Digital Signal Processing 30 (2014) 27–36

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

State estimation for discrete-time Markov jump linear systems with multiplicative noises and delayed mode measurements [☆]

Wei Liu [a,*], Guangzhen Hu [b]

[a] School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454003, PR China
[b] School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan 114051, PR China

ARTICLE INFO

Article history:
Available online 13 April 2014

Keywords:
State estimation
Discrete-time
Markov jump linear systems
Multiplicative noises
Delayed mode measurements

ABSTRACT

In this paper, the state estimation problem for discrete-time Markov jump linear systems affected by multiplicative noises is considered. The available measurements for the system under consideration have two components: the first is the model measurement and the second is the output measurement, where the model measurement is affected by a fixed amount of delay. Using Bayes' rule and some results obtained in this paper, a novel suboptimal state estimation algorithm is proposed in the sense of minimum mean-square error under a lot of Gaussian hypotheses. The proposed algorithm is recursive and does not increase computational and storage load with time. Computer simulations are carried out to evaluate the performance of the proposed algorithm.

1. Introduction

Discrete-time Markov jump linear systems (DTMJLSs) are discrete-time systems with parameters that vary with discrete-time according to a finite-state Markov chain. DTMJLSs are an important class of stochastic time-variant systems because they have been used to model a wide variety of practical systems [1] with the behavior of physical processes subject to random abrupt changes in structure.

The state estimation problem for this class of systems is a fundamental issue and has received much research interest in recent years due to its application backgrounds such as tracking of maneuvering object, image processing, telecommunications and economics (see, for example [2–4]). The state estimation problem for DTMJLSs that have neither multiplicative noises nor delayed mode measurements was considered for two main cases. The first case is that both the output and mode can be measured. Optimal state estimation algorithm is obtained from Kalman filter for time varying systems [5–7]. The second case is that only the output is measured. Optimal state estimation algorithm in this case requires exponential complexity of $O(N^T)$, where N denotes the possible realizations of a finite-state Markov chain and T denotes the number of measurements [8,9]. As a result, suboptimal algorithm has to be considered to limit the computational requirements. The generalized pseudo Bayesian (GPB) methods [9,10] and the interacting multiple-model (IMM) algorithm [11] are the most popular suboptimal algorithms. Using different Gaussian hypotheses, they were obtained by summing the weighted mode conditional estimates. Other suboptimal algorithms can be found in [12–20]. Little research has been done for the case when DTMJLSs are affected by delayed output measurements. In [6], the state estimation problem for DTMJLSs with delayed mode measurements was considered, and an optimal algorithm was developed. The results of [6] were extended in [7] to consider both delayed output measurements and delayed mode measurements. However, the above state estimation algorithms for DTMJLSs with delayed mode measurements do not consider the case that this class of systems are affected by multiplicative noises. Multiplicative noise exists in many important cases, for example, fading or reflection of the transmitted signal over ionospheric channel, and certain situations involving sampling, gating, or modulation [21]. The state estimation problem for systems with multiplicative noises has received considerable research interest and many results have been reported (see, for example, [21–26]). The state estimation problem for discrete-time linear systems with multiplicative noises has been studied in [21–24]. In [25], the robust Kalman filtering problem for discrete time-varying uncertain systems with multiplicative noises has been investigated. More recently, the state estimation problem for multi-sensor systems with multiplicative noises and missing measurements has been discussed in [26]. Unfortunately, the aforementioned literature for systems with multiplicative noises does not consider the case of DTMJLSs with delayed mode measurements. To the best of our knowledge,

☆ Supported by the Key Project of Henan Educational Committee (13A413332) and the Doctoral Foundation of Henan Polytechnic University (648713).

