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Multidimensional scaling approach for node localization using received signal strength measurements

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ABSTRACT

Node localization has played an important role in wireless sensor networks. In this paper, cooperative localization using received signal strength (RSS) measurements is addressed. The technique of weighted multidimensional scaling (WMDS) which relies on pairwise distance information between nodes is utilized in our algorithm development. Assuming that the transmit power is available, we first convert the original nonlinear localization problem to a system of linear equations, leading to computational attractiveness. It is also proved that the positioning accuracy of the WMDS solution attains the Cramér–Rao lower bound at sufficiently small noise conditions. Furthermore, the proposed method is extended to the unknown transmit power case by exploiting the ratio of squared distance estimates extracted from the RSS information. The effectiveness of the WMDS approach is demonstrated via comparison with several conventional RSS-based positioning methods.

1. Introduction

Node localization is a fundamental task for the operation and management of wireless sensor network (WSN) applications because most of the nodes are arbitrarily placed with their positions being unknown [1–3]. In a typical WSN, there are a few number of nodes with *a priori* known positions, or the so-called anchors, whose coordinates may be obtained using the global positioning system. The objective of node positioning is to locate the remaining sensors with the use of the pairwise measurements between the nodes, including the anchors. There are two localization approaches, namely, non-cooperative and cooperative. In the former, only measurements between unknown-position nodes and anchors are utilized for positioning, while the latter uses all available measurements, which involve those among the unknown-position nodes. Since the unknown-position nodes dominate, it is expected that the cooperative approach will yield higher localization accuracy especially for a large network.

Time-of-arrival (TOA), time-difference-of-arrival (TDOA), received signal strength (RSS) and angle-of-arrival (AOA) are popular measurement models for positioning [4]. The first three models can provide pairwise distance estimates while the last one contains direction information. Among them, the RSS-based positioning approach, which employs the propagation path loss of signal traveling from one sensor to another, has been the subject of great interest because of its low complexity and cost in hardware [1]. However, it is still a challenging task to locate the nodes because the RSS information is highly nonlinear with the unknown coordinates and the measurement noise is of multiplicative nature [5]. Maximum likelihood (ML) estimator is a straightforward but highly nonlinear algorithm for RSS-based localization [6–8]. Although the ML method can obtain asymptotically optimal solution when the statistics of the measurement errors are known, its objective function contains multiple maxima and minima, indicating that global convergence is not guaranteed. Semidefinite relaxation (SDR) is another popular candidate and its main idea is to approximate the nonlinear ML problem to a convex program. The corresponding proposals include norm approximation [9], unscented transformation [10] and Taylor series expansion [11], but they can only provide suboptimal performance because of relaxing the original objective function. On the other hand, target localization can be formulated as a sparse signal recovery problem [12,13] and this sparsity-based approach can achieve high estimation accuracy even at larger noise conditions. Due to the low complexity of linear operations, linear least squares (LLS) methods have also been proposed for RSS-based positioning particularly for a single unknown-position node [14–20]. These LLS algorithms can be considered as non-cooperative methods in WSNs because they only exploit the RSS measurements between the unknown-position nodes and anchors. In order to attain a higher performance with low computational complexity, our aim is to devise a linear and

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cooperative algorithm for RSS-based node localization where all pairwise measurements are utilized.

Multidimensional scaling (MDS) has been a computationally attractive technique for analyzing experimental data in psychology, geography and molecular biology [21,22]. In [23], the MDS is applied for single-source localization via transforming all pairwise TOA information into the relative coordinates of sensors. The MDS methodology has also been formulated as subspace techniques in [24,25]. As the performance of [23–25] is suboptimal, a weighted MDS (WMDS) algorithm is developed in [26,27] to increase the positioning accuracy by employing a proper weighting matrix on the resultant system of linear equations. The WMDS approach has been extended to cooperative positioning of multiple nodes using TOA measurements in [28]. In this work, the WMDS technique is exploited to devise accurate and computationally efficient algorithms for RSS-based positioning.

The rest of the paper is organized as follows. The problem of locating multiple sensors using RSS measurements is formulated in Section 2 and the WMDS algorithm [28] for TOA-based localization is reviewed in Section 3. Node positioning algorithms based on the RSS measurements are developed in Section 4 where the cases of known and unknown transmit power are investigated. It is also proved that for the known transmit power scenario, the localization performance of the WMDS method can attain the Cramér–Rao lower bound (CRLB) at sufficiently small noise conditions. Simulation results are included in Section 5 to evaluate the performance of the proposed estimators by comparing with the SDR [11], ML and non-cooperative LLS [18,19] methods as well as CRLB. Finally, conclusions are drawn in Section 6.

