Automatica 68 (2016) 194–202

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Performance output tracking for a wave equation subject to unmatched general boundary harmonic disturbance[☆]

Wei Guo[a], Bao-Zhu Guo[b,c,d]

[a] School of Statistics, University of International Business and Economics, Beijing 100029, China
[b] School of Mathematical Science, Shanxi University, Taiyuan 030006, China
[c] Key Laboratory of System and Control, Academy of Mathematics and Systems Science, Academia Sinica, Beijing 100190, China
[d] School of Computer and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, Johannesburg, South Africa

ARTICLE INFO

Article history:
Received 6 February 2015
Received in revised form
13 November 2015
Accepted 1 January 2016
Available online 22 February 2016

Keywords:
Wave equation
Disturbance rejection
Performance output tracking

ABSTRACT

In this paper, we consider performance output reference tracking for a wave equation with general harmonic disturbance at one end and the control at the other end. We first design a state reference system determined completely by the measured output and the reference signal where the parameter update law is presented. An adaptive servomechanism output feedback control is then designed. It is shown that the closed-loop system is well-posed. Three control objectives are achieved: (a) the performance output is tracking the reference signal; (b) all the internal-loops are bounded; (c) when the disturbance and reference are disconnected, the closed-loop is exponentially stable. Finally, the state of system is shown to be tracking the reference state and the updated parameters are convergent to their true values.

1. Introduction

A classical problem in control theory is to achieve servo action or output regulation, that is, to design a controller so that the output of closed-loop system tracks asymptotically a reference signal regardless of external disturbance and initial state. Many classical output regulation results for finite-dimensional systems, for instance, Callier & Desoer, 1980, Davison, 1976, Desoer & Lin, 1985, Francis, 1977, Francis & Wonham, 1976 and Isidori & Byrnes, 1990 have been generalized to the infinite-dimensional systems, like Byrnes, Laukó, Gilliam, and Shubov (2000), Deutscher (2011), Hämäläinen and Pohjolainen (2010), Ke, Logemann, and Rebarber (2009), Maidi, Diaf, and Corriou (2008), Rebarber and Weiss (2003), Pisano, Orlov, and Usai (2011) and Schumacher (1983), among many others. However, the performance output tracking is not sufficiently addressed in the context of infinite-dimensional systems like that for finite-dimensional linear systems, see, for instance, Lewis, Vrabie, and Syrmos (2012, p. 315).

In this paper, we consider performance output tracking for a one-dimensional wave equation with general harmonic disturbance at one end and control at the other end, which is described by the following PDE:

$$\begin{cases} y_{tt}(x,t) = y_{xx}(x,t), \quad x \in (0,1),\ t>0, \\ y_x(0,t) = U_0(t) + d(t), \quad t \ge 0, \\ y_x(1,t) = U(t), \quad t \ge 0, \\ y(x,0) = y_0(x), \qquad y_t(x,0) = y_1(x), \quad 0 \le x \le 1, \\ y_{out}(t) = y_t(0,t), \end{cases} \tag{1}$$

where and henceforth $y'(x,t)$ or $y_x(x,t)$ denotes the derivative of $y(x,t)$ with respect to x and $\dot{y}(x,t)$ or $y_t(x,t)$ the derivative with respect to t, $U_0(t)$ and $U(t)$ are the input (control), $y_{out}(t)$ is the measured output, (y_0, y_1) is the initial value; $d(t)$ represents the general harmonic disturbance which has the following form:

$$d(t) = \sum_{j=1}^{m} \left(\bar{\theta}_j \sin \alpha_j t + \bar{\vartheta}_j \cos \alpha_j t \right),$$

where $\alpha_j \neq 0, j = 1, 2, \ldots, m$ are (known) frequencies of the harmonic disturbance, $\bar{\theta}_j$ and $\bar{\vartheta}_j, j = 1, 2, \ldots, m$ are (unknown) amplitudes of the harmonic disturbance. System (1) is a typical non-collocated Neumann control problem: control is on one end and the output is on the other end. In addition, the disturbance and control are not matched ($U(t)$ and $d(t)$, see (2) later). It is noted that our disturbance here is more general than the finite sum of periodic

[☆] This work was supported by the National Natural Science Foundation of China (61374088, 61273129), the Fundamental Research Funds for the Central Universities in UIBE (15JQ01) and the National Research Foundation of South Africa. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Thomas Meurer under the direction of Editor Miroslav Krstic.

E-mail addresses: guowei74@126.com (W. Guo), bzguo@iss.ac.cn (B.-Z. Guo).

http://dx.doi.org/10.1016/j.automatica.2016.01.041
0005-1098/

harmonic disturbance that can be considered as an approximation of the periodic disturbance signal in terms of Fourier expansion.