* Corresponding author.
E-mail address: intervalm@163.com (W. Liu).

http://dx.doi.org/10.1016/j.dsp.2014.04.002
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the state estimation problem for DTMJLSs with both multiplicative noises and delayed mode measurements has not been investigated. When multiplicative noises appear and influence the DTMJLSs with delayed mode measurements, previous designs have ignored this effect and, as a result, the existing algorithms were implemented without considering multiplicative noise. When multiplicative noises are not considered in the model, unacceptable performance can often result. Hence, it is necessary to consider the case that the DTMJLSs with delayed mode measurements are affected by multiplicative noises.

In this paper, we consider the state estimation problem of DTMJLSs with multiplicative noises and delayed mode measurements. The main objective of this paper is to develop a state estimation algorithm for the system under consideration. Due to the existence of multiplicative measurement noises, the system under consideration is a non-Gaussian system. It is well known that the optimal state estimation in the sense of minimum mean-square error (MMSE) for non-Gaussian systems is nonlinear infinite dimensional and is hard to compute actually. Hence, we propose a novel suboptimal algorithm. In order to propose the new suboptimal algorithm, we first obtain an optimal algorithm for a DTMJLS with multiplicative measurement noises in the standard case (i.e. the output and model measurements are available with no delay). Then, the above new optimal algorithm and Bayes' rule are applied to the state estimation of the system under consideration so that a suboptimal algorithm is obtained by summing the weighted mode-conditional estimates under a lot of Gaussian hypotheses. The proposed suboptimal algorithm is recursive and finite dimensionally computable. It is worth noting that, for the computation of weights, the method used in this paper is new and different from the one used in [6] and [7]. In this paper, the weights are computed in a recursive form obtained from Bayes' rule. However, the computation of the weights in [6] and [7] requires the continued multiplication of many transition probabilities and conditional density functions, which leads to larger storage load.

This paper is organized as follows. In Section 2, the problem under consideration is formulated, and an auxiliary lemma is presented. A suboptimal algorithm is proposed in Section 3. In Section 4, the performance of the proposed algorithm is illustrated via two examples of scalar dynamic systems and an example of maneuvering target tracking. Concluding remarks are made in Section 5.

Notation. For a general random process Z_k, the history of the process from time 0 up to time k is denoted by Z^k, that is $Z^k = \{Z_0, Z_1, \cdots, Z_k\}$. The sum $\sum_{i_a=1}^{N}\sum_{i_{a+1}=1}^{N}\cdots\sum_{i_b=1}^{N}$ is denoted by $\sum_{i_a^b}$ where a and b are integers such that $a \le b$. Let $\hat{E}[X|Y=y]$ denotes the linear MMSE estimate of X given $Y=y$ where X and Y are random vectors. For a random vector X, $\mathrm{Var}(X)$ denotes the covariance matrix of X.

2. Problem formulation and auxiliary lemma

Consider the following discrete-time Markov jump linear system

$$x_{k+1} = A(\theta_{k+1})x_k + B(\theta_{k+1})\omega_{k+1} + F(\theta_{k+1})u_{k+1}, \tag{1}$$

$$y_k = \left(C(\theta_k) + \sum_{\mu=1}^{M}\tilde{C}^{\mu}(\theta_k)\zeta_k^{\mu}\right)x_k + D(\theta_k)\upsilon_k, \quad k=0,1,\cdots \tag{2}$$

where $x_k \in \mathrm{R}^n$ is the unknown state; θ_k is a known discrete-time Markov chain with finite state space $\{1,2,\cdots,N\}$ and transition probability

$$p_{ij} \triangleq P(\theta_{k+1}=j|\theta_k=i), \quad i,j=1,2,\cdots,N; \tag{3}$$

$\omega_k \in \mathrm{R}^m$ is the additive process noise; $u_k \in \mathrm{R}^s$ is the exogenous input; $y_k \in \mathrm{R}^p$ is the measurement; $\upsilon_k \in \mathrm{R}^q$ is the additive measurement noise; $\zeta_k^{\mu} \in \mathrm{R}$ is the multiplicative measurement noise; and $A(\theta_k)$, $B(\theta_k)$, $C(\theta_k)$, $\tilde{C}^{\mu}(\theta_k)$, $D(\theta_k)$ and $F(\theta_k)$ are time-varying matrices of appropriate dimensions; and the initial state x_0 is a Gaussian random vector with mean $\bar{x}_0$ and covariance matrix $\bar{P}_0$.

