Automatica 68 (2016) 184–193

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stabilization and robustness analysis for a chain of exponential integrators using strict Lyapunov functions ☆

Michael Malisoff[a], Miroslav Krstic[b,1]

[a] Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803-4918, USA

[b] Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093-0411, USA

ARTICLE INFO

Article history:
Received 27 July 2015
Received in revised form
10 November 2015
Accepted 15 January 2016
Available online 22 February 2016

Keywords:
Strict Lyapunov functions
Input-to-state stability

ABSTRACT

We provide a new method for building strict Lyapunov functions for two dimensional chains of exponential integrators, using nested exponential functions. One challenge is that the right sides of the systems saturate, so they are not completely controllable. The strictness of the Lyapunov functions is key to proving input-to-state stability (or ISS) properties with respect to additive uncertainty on the controls. We show how a large class of tracking problems for nonlinear systems with positivity constraints on the states can be solved using our theory.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

This paper continues our search (begun in Malisoff & Mazenc, 2009; Mazenc & Malisoff, 2006, 2010; Mazenc, Malisoff, & Bernard, 2009) for new constructions of strict Lyapunov functions that can be used to prove stability and robustness properties for nonlinear control systems. In some cases, stabilization problems can be solved with the help of nonstrict Lyapunov functions, which are proper and positive definite functions whose time derivatives are nonpositive along all solutions of the closed loop system. By proper and positive definiteness of a function V, we mean that V is zero at the equilibrium, positive at all other states, and satisfies $V(Z) \to \infty$ as $|Z| \to \infty$ or as Z approaches the boundary of the state space. However, nonstrict Lyapunov functions by themselves are insufficient to solve asymptotic stabilization problems, since they do not ensure convergence to the equilibrium. Instead, one often uses nonstrict Lyapunov functions in conjunction with LaSalle invariance or a Matrosov approach; see Khalil (2002) and Matrosov (1962).

However, even if one uses LaSalle invariance or standard Matrosov approaches, there is usually no guarantee of robustness, e.g., with respect to control or model uncertainty. This helped motivate the 'strictification' approach from Malisoff and Mazenc (2009), which converts nonstrict Lyapunov functions into strict ones. A strict Lyapunov function is a proper and positive definite function whose time derivative is negative along all trajectories of the closed loop system at all points outside the equilibrium. Strict Lyapunov functions allow us to robustify controls, e.g., to prove robustness in the key sense of ISS; see Khalil (2002).

To see how this 'robustification' approach can be done in the special case of time invariant nonlinear control affine systems of the form $\dot{z} = f(z)+g(z)u(z)$ with state space $\mathbb{R}^n$ for any n, assume that we found a control $u(z)$ such that the closed loop system is globally asymptotically stable to the origin, and that we have a strict Lyapunov function v for the closed loop system such that $-\dot{v}(z)$ is a proper and positive definite function, or equivalently, there is a class $\mathcal{K}_\infty$ function α such that $\dot{v}(z) \le -\alpha(|z|)$ holds along all trajectories of the closed loop system; see Khalil (2002). Assume that there are actuator errors, which we model by $\dot{z} = f(z)+g(z)(u(z)+\delta)$ where δ is an unknown measurable essentially bounded function. Then, by a simple application of the triangle inequality that uses the control affineness, the closed loop system $\dot{z} = f(z) + g(z)(u^\sharp(z) + \delta)$ is input-to-state stable when we use the augmented controller $u^\sharp(z) = u(z) - (\nabla v(z)g(z))^\top$, i.e., we subtract off the Lie derivative $L_g v(z) = (\nabla v(z)g(z))^\top$; see Sections 5 and 8 for more on robustifying controls.

