting factors and the performance bound depending on the system model. Although HIF algorithm is similar with that of the Kalman filter, it is not a Bayesian approach because the estimate is not obtained probabilistically in a form of (3). Although the HIF approach is very useful with satisfactory performance (particularly in unknown noise statistics scenario), it is not well-known to researchers. In this paper, we explain the step by step algorithm of HIF (specifically, its extended version for non-linear problems) with a number of example problems where HIF can be successfully applied. Besides, we assess the performance of extended HIF (EHIF) with that of particle filtering [13–18], unscented Kalman filtering [19–22]. Consequently, we expect the contribution of this tutorial to easy, disseminative applications of EHIF for many important problems where, particularly, the Kalman filtering could have been successfully applied if noise statistics were known. Further reading related with EHIF can be referred to [23,24].

For readability facilitation, the list of abbreviations used in this paper is provided as follows:

List of abbreviations

- ACK: acknowledgment
- BER: bit error rate
- CIR: channel impulse response
- CFO: carrier frequency offset
- CTS: clear-to-send
- CRPF: cost reference particle filter
- CSMA/CA: carrier sense multiple access with collision avoidance
- CUSUM: cumulative summary
- DCF: distributed coordination function
- DIFS: distributed inter-frame space
- EHIF: extended H infinity filter
- EKF: extended Kalman filter
- GPF: Gaussian particle filter
- HIF: H infinity filter
- ISI: inter-symbol interference
- KF: Kalman filter
- MAC: medium access control
- MMSE: minimum mean squared error
- MSE: mean squared error
- OFDM: orthogonal frequency division multiplexing
- PF: particle filter
- PHY: physical layer
- PMF: probability mass function
- QPSK: quadrature phase-shift keying
- RMSE: root mean squared error
- UKF: unscented Kalman filter
- RTS: request-to-send
- SIFS: short inter-frame space
- SNR: signal to noise ratio
- SPF: sequential importance resampling particle filter
- WLAN: wireless local area network

2. Extended H infinity filtering

The original, continuous-time H infinity filter (HIF) was employed in control area, and HIF has not been widely employed due to its high level of mathematical understanding and the requirement of a good system modeling. Recently proposed HIF applications are designed to guarantee the H infinity norm, which is defined in (5) below (this is for the discrete-time case), less than a prescribed value based on noise signals and resulting estimation errors. In this estimation, the noise source can be arbitrary while it is bounded by a certain threshold, and the worst possible amplification of an error signal is minimized; therefore, it can be thought of as a minimax approach. The approach is similar with that of zero-sum game where minimax solution tries to minimize the maximum expected point-loss independent of the opponent's strategy. In the game of HIF, the filter designer prepares for the worst case that the opponent player (factors for noise that incurs error) can provide. In other words, the goal of the filter is to obtain a uniformly small estimation error for any combinations of state process noise, measurement noise, and any initial states. In the game of the discrete time HIF (so called "quadratic difference game" [25] whereas it is called "quadratic differential game" for continuous cases [26]), the filter minimizes the estimated state error while the noise factors are maximized. Therefore, the minimizer obtains the optimal filtered estimate while the maximizer tries to give the combination of "the worst-case disturbance" and "the worst initial error condition." Consequently, HIF does not require the prior knowledge of noise statistics, and deals with deterministic noisy disturbance in its applications [25] as opposed to the case of the Kalman filtering. Whereas the EKF minimizes mean squared error (MSE) of the estimate (equivalently, minimizing the variance of the estimation error), EHIF is designed to minimize the worst possible error. In the game, the loss (cost) for the estimator is a measure of performance (which needs to be minimized for better performance) in this filter. Discrete-time HIF can be interpreted as a minimax problem where the estimator, i.e. numerator of (5) below, plays against the exogenous inputs and the initial state uncertainty, i.e. denominator of (5) below. Accordingly, the cost function for the designer in discrete-time HIF is defined as follows:

$$J = \frac{\sum_{k=0}^{N-1} \|\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k\|^2_{\chi_k}}{\|\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0\|^2_{P_0^{-1}} + \sum_{k=0}^{N-1}(\|u_k\|^2_{W_k^{-1}} + \|w_k\|^2_{V_k^{-1}})}, \tag{5}$$

where N is the number of total time steps, $\hat{\boldsymbol{x}}_k$ is the estimated state at time step k, u_k is the state noise, w_k is the observation noise, and $\boldsymbol{x}_0$ is the initial state, respectively. χ_k, P_k, W_k and V_k are weighting factors, and $\|\cdot\|$ denotes the vector norm, i.e. $\|u_k\|^2_{W_k^{-1}}$ implies $u_k^\top W^{-1} u_k$ where $\top$ denotes the matrix transpose. The way how the weighting factors are determined is that, for instance: if it is known that the second element of w_k is small, then $V_k(2,2)$ is chosen to be small compared to other elements. If the state ($\boldsymbol{x}$) to be estimated is a scalar, we need a simpler preliminary tuning process than a vector case for satisfactory tracking performance.

Direct minimization of J is not tractable; therefore, the performance bound γ is introduced, and it satisfies

$$\sup J < \gamma^{-1}, \tag{6}$$

where, "sup" denotes "supremum." Then, J' is defined as

$$J' = -\gamma^{-1}\|\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0\|^2_{P_0^{-1}} + \sum_{k=0}^{N-1}\big[\|\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k\|^2_{\chi_k} - \gamma^{-1}\big(\|u_k\|^2_{W_k^{-1}} + \|w_k\|^2_{V_k^{-1}}\big)\big], \tag{7}$$

and, the problem becomes a matter of solving the following minimax problem:

$$\min_{\hat{\boldsymbol{x}}_k}\Big(\max_{u_k, w_k, \boldsymbol{x}_0} J'\Big) \tag{8}$$

The EHIF approach which solves above minmax problem is given by [25]

$$\hat{\boldsymbol{x}}_k = g(\hat{\boldsymbol{x}}_{k-1}) + H_k\big[\boldsymbol{y}_k - h(\hat{\boldsymbol{x}}_{k-1})\big] \tag{9}$$