



Brief paper

Distributed event-triggered coordination for average consensus on weight-balanced digraphs[☆]

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ABSTRACT

This paper proposes a novel distributed event-triggered algorithmic solution to the multi-agent average consensus problem for networks whose communication topology is described by weight-balanced, strongly connected digraphs. The proposed event-triggered communication and control strategy does not rely on individual agents having continuous or periodic access to information about the state of their neighbors. In addition, it does not require the agents to have a priori knowledge of any global parameter to execute the algorithm. We show that, under the proposed law, events cannot be triggered an infinite number of times in any finite period (i.e., no Zeno behavior), and that the resulting network executions provably converge to the average of the initial agents' states exponentially fast. We also provide weaker conditions on connectivity under which convergence is guaranteed when the communication topology is switching. Finally, we also propose and analyze a periodic implementation of our algorithm where the relevant triggering functions do not need to be evaluated continuously. Simulations illustrate our results and provide comparisons with other existing algorithms.

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1. Introduction

This paper studies the multi-agent average consensus problem, where a group of agents seek to agree on the average of their initial states. Due to its numerous applications in networked systems, many algorithmic solutions exist to this problem; however, a majority of them rely on agents having continuous or periodic availability of information from other agents. Unfortunately, this assumption leads to inefficient implementations in terms of energy consumption, communication bandwidth, congestion, and processor usage. Motivated by these observations, our main goal here is the design of a provably correct distributed event-triggered strategy that prescribes when communication and control updates should occur so that the resulting asynchronous network executions still achieve average consensus.

Literature review: Triggered control seeks to understand the trade-offs between computation, communication, sensing, and actuator effort in achieving a desired task with a guaranteed level of performance. Early works (Åström & Bernhardsson, 2002) consider tuning controller executions to the state evolution of a given system, but the ideas have since then been extended to consider other tasks, see Heemels, Johansson, and Tabuada (2012) and references therein for a recent overview. Among the many references in the context of multi-agent systems, Mazo and Tabuada (2011) specify the responsibility of each agent in updating the control signals, Wang and Lemmon (2011) consider network scenarios with disturbances, communication delays, and packet drops, and Stöcker, Vey, and Lunze (2013) study decentralized event-based control that incorporates estimators of the interconnection signals among agents. Several works have explored the application of event-triggered ideas to the acquisition of information by the agents. To this end, Heemels and Donkers (2013), Meng and Chen (2013) and Xie, Liu, Wang, and Jia (2009) combine event-triggered controller updates with sampled data that allows for the periodic evaluation of the triggers. Zhong and Cassandras (2010) drop the need for periodic access to information by considering event-based broadcasts, where agents decide with local information only when to obtain further information about neighbors. Self-triggered control (Anta & Tabuada, 2010; Wang &

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Lemmon, 2009) relaxes the need for local information by deciding when a future sample of the state should be taken based on the available information from the last sampled state. Team-triggered coordination (Nowzari & Cortés, 2016) combines the strengths of event- and self-triggered control into a unified approach for networked systems.

The literature on multi-agent average consensus is vast, see e.g., Mesbahi and Egerstedt (2010), Olfati-Saber, Fax, and Murray (2007), Ren and Beard (2008) and references therein. Olfati-Saber and Murray (2004) introduce a continuous-time algorithm that achieves asymptotic convergence to average consensus for both undirected and weight-balanced directed graphs. Dimarogonas, Frazzoli, and Johansson (2012) build on this algorithm to propose a Lyapunov-based event-triggered strategy that dictates when agents should update their control signals but its implementation relies on each agent having perfect information about their neighbors at all times. The work (Seybotha, Dimarogonas, & Johansson, 2013) uses event-triggered broadcasting with time-dependent triggering functions to provide an algorithm where each agent only requires exact information about itself, rather than its neighbors. However, its implementation requires knowledge of the algebraic connectivity of the network. In addition, the strictly time-dependent nature of the thresholds makes the network executions decoupled from the actual state of the agents. Closer to our treatment here, Garcia, Cao, Yu, Antsaklis, and Casbeer (2013) propose an event-triggered broadcasting law with state-dependent triggering functions where agents do not rely on the availability of continuous information about their neighbors (under the assumption that all agents have initial access to a common parameter). This algorithm works for networks with undirected communication topologies, tolerates quantized communication, and guarantees that all inter-event times are strictly positive, but does not discard the possibility of an infinite number of events happening in a finite time period. The work (Fan, Liu, Feng, & Wang, 2015) proposes a self-triggered algorithm for this problem that is guaranteed to avoid Zeno executions. We consider here a more general class of communication topologies described by weight-balanced, directed graphs. The works (Gharesifard & Cortés, 2012; Rikos, Charalambous, & Hadjicostis, 2014) present provably correct distributed strategies that, given a directed communication topology, allow a network of agents to find such weight edge assignments.