We now have three assumptions.

1. ω_k, υ_k and ζ_k^{μ} are zero-mean normalized white Gaussian noise sequences.
2. ω_k is independent of υ_i and ζ_i^{μ}, $i=0,1,\cdots,k$. υ_k is independent of ζ_i^{μ}, and ζ_i^{μ} is independent of ζ_i^{j}, $j=1,2,\cdots,M$, $j \ne \mu$.
3. x_0 is independent of ω_k, υ_k and ζ_k^{μ}, and θ_k is independent of x_0, ω_k, υ_k and ζ_k^{μ}.

Assuming that the state vector x_k is not known, and that at the current time the data available include the output measurements up to time k and mode measurements up to time $k-h$ where h is a positive integer representing the delays affecting the mode measurement. The objective of this paper is to design a recursive algorithm to estimate the state x_k using available measurements in the sense of MMSE. More precisely, given a measurement sequence y^k, θ^{k-h}, we want to compute $E[x_k|y^k, \theta^{k-h}]$ that is the MMSE estimate of x_k given y^k, θ^{k-h}.

Remark 1. Since the measurement y_i, $i=0,1,\cdots,k$, contains the multiplicative measurement noises $\zeta_i^1, \zeta_i^2, \cdots, \zeta_i^M$, the Gaussianity of y_i does not hold. As a result, the system under consideration is a non-Gaussian system. It is well known that the optimal conditional mean state estimates of non-Gaussian systems is nonlinear infinite dimensional and is difficult to compute actually. Hence, suboptimal algorithm should be considered to approximately compute $E[x_k|y^k, \theta^{k-h}]$.

Lemma 1. *(See [27].) Under the case when $f(X|Y)$ obeys Gaussian distribution, the linear MMSE estimate of X given $Y=y$ is equivalent to the MMSE estimate of X given $Y=y$, that is*

$$\hat{E}[X|Y=y] = E[X|Y=y]. \tag{4}$$

3. Proposed algorithm

In this section, we obtain a suboptimal state estimation algorithm for the discrete-time Markov jump linear system under consideration in the sense of MMSE.

3.1. Standard case

We first discuss the state estimation problem for DTMJLSs affected by multiplicative noises in the standard case. That is the output and model measurements are available with no delay.

For notational simplicity, let $\hat{x}_k^{\theta} \triangleq \hat{E}[x_k|y^k, \theta^k]$, $P_k^{\theta} \triangleq E[(x_k^{\theta} - \hat{x}_k^{\theta})(x_k^{\theta} - \hat{x}_k^{\theta})^T]$, $\bar{x}_k^{\theta} \triangleq E[x_k^{\theta}]$ and $\bar{X}_k^{\theta} \triangleq E[x_k^{\theta}(x_k^{\theta})^T]$ where x_k^{θ} is the random vector x_k in the case when the Markov chain has the state θ^k. The optimal estimator in the standard case is summarized in the following theorem.

Theorem 1. *When there is no delay in the mode measurement (2), namely, $h=0$, the linear MMSE estimate $\hat{x}_k^{\theta}$ and corresponding covariance matrix P_k^{θ} can be recursively computed by the following equalities:*

$$\begin{aligned}\bar{X}_k^{\theta} &= A(\theta_k)\bar{X}_{k-1}^{\theta}A(\theta_k)^T + B(\theta_k)B(\theta_k)^T + F(\theta_k)u_k u_k^T F(\theta_k)^T \\ &\quad + A(\theta_k)\bar{x}_{k-1}^{\theta}u_k^T F(\theta_k)^T + \left(A(\theta_k)\bar{x}_{k-1}^{\theta}u_k^T F(\theta_k)^T\right)^T,\end{aligned} \tag{5}$$