However, to implement the control $u^\sharp$, one needs an explicit closed form formula for the gradient ∇v of the strict Lyapunov function in the formula for $u^\sharp$, and this was another motivation

☆ Supported by NSF-ECCS Grants 1408376 (Krstic), 1102348 (Malisoff), and 1408295 (Malisoff). The material in this paper was presented at the 54th IEEE Conference on Decision and Control, December 15–18, 2015, Osaka, Japan; see Remark 1 for a comparison between the conference version and this paper. This paper was recommended for publication in revised form by Associate Editor Hyungbo Shim under the direction of Editor Andrew R. Teel.

E-mail addresses: malisoff@lsu.edu (M. Malisoff), krstic@ucsd.edu (M. Krstic).

1 Tel.: +1 858 534 5670; fax: +1 858 822 3107.

http://dx.doi.org/10.1016/j.automatica.2016.01.066
0005-1098/© 2016 Elsevier Ltd. All rights reserved.

for the strictification approach, but there are other motivations. For instance, having strict Lyapunov functions in closed form often makes it possible to find explicit formulas for the comparison functions in ISS estimates, and strict Lyapunov function constructions also make it possible to quantify the effects of input or state delays, and to perform backstepping.

In this paper, we use a Matrosov type strictification to prove a stabilization result for a controlled chain of exponential integrators in which the right sides of both equations saturate, so the system is not completely controllable. While the system is time invariant, many interesting tracking problems for time varying bilinear systems can be transformed into chains of exponential integrators that are covered by our theory, including cases where there are positivity constraints on the state values and delays; see Sections 7–8. Our strict Lyapunov function for the closed loop system allows us to prove robustness under uncertainty and delays, including ISS with respect to additive uncertainties on the control for cases where the system is not control affine; see Section 5. Our strictification uses just one auxiliary function, but the choices of the functions needed for the strictification are not obvious, which makes our work novel and interesting. We believe that our new approach for overcoming these challenges has the potential for many other applications to higher dimensional systems that are also not completely controllable.

2. Chain of exponential integrators model

We begin by studying systems of the form

$$\begin{cases}\dot{x} = 1 - e^{y}\\ \dot{y} = D^{*}(1 - e^{u})\end{cases} \tag{1}$$

having the state space $\mathbb{R}^2$, where u is the control and D^* is a positive constant (but see below for cases where the 1's in (1) are replaced by more general nonlinear functions $\mathcal{M}_1$ and $\mathcal{M}_2$, Sections 5–6 for cases with uncertainties and time delays, and Sections 7–8 for ways to transform many interesting systems with positivity constraints on the states into the form (1)). The global stabilization of (1) is nontrivial. By contrast, the local stabilization is easy, since any linear feedback $u = k_1 x + k_2 y$ for suitable constant k_i's will do. For global stabilization, we have to contend with the fact that the right sides of both equations in (1) saturate on the upper end at 1, so the system is not completely controllable. However, the system is not exponentially unstable in open loop. Instead, it is polynomially unstable, e.g., $x(t) = x_0 + t(1 - e^{y(0)})$ when $u = 0$.

Attempting stabilization with backstepping is fruitless, because backstepping would require both positive and negative values of e^u, so backstepping is too aggressive. There are valuable results in the literature on bounded backstepping that lead to controls that satisfy input constraints, including more general situations where the dynamics are time varying or have delays; see Mazenc and Bowong (2004) and Mazenc and Malisoff (2015). However, these results either do not ensure ISS, or do not provide the strict Lyapunov functions for (1) that are essential for what follows.

On the other hand, we can build controls that stabilize the equilibrium $(0, 0)$ of (1), and corresponding nonstrict Lyapunov functions for the closed loop system, on $\mathbb{R}^2$. For instance, we can use the following nonstrict Lyapunov function from Malisoff and Krstic (2015):

$$V_1(x, y) = x + \exp(-x) - 1 + \frac{1}{D^*}(\exp(y) - y - 1). \tag{2}$$

In fact, the time derivative of (2) along all trajectories of (1) in closed loop with the control

$$u = y - x \tag{3}$$

satisfies

$$\dot{V}_1 = -\exp(-x)(\exp(y) - 1)^2 \tag{4}$$

and then stability of the closed loop system follows from LaSalle's theorem. However, (2) is not a strict Lyapunov function, since its time derivative (4) is zero at all points where $y = 0$, so (2) is not amenable to the robustification objective we discussed in the introduction.