First, it is observed that by a direct output feedback $U_0(t) = qy_t(0,t)$, $q > 0$, we obtain the following system:

$$\begin{cases} y_{tt}(x,t) = y_{xx}(x,t), \quad x \in (0,1),\ t > 0, \\ y_x(0,t) = qy_t(0,t) \\ \qquad + \sum_{j=1}^{m} \left(\bar{\theta}_j \sin \alpha_j t + \bar{\vartheta}_j \cos \alpha_j t\right), \quad t \geq 0, \\ y_x(1,t) = U(t), \quad t \geq 0, \\ y(x,0) = y_0(x), \quad y_t(x,0) = y_1(x), \quad 0 \leq x \leq 1, \\ y_{out}(t) = y_t(0,t). \end{cases} \tag{2}$$

The advantage of system (2) is that when the disturbance and control are disconnected, the system is exponentially stable except the rigid motion which is produced from the zero eigenvalue for two free end wave equation. This is also the reason behind to introduce the control $U_0(t)$.

For a given reference signal $y_{ref}(t)$, we are expected to design an output feedback control for system (2) so that

$$y(1,t) \to y_{ref}(t) \quad \text{as } t \to \infty \tag{3}$$

where $y(1,t)$ is considered as a performance output. It is obvious that $y(1,t)$, although measurable, is not identical to all measured outputs which include $y_t(0,t)$.

It is noted that the adaptive servomechanism has been applied to infinite-dimensional systems in earlier paper (Logemann & Ilchmann, 1994). In paper (Kobayashi & Oya, 2002), an adaptive servomechanism control is designed for a class of distributed parameter system where the input and output operators are collocated and the disturbance is matched with control.

The main result of this paper claims that for any given frequencies $\alpha_j \neq 0, j = 1, 2, \ldots, m$, one can always construct an adaptive servomechanism to achieve both unknown parameter estimation and performance output reference tracking.

The rest of the paper is organized as follows. In next section, we first design a reference model with parameter updated law from the measured output. An adaptive control law is thus designed. The main results are stated in this section. Section 3 is devoted to the proof of main results. In Section 4, we present some illustrative simulation results.

2. Adaptive servomechanism design and main results

Our reference model is completely inspired from model reference adaptive control approach in identifying the unknown parameters. For the reference signal $y_{ref}(t)$, we design the following reference model:

$$\begin{cases} \widehat{y}_{tt}(x,t) = \widehat{y}_{xx}(x,t), \\ \widehat{y}_x(0,t) = \sum_{j=1}^{m} \left(\theta_j(t) \sin \alpha_j t + \vartheta_j(t) \cos \alpha_j t\right) + q\widehat{y}_t(0,t), \\ \widehat{y}(1,t) = y_{ref}(t), \\ \dot{\theta}_j(t) = -r_j(y_t(0,t) - \widehat{y}_t(0,t)) \sin \alpha_j t, \\ \dot{\vartheta}_j(t) = -l_j(y_t(0,t) - \widehat{y}_t(0,t)) \cos \alpha_j t, \\ \theta_j(0) = \theta_{j0}, \quad \vartheta_j(0) = \vartheta_{j0}, \quad j = 1, 2, \ldots, m, \\ \widehat{y}(x,0) = \widehat{y}_0(x), \quad \widehat{y}_t(x,0) = \widehat{y}_1(x), \end{cases} \tag{4}$$

where $r_j, l_j, j = 1, 2, \ldots, m$ are positive constants, $\theta_j(t)$ is the updated parameter for unknown parameter $\bar{\theta}_j$, and $\vartheta_j(t)$ is for $\bar{\vartheta}_j$. It is seen that system (4) is completely determined by the measured output of system (2) and the reference signal $y_{ref}(t)$ only. There are some advantages for system (4): (a) The right boundary of (4) produces the reference signal $y_{ref}(t)$; (b) system (4) is exponentially stable when the disturbance and reference are disconnected in the sense $y_{ref}(t) = 0$ and $\sum_{j=1}^{m}\left(\theta_j(t) \sin \alpha_j t + \vartheta_j(t) \cos \alpha_j t\right) = 0$; (c) when the disturbance and reference are connected, system (4) is always bounded which will be discussed later. The aim of designing (4) is to make system (2) track (4) and as a result, $y(1,t)$ tracks $\widehat{y}(1,t) = y_{ref}(t)$.

