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High accuracy and low complexity adaptive Generalized Sidelobe Cancelers for colored noise scenarios

Ricardo Kehrle Miranda [a], João Paulo C.L. da Costa [a], Felix Antreich [b]

[a] *Laboratory of Array Signal Processing (LASP), Department of Electrical Engineering, University of Brasília, Brazil*
[b] *Institute of Communications and Navigation, German Aerospace Center (DLR), Germany*

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ABSTRACT

The Generalized Sidelobe Canceler (GSC) is a beamforming scheme which is applied in many fields such as audio, RADAR, SONAR and telecommunications. Recently, the adaptive Reduced Rank GSC (RR-GSC) has been proposed for applications with a large number of sensors. Due to its dimensionality reduction step, the adaptive RR-GSC achieves an enhanced performance in comparison with the standard GSC. However, both standard GSC and RR-GSC have their performance drastically degraded in the presence of colored noise.

In this paper, we propose to extend further the GSC and the RR-GSC for colored noise scenarios. As shown in this paper, such improvement in colored noise scenarios can be obtained by incorporating a stochastic or a deterministic prewhitening step in the GSC and RR-GSC algorithms. Since the prewhitening increases the computational complexity, a block-wise reduced rank stochastic gradient GSC beamformer is also proposed. The block-wise step allows only one prewhitening step per block while in the previous schemes one per sample was needed. Another proposed advance in colored noise scenarios is the incorporation of the Vandermonde Invariance Transform (VIT). The VIT works as a pre-beamformer which reduces the interferent power of the undesired sources and the colored noise effect. We show by means of simulations the improved results even for highly correlated scenarios.

1. Introduction

Beamforming is an important topic in array signal processing and has applications in several fields such as RADAR [1], SONAR [2], telecommunications [3] and audio [4]. In the literature, there are several adaptations of Direction of Arrival (DOA) estimation schemes for colored noise scenarios [5–7], and once the DOA information is obtained, it can be introduced to the beamformer. The addition of such constraints led to the development of beamformers such as the Direct Form Processor (DFP), which includes the Linearly Constrained Minimum Variance (LCMV) and Linearly Constrained Constant Modulus (LCCM) [8], and the Generalized Sidelobe Canceler (GSC) [9]. For real time applications, the necessity for adaptive algorithms grows and, with this need, adaptive versions of the GSC were proposed in earlier works [10,11,8].

However, when the number of elements in a sensor array is high, these algorithms suffer from computational complexity increase. Therefore, recently, adaptive reduced rank DFP and GSC schemes were also proposed in order to reduce the dimensionality of the adaptive filters. The rank reduction step also has a noise removal effect, thus showing an enhanced performance [11,8]. These works use the constant modulus (CM) cost function [10], as it was shown to have a better accuracy for constant envelope signals. Yet, adaptive beamforming techniques using the GSC usually assume uncorrelated white noise in the receivers, which is not realistic.

For colored noise scenarios, prewhitening schemes have been successfully applied in combination with DOA estimation [5,6] and audio signal processing schemes [12]. The prewhitening schemes are divided into stochastic [6,12] and deterministic prewhitening [5]. In deterministic prewhitening, the noise may have a specific structure which can be exploited, while in the stochastic prewhitening, no structure is assumed. Moreover, there are also multidimensional prewhitening schemes for the case that the data has a tensor structure [13].

In this work, we propose to extend the least mean squares GSC (LMS-GSC) and the Reduced Rank LMS-GSC (RR-LMS-GSC) for colored noise scenarios by incorporating a prewhitening step. We propose the prewhitened GSC schemes considering the deterministic prewhitening [5] and the stochastic prewhitening [6,12]. The colored noise is usually concentrated in certain direction. Therefore,

E-mail addresses: rickehrle@unb.br (R.K. Miranda), joaopaulo.dacosta@ene.unb.br (J.P.C.L. da Costa), felix.antreich@dlr.de (F. Antreich).
URL: http://www.redes.unb.br/lasp (R.K. Miranda).

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to further enhance the GSC, the VIT [14] is also applied as a preprocessing step. The colored noise can be also concentrated close to the desired signal direction, therefore a prewhitening step is also still needed along the VIT. The stochastic prewhitening needs the computation of one SVD at each iteration. In order to reduce the complexity of the stochastic prewhitening, we propose the block-wise reduced rank stochastic gradient GSC (BW-RR-GSC) beamformer.