The system (1) naturally arises in the study of systems with positivity constraints on the states, such as

$$\begin{cases}\dot{X} = (1 - Y)X\\ \dot{Y} = (D^* - D)Y\end{cases} \tag{5}$$

on the state space $(0, \infty)^2$, by setting $x(t) = \ln(X(t)/X_r)$, $y(t) = \ln(Y(t))$, and $u(t) = \ln(D(t)/D^*)$ for any constant $X_r > 0$ (but see Section 7 for much more general systems with positivity constraints). In fact, the preceding argument proves the following, by setting $u = y - x$:

Theorem 1. *For all constants $X_r > 0$, the system (5), in closed loop with the positive valued control $D = D^* Y X_r / X$, has the positively invariant set $(0, \infty)^2$ and is globally asymptotically stable to $(X_r, 1)$.* □

Before providing our general theory, we illustrate Theorem 1 using simulations for (5) with the control from Theorem 1. We choose $X_r = 1$ and $D^* = 5$, with the initial condition $(X(0), Y(0)) = (e^3, e^{-2})$, which corresponds to the initial condition $(x(0), y(0)) = (3, -2)$ in the transformed variables on $\mathbb{R}^2$. In Fig. 1, we plot the corresponding trajectories for $X(t)$, $Y(t)$ and $D(t)$ as solid lines, with the set point levels given as dotted lines. Our simulations illustrate our controller's ability to ensure asymptotic convergence of the state vector toward the equilibrium $(1, 1)$.

To examine the overshoots in Fig. 1, we also provide a phase portrait in Fig. 2 that shows four trajectories of the system in closed loop with $D = D^* Y X_r / X$, superimposed by six level curves of our nonstrict Lyapunov function V_1 from (2). We choose $X_r = 1$ and $D^* = 5$. Clockwise from the top in Fig. 2, the solid lines show the solutions $(X(t), Y(t))$ on the time interval $[0, 2.5]$ and the initial conditions (e^4, e^3), (e^3, e^2), (e^3, e^{-2}), and (e^2, e^{-3}), respectively. The dotted lines in Fig. 2 show level curves of the nonstrict Lyapunov function V_1 from (2) with $x(t) = \ln(X(t)/X_r)$ and $y(t) = \ln(Y(t))$, with $V_1 = L$ for the following values of L: 0.01, 0.1, 0.3, 0.5, 0.75, and 1.45. The figure shows convergence of the solutions towards $(1, 1)$ and the crossings through the level curves of V_1. See Section 7 for large classes of systems with positivity constraints, delays, and uncertainties and other changes of variables that are covered by our theory. Such results require strict Lyapunov functions. This motivates the next section, which transforms (2) into a strict Lyapunov function for a suitable generalization of (1).

3. Main strict Lyapunov function construction

We construct an explicit strict Lyapunov function for a broad class of systems of the form

$$\begin{cases}\dot{x} = \mathcal{M}_1(x, y) - e^{y}\\ \dot{y} = D^{*}(\mathcal{M}_2(x, y) - e^{u})\end{cases} \tag{6}$$

(but see below for many other systems with positive state constraints). The results in this section cover the system (1) in closed loop with (3) on $\mathbb{R}^2$, which plays a key role in our robustness and delays analysis in Sections 5–8. We again allow D^* to be any positive constant, and we assume that $\mathcal{M}_1$ and $\mathcal{M}_2$ are any locally Lipschitz functions that admit constants $\bar{m}_1 \geq 0$ and $\bar{m}_2 \geq 0$ such that

Download English Version:

https://daneshyari.com/en/article/695208

Download Persian Version:

https://daneshyari.com/article/695208

Daneshyari.com