Notation: We use $[\mathbf{A}]_{:,i}$, $[\mathbf{A}]_{:,i:j}$, $[\mathbf{A}]_{i:j,:}$ and $[\mathbf{A}]_{i:j,l:m}$ to represent the ith column, ith to jth columns, ith to jth rows, and entries in the intersection of ith to jth rows and lth to mth columns, of matrix $\mathbf{A}$, respectively. The ^{T}, $^{-1}$, † and $\otimes$ denote the matrix transpose, inverse, pseudo-inverse and Kronecker product, respectively, while $\mathbb{E}$ is the expectation operator. The vec($\mathbf{A}$) corresponds to the vectorization of $\mathbf{A}$, diag$(a_1, a_2, \cdots, a_k)$ is a diagonal matrix with diagonal elements $a_1, a_2, \cdots, a_k$, and blkdiag$(\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_k)$ is block diagonal matrix with matrices $\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_k$. The $\mathbf{1}_i$, $\mathbf{0}_{i\times j}$, $\mathbf{I}_i$ and $\mathbf{e}_i$ represent $i \times 1$ vector with all elements 1, $i \times j$ zero matrix, $i \times i$ identity matrix and the ith column of $\mathbf{I}_{0.5(M+N)(M+N-1)}$, respectively.

2. Problem formulation

We consider a network consisting of $(M + N)$ sensors in a two-dimensional space. Let $\mathbf{x}_i = [x_i \;\; y_i]^T$, $i = 1, 2, \cdots, M, M + 1, \cdots, M + N$, be the coordinates of the ith node. Without loss of generality, it is assumed that the first M sensors are the anchors while the coordinates of the last N nodes are the unknown parameters of interest. The distance between the ith and jth sensors, denoted by $d_{i,j}$, is

$$d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \quad i, j = 1, 2, \cdots, M + N. \tag{1}$$

According to [6,29], the RSS measurements are related to $\{d_{i,j}\}$ as:

$$P_{i,j} = P_0 - 10\alpha \log_{10}(d_{i,j}) + n_{i,j}, \quad i, j = 1, 2, \cdots, M + N, \tag{2}$$

where $P_{i,j}$ denotes the averaged power in decibel-milliwatts (dBm) with signal being transmitted from the ith sensor and received at the jth sensor, P_0 is the transmit power or the measured signal strength at 1 meter distance in dBm, and α represents the path-loss factor that measures the rate at which the RSS decreases with distance. The $\{n_{i,j}\}$ are the average shadow fadings which are modeled as uncorrelated zero-mean Gaussian variables with variances $\{\sigma_{i,j}^2\}$. It is assumed that P_0, α and/or $\{\sigma_{i,j}^2\}$ are known *a priori* through a testing and calibration campaign [1,14]. Considering that all pairwise RSS measurements are available and $P_{i,j} = P_{j,i}$ [6], and denoting the set $\boldsymbol{\Theta} = \{(1, M + 1), \cdots, (1, M + N), (2, M + 1), \cdots, (2, M + N), \cdots, (M, M + 1), \cdots, (M, M + N), (M + 1, M + 2), \cdots, (M + 1, M + N), (M + 2, M + 3), \cdots, (M + 2, M + N), \cdots, (M + N - 1, M + N)\}$, the task is to estimate the positions of the nodes $\{\mathbf{x}_i \mid i = M + 1, M + 2, \cdots, M + N\}$ using the known coordinates of anchors $\{\mathbf{x}_i \mid i = 1, 2, \cdots, M\}$, path-loss factor α and RSS measurements $\{P_{i,j} \mid i, j \in \boldsymbol{\Theta}\}$. We first tackle the problem when P_0 is available and then extend our development to the unknown transmit power scenario.

3. Review of WMDS algorithm [28]

In this section, the WMDS algorithm where TOA measurements are utilized for collaboratively locating multiple nodes is reviewed. In Section 4, this technique will then be modified for RSS-based positioning via converting $\{P_{i,j}\}$ to distance measurements.