This paper is divided into 6 sections including this introduction. In Section 2 we present the data model for colored noise. In Section 3 the classic beamformer designs for the LMS-GSC and RR-LMS-GSC are briefly introduced. Then, in Section 4 we propose our high accuracy and low complexity GSC schemes by incorporating prewhitening steps, the VIT and a block-wise modification for colored noise scenarios. In Section 5, simulations are shown and the results are drawn. Finally, Section 6 makes the conclusions about the work.

Notation Scalars are denoted by lower-case letters $(a, b, \ldots)$, vectors are written as boldface lower-case letters $(\mathbf{a}, \mathbf{b}, \ldots)$ and matrices as boldface capitals $(\mathbf{A}, \mathbf{B}, \ldots)$. The superscripts $^{\mathrm{T}}$, $^{\mathrm{H}}$ and * represent transpose, Hermitian transpose and complex conjugate of a term, respectively. The operator $\mathrm{E}\{\cdot\}$ stands for the expected value operation.

2. Data model

We assume that d sources are transmitting different symbols at the n-th time instant. Since the sources are far away from the receiver, the narrowband wave fronts are considered planar. We assume a Uniform Linear Array (ULA) with M isotropic sensor elements with an inter-element spacing of Δ wavelengths. Therefore, we can mathematically represent the received symbols as

$$\mathbf{x}(n) = \mathbf{a}(\theta_0)s(n) + \mathbf{A}_{\mathrm{int}}(\boldsymbol{\theta}_{\mathrm{int}})\mathbf{s}_{\mathrm{int}}(n) + \mathbf{n}^{(\mathrm{c})}(n), \tag{1}$$

where $\mathbf{x}(n) = [x_0(n), \ldots, x_{M-1}(n)]^{\mathrm{T}}$ is the vector containing the received symbols at time instant n, $s(n)$ is the desired signal, $\mathbf{s}_{\mathrm{int}}(n)$ is a vector with the interference symbols from the $d-1$ interferes and $\mathbf{n}^{(\mathrm{c})}(n)$ contains colored noise samples at the sensor elements. Note that $\mathbf{n}^{(\mathrm{c})}(n) = \mathbf{L}\mathbf{n}(n)$, where $\mathbf{n}(n)$ contains i.i.d. noise samples with Circularly Symmetric Complex Gaussian (CSCG) distributions. The matrix $\mathbf{L} \in \mathbb{C}^{M\times M}$ stands for the correlation matrix. For the special case where $\mathbf{L}$ is the identity matrix the noise becomes white at the sensors. The vector $\mathbf{a}(\theta_0)$ is the steering vector with a Vandermonde structure for the desired signal, where the elements of the vector $\mathbf{a}(\theta_0)$ are a function of the DOA of the desired signal defined as θ_0 and are arranged in a column. The matrix $\mathbf{A}_{\mathrm{int}}(\boldsymbol{\theta}_{\mathrm{int}}) \in \mathbb{C}^{M\times d-1}$ is the steering matrix containing all the steering vectors of the interfering signals where their corresponding DOAs are comprised in the vector $\boldsymbol{\theta}_{\mathrm{int}} \in \mathbb{C}^{d-1\times 1}$. The DOAs might also be represented by the spatial frequencies, i.e. spatially related phase delays, $\phi_0 = -2\pi\Delta\sin\theta_0$ and $\boldsymbol{\phi}_{\mathrm{int}} = -2\pi\Delta\sin\boldsymbol{\theta}_{\mathrm{int}} \in \mathbb{C}^{d-1\times 1}$, respectively.

More generally, assuming a sliding window in which at time n a snapshot of the current and the $N-1$ previously transmitted symbols are allocated into a vector $\mathbf{s}$ and collecting the interfering signals into a matrix $\mathbf{S}_{\mathrm{int}}(n) \in \mathbb{C}^{N\times d-1}$ we can rewrite our model in a compact format

$$\mathbf{X}(n) = \mathbf{a}(\theta_0)\mathbf{s}^{\mathrm{T}}(n) + \mathbf{A}_{\mathrm{int}}(\boldsymbol{\theta}_{\mathrm{int}})\mathbf{S}_{\mathrm{int}}^{\mathrm{T}}(n) + \mathbf{N}^{(\mathrm{c})} \in \mathbb{C}^{M\times N}, \tag{2}$$

where $\mathbf{X}(n) = [\mathbf{x}(n-N+1), \ldots, \mathbf{x}(n)]$ and $\mathbf{N}^{(c)} = \mathbf{L}\cdot\mathbf{N} \in \mathbb{C}^{M\times N}$. The matrix $\mathbf{N} \in \mathbb{C}^{M\times N}$ contains the N white noise samples for all M sensors in the same manner as $\mathbf{X}$ contains N signal plus noise samples from the M sensors. The variable $\mathbf{s}(n) \in \mathbb{C}^{N\times 1}$ has the N latest samples for the desired signal and $\mathbf{S}_{\mathrm{int}} \in \mathbb{C}^{N\times d-1}$ has the N latest samples for the $d-1$ interfering sources.