Statement of contributions: Our main contribution is the design and analysis of novel event-triggered broadcasting and controller update strategies to solve the multi-agent average consensus problem over weight-balanced digraphs. With respect to the conference version of this work (Nowzari & Cortés, 2014), the present manuscript introduces new trigger designs, extends the treatment from undirected graphs to weight-balanced digraphs, and provides a comprehensive technical treatment. Our proposed law does not require individual agents to have continuous access to information about the state of their neighbors and is fully distributed in the sense that it does not require any a priori knowledge by agents of global network parameters to execute the algorithm. Our Lyapunov-based design builds on the evolution of the network disagreement to synthesize triggers that agents can evaluate using locally available information to make decisions about when to broadcast their current state to neighbors. In our design, we carefully take into account the discontinuities in the information available to the agents caused by broadcasts received from neighbors and their effect on the feasibility of the resulting implementation. Our analysis shows that the resulting asynchronous network executions are free from Zeno behavior, i.e., only a finite number of events are triggered in any finite time period, and exponentially converge to agreement on the average of all agents' initial states over weight-balanced, strongly connected

digraphs. We also provide a lower bound on the exponential convergence rate and characterize the asymptotic convergence of the network under switching topologies that remain weight-balanced and are jointly strongly connected. Lastly, we propose a periodic implementation of our event-triggered design that has agents check the triggers periodically and characterize the sampling period that guarantees correctness. Various simulations illustrate our results.

2. Preliminaries

This section introduces some notational conventions and notions on graph theory. Let \mathbb{R} , $\mathbb{R}_{>0}$, $\mathbb{R}_{\geq 0}$, and $\mathbb{Z}_{>0}$ denote the set of real, positive real, nonnegative real, and positive integer numbers, respectively. We denote by $\mathbf{1}_N$ and $\mathbf{0}_N \in \mathbb{R}^N$ the column vectors with entries all equal to one and zero, respectively. We let $\|\cdot\|$ denote the Euclidean norm on \mathbb{R}^N . We let $\text{diag}(\mathbb{R}^N) = \{x \in \mathbb{R}^N \mid x_1 = \dots = x_N\} \subset \mathbb{R}^N$ be the agreement subspace in \mathbb{R}^N . For a finite set S , we let $|S|$ denote its cardinality. Given $x, y \in \mathbb{R}$, Young's inequality (Hardy, Littlewood, & Polya, 1952) states that, for any $\varepsilon \in \mathbb{R}_{>0}$,

$$xy \leq \frac{x^2}{2\varepsilon} + \frac{\varepsilon y^2}{2}. \quad (1)$$

A weighted directed graph (or weighted digraph) $\mathcal{G} = (V, E, W)$ is comprised of a set of vertices $V = \{1, \dots, N\}$, directed edges $E \subset V \times V$ and weighted adjacency matrix $W \in \mathbb{R}_{\geq 0}^{N \times N}$. Given an edge $(i, j) \in E$, we refer to j as an out-neighbor of i and i as an in-neighbor of j . The sets of out- and in-neighbors of a given node i are $\mathcal{N}_i^{\text{out}}$ and $\mathcal{N}_i^{\text{in}}$, respectively. The weighted adjacency matrix $W \in \mathbb{R}^{N \times N}$ satisfies $w_{ij} > 0$ if $(i, j) \in E$ and $w_{ij} = 0$ otherwise. A path from vertex i to j is an ordered sequence of vertices such that each intermediate pair of vertices is an edge. A digraph \mathcal{G} is strongly connected if there exists a path from all $i \in V$ to all $j \in V$. The out- and in-degree matrices D^{out} and D^{in} are diagonal matrices where

$$d_i^{\text{out}} = \sum_{j \in \mathcal{N}_i^{\text{out}}} w_{ij}, \quad d_i^{\text{in}} = \sum_{j \in \mathcal{N}_i^{\text{in}}} w_{ji},$$

respectively. A digraph is weight-balanced if $D^{\text{out}} = D^{\text{in}}$. The (weighted) Laplacian matrix is $L = D^{\text{out}} - W$. Based on the structure of L , at least one of its eigenvalues is zero and the rest of them have nonnegative real parts. If the digraph \mathcal{G} is strongly connected, 0 is simple with associated eigenvector $\mathbf{1}_N$. The digraph \mathcal{G} is weight-balanced if and only if $\mathbf{1}_N^T L = \mathbf{0}_N^T$ if and only if $L_s = \frac{1}{2}(L + L^T)$ is positive semidefinite. For a strongly connected and weight-balanced digraph, zero is a simple eigenvalue of L_s . In this case, we order its eigenvalues as $\lambda_1 = 0 < \lambda_2 \leq \dots \leq \lambda_N$, and note the inequality

$$x^T L x \geq \lambda_2(L_s) \left\| x - \frac{1}{N} (\mathbf{1}_N^T x) \mathbf{1}_N \right\|^2, \quad (2)$$

for all $x \in \mathbb{R}^N$. The following property will also be of use later,

$$\lambda_2(L_s) x^T L x \leq x^T L_s^T x \leq \lambda_N(L_s) x^T L x. \quad (3)$$

This can be seen by noting that L_s is diagonalizable and rewriting $L_s = S^{-1} D S$, where D is a diagonal matrix containing the eigenvalues of L_s .

3. Problem statement

We consider the multi-agent average consensus problem for a network of N agents. We let \mathcal{G} denote the weight-balanced, strongly connected digraph describing the communication topology of the network. Without loss of generality, we use the

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