Let $\varepsilon(x,t) = y(x,t) - \widehat{y}(x,t)$ be the error between system (2) and state reference system (4). Then $\varepsilon(x,t)$ is governed by

$$\begin{cases} \varepsilon_{tt}(x,t) = \varepsilon_{xx}(x,t), \\ \varepsilon_x(0,t) = q\varepsilon_t(0,t) + \sum_{j=1}^{m} \left(\widetilde{\theta}_j(t) \sin \alpha_j t + \widetilde{\vartheta}_j(t) \cos \alpha_j t\right), \\ \varepsilon_x(1,t) = U(t) - \hat{y}_x(1,t), \\ \dot{\widetilde{\theta}}_j(t) = r_j \varepsilon_t(0,t) \sin \alpha_j t, \\ \dot{\widetilde{\vartheta}}_j(t) = l_j \varepsilon_t(0,t) \cos \alpha_j t, \\ \widetilde{\theta}_j(0) = \bar{\theta}_j - \theta_{j0} = \widetilde{\theta}_{j0}, \quad \widetilde{\vartheta}_j(0) = \bar{\vartheta}_j - \vartheta_{j0} = \widetilde{\vartheta}_{j0}, \end{cases} \tag{5}$$

$$\begin{cases} \varepsilon(x,0) = y_0(x) - \widehat{y}_0(x) = \varepsilon_0(x), \\ \varepsilon_t(x,0) = y_1(x) - \widehat{y}_1(x) = \varepsilon_1(x), \end{cases}$$

where $\widetilde{\theta}_j(t) = \bar{\theta}_j - \theta_j(t)$, $\widetilde{\vartheta}_j(t) = \bar{\vartheta}_j - \vartheta_j(t)$, $j = 1, 2, \ldots, m$, are the parameter errors.

We propose the following output feedback control:

$$\begin{aligned} U(t) &= -c_1\varepsilon(1,t) + \hat{y}_x(1,t) \\ &= -c_1[y(1,t) - y_{ref}(t)] + \hat{y}_x(1,t), \end{aligned} \tag{6}$$

where $c_1 > 0$ is a design parameter and $\varepsilon(1,t) = y(1,t) - y_{ref}(t)$ is the performance output tracking error.

Then system (5) corresponding to the feedback control (6) becomes

$$\begin{cases} \varepsilon_{tt}(x,t) = \varepsilon_{xx}(x,t), \\ \varepsilon_x(0,t) = q\varepsilon_t(0,t) + \sum_{j=1}^{m} \left(\widetilde{\theta}_j(t) \sin \alpha_j t + \widetilde{\vartheta}_j(t) \cos \alpha_j t\right), \\ \varepsilon_x(1,t) = -c_1\varepsilon(1,t), \\ \dot{\widetilde{\theta}}_j(t) = r_j \varepsilon_t(0,t) \sin \alpha_j t, \\ \dot{\widetilde{\vartheta}}_j(t) = l_j \varepsilon_t(0,t) \cos \alpha_j t, \\ \widetilde{\theta}_j(0) = \widetilde{\theta}_{j0}, \quad \widetilde{\vartheta}_j(0) = \widetilde{\vartheta}_{j0}, \quad j = 1, 2, \ldots, m \\ \varepsilon(x,0) = \varepsilon_0(x), \quad \varepsilon_t(x,0) = \varepsilon_1(x). \end{cases} \tag{7}$$

A motivation for the design of parameter update laws $\theta_j(t)$ and $\vartheta_j(t), j = 1, 2, \ldots, m$ is that the closed-loop system (7) has the Lyapunov function following:

$$\begin{aligned} E_\varepsilon(t) &= \frac{1}{2}\int_0^1 [\varepsilon_t^2(x,t) + \varepsilon_x^2(x,t)]dx + \frac{c_1}{2}\varepsilon^2(1,t) \\ &\quad + \frac{1}{2}\sum_{j=1}^{m}\left[\frac{\widetilde{\theta}_j^2(t)}{r_j} + \frac{\widetilde{\vartheta}_j^2(t)}{l_j}\right]. \end{aligned} \tag{8}$$

Actually, a simple formal computation along the solution of (7) yields

$$\begin{aligned} \dot{E}_\varepsilon(t) &= \varepsilon_t(x,t)\varepsilon_x(x,t)|_0^1 + c_1\varepsilon(1,t)\varepsilon_t(1,t) \\ &\quad + \sum_{j=1}^{m}\left[\frac{1}{r_j}\widetilde{\theta}(t)\dot{\widetilde{\theta}}(t) + \frac{1}{l_j}\widetilde{\vartheta}_j(t)\dot{\widetilde{\vartheta}}_j(t)\right] \\ &= -q\varepsilon_t^2(0,t) \leq 0. \end{aligned} \tag{9}$$

Let $V = H^3(0,1) \cap D(A)$ with A being an operator defined in $L^2(0,1)$ by

$$\begin{cases} A\phi(x) = -\phi''(x), \quad \forall \phi(\cdot) \in D(A), \\ D(A) = \Big\{\phi(\cdot) \in H^2(0,1), \phi'(0) = 0, \\ \qquad \phi'(1) = -c_1\phi(1)\Big\}. \end{cases} \tag{10}$$