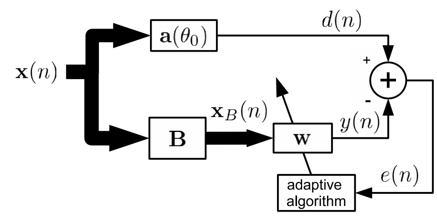

Fig. 1. LMS-GSC block diagram.

Here, we assume that the received symbols $\mathbf{X}(n)$ and the DOA of the desired signal θ_0 are known at the receiver and we desire to find $\hat{s}(n)$, which is an estimate of $s(n)$. To find the DOA, we refer to [5–7] or alternatively we can assume that the position of the transmitter with respect to the receiver is known.

3. State-of-the-art beamformer designs

This section is divided into two subsections. In Section 3.1, we review the standard LMS-GSC beamformer, while in Section 3.2, we review the reduced rank LMS-GSC (RR-LMS-GSC).

3.1. LMS-GSC

The GSC algorithm turns a constrained problem into an unconstrained problem by introducing a blocking matrix, which is the orthogonal complement of the constraint $\mathbf{a}(\theta_0)$. In our case, the constraint is formed based on the steering vector of the desired signal, which can be estimated via [5–7].

In Fig. 1, the input signal $\mathbf{x}(n)$ passes through a beam pointed at the desired signal direction θ_0 generating $d(n) = \mathbf{a}^{\mathrm{H}}(\theta_0)\mathbf{x}(n)$. The same input signal also passes through a blocking matrix $\mathbf{B}$ which is the orthogonal complement of the constraint $\mathbf{a}(\theta_0)$. Consequently, $\mathbf{B}$ blocks the desired signal and let ideally only $\mathbf{A}_i(\theta_i)\mathbf{s}_i(n)$ pass. The filter $\mathbf{w}$ should then be adjusted so that it generates the interference signal $y(n)$ that is subtracted from the desired signal $d(n)$.

In Fig. 1, $y(n)$ is given by

$$y(n) = \mathbf{w}^{\mathrm{H}}\mathbf{x}_B(n), \tag{3}$$

where $\mathbf{x}_B(n) = \mathbf{B}\mathbf{x}(n)$. As shown in Fig. 1, the error signal $e(n)$ is used by the adaptive algorithm to adjust $\mathbf{w}$. Once $\mathbf{w}$ converges, then we have that $\hat{s}(n) = e(n)$. Since $e(n)$ is free from interference it is also the system's output signal.

The adaptation of $\mathbf{w}$ is computed via stochastic gradient of the following cost function

$$J_{\mathrm{lms}}(\mathbf{w}) = E\left\{\left|d(n) - \mathbf{w}^{\mathrm{H}}\mathbf{x}_B(n)\right|^2\right\} \tag{4}$$

which gives the update rule for the adaptive part

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_{\mathrm{lms}}\nabla_{\mathbf{w}}J_{\mathrm{lms}}(\mathbf{w}) \tag{5}$$

with μ_{lms} being the step size for the LMS-GSC.

We use the instantaneous estimates $\hat{\mathbf{R}}_{\mathbf{xx}} = \mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n)$ and $\hat{\mathbf{r}}_{d\mathbf{x}} = d(n)\mathbf{x}(n)$ [9] to find the stochastic gradient:

$$\hat{\nabla}_{\mathbf{w}}J_{\mathrm{lms}} = 2\mathbf{B}\mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n)\mathbf{B}^{\mathrm{H}}\mathbf{w} - 2\mathbf{B}d(n)\mathbf{x}(n). \tag{6}$$

Now the stochastic gradient is inserted into LMS update rule for the GSC [9]:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_{\mathrm{lms}}\mathbf{B}\mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n)\big(\mathbf{a}(\theta_0) - \mathbf{B}^{\mathrm{H}}\mathbf{w}\big). \tag{7}$$