Let $\mathbf{X} = [\mathbf{X}_a \;\; \mathbf{X}_s]$ be the matrix of coordinates where $\mathbf{X}_a = [\mathbf{x}_1 \; \mathbf{x}_2 \; \cdots \; \mathbf{x}_M]$ collects the known coordinates of all anchors while $\mathbf{X}_s = [\mathbf{x}_{M+1} \; \mathbf{x}_{M+2} \; \cdots \; \mathbf{x}_{M+N}]$ contains the unknowns to be determined. Without loss of generality, we assume that $\mathbf{X}_a \mathbf{1}_M = \mathbf{0}_{2\times 1}$ which is fulfilled by a simple translation using $\mathbf{x}_i - \sum_{i=1}^{M} \mathbf{x}_i / M$, $i = 1, 2, \cdots, M$, and thus the actual sensor positions can be recovered in a similar manner. The classical MDS algorithm which employs the centroid of all sensors, denoted by $\mathbf{x}_c = \sum_{i=1}^{M+N} \mathbf{x}_i / (M + N)$, as the origin, is based on the following matrix $\mathbf{B}$:

$$\mathbf{B} = \mathbf{X}_c^T \mathbf{X}_c = \mathbf{J}\mathbf{X}^T\mathbf{X}\mathbf{J} = -0.5\mathbf{J}\mathbf{D}\mathbf{J}, \tag{3}$$

where

$$\mathbf{X}_c = \mathbf{X} - \mathbf{x}_c \mathbf{1}_{M+N} \tag{4}$$

$$\mathbf{J} = \mathbf{I}_{M+N} - \frac{1}{M+N}\mathbf{1}_{M+N}\mathbf{1}_{M+N}^T \tag{5}$$

$$\mathbf{D} = \begin{bmatrix} 0 & d_{1,2}^2 & d_{1,3}^2 & \cdots & d_{1,M}^2 & d_{1,M+1}^2 & \cdots & d_{1,M+N}^2 \\ d_{2,1}^2 & 0 & d_{2,3}^2 & \cdots & d_{2,M}^2 & d_{2,M+1}^2 & \cdots & d_{2,M+N}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \cdots \\ d_{M,1}^2 & d_{M,2}^2 & d_{M,3}^2 & \cdots & 0 & d_{M,M+1}^2 & \cdots & d_{M,M+N}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \cdots \\ d_{M+N,1}^2 & d_{M+N,2}^2 & d_{M+N,3}^2 & \cdots & d_{M+N,M}^2 & d_{M+N,M+1}^2 & \cdots & 0 \end{bmatrix}. \tag{6}$$

We see that $\mathbf{B}$ and $\mathbf{D}$ are characterized by the pairwise distances. Note that $\mathbf{X}^T\mathbf{X} \neq -0.5\mathbf{D}$ because $\mathbf{D}$ is rank deficient. Aiming to decompose (3) into known and unknown components, we separate $\mathbf{J}$ into upper and lower parts so that $\mathbf{XJ}$ can be rewritten as

$$\mathbf{XJ} = [\,\mathbf{X}_a \quad \mathbf{X}_s\,] \begin{bmatrix} \mathbf{J}_a \\ \mathbf{J}_s \end{bmatrix} = \mathbf{X}_a\mathbf{J}_a + \mathbf{X}_s\mathbf{J}_s = \check{\mathbf{X}}_s\check{\mathbf{X}}_a, \tag{7}$$

where

$$\mathbf{J}_a = [\mathbf{J}]_{1:M,:} = \left[\mathbf{I}_M - \mathbf{1}_M\mathbf{1}_M^T/(M+N) \quad -\mathbf{1}_M\mathbf{1}_N^T/(M+N)\right] \tag{8}$$

$$\mathbf{J}_s = [\mathbf{J}]_{(M+1):(M+N),:} = \left[-\mathbf{1}_N\mathbf{1}_M^T/(M+N) \quad \mathbf{I}_N - \mathbf{1}_N\mathbf{1}_N^T/(M+N)\right] \tag{9}$$

$$\check{\mathbf{X}}_s = [\mathbf{I}_2 \quad \mathbf{X}_s] \tag{10}$$

$$\check{\mathbf{X}}_a = \begin{bmatrix} \mathbf{X}_a\mathbf{J}_a \\ \mathbf{J}_s \end{bmatrix}. \tag{11}$$

Note that $\mathbf{X}_a\mathbf{J}_a = [\mathbf{X}_a \;\; \mathbf{0}_{2\times N}]$ with the use of $\mathbf{X}_a\mathbf{1}_M = \mathbf{0}_{2\times 1}$. Employing