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Delay-dependent robust L_2–L_∞ filter design for uncertain neutral stochastic systems with mixed delays ☆

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ABSTRACT

This paper is concerned with the problem of the robust L_2–L_∞ filter design for uncertain neutral stochastic systems with mixed delays. By constructing a modified Lyapunov–Krasovskii functional, some novel delay-dependent exponential stability criteria for uncertain neutral stochastic systems with mixed delays are established in terms of linear matrix inequality. And the obtained stability criteria pave the way for designing the robust L_2–L_∞ filter to guarantee the robustly exponential stability for the filtering error systems with a prescribed L_2–L_∞ performance level for all admissible uncertainties. Finally, three illustrative numerical examples are given to show the effectiveness of the obtained results.

1. Introduction

The L_2–L_∞ filtering problem has been widely discussed over the past decades and its applications into a variety of areas such as signal processing, signal estimation, pattern recognition, communications, control application and many practical control systems have been studied. The problem of filtering can be briefly described as the design of an estimator from the measured output to estimate the state of the given systems. One of its main advantages is that it is insensitive to the exact knowledge of the statistics of the noise signals. Thus, the problem on the L_2–L_∞ filter design for the control systems has been attracting much attention over the last ten years and some valuable results on this issue were reported in the literatures, see e.g. [1,2,7,9–11,17–21,23,28,31,33,34] and the references therein. Time delay and system uncertainties are often the two main sources of instability, oscillation and poor performance of the control system, which are encountered in various engineering systems, such as communication, electronics, hydraulic and chemical systems [41,43–45]. Recently, more attention has been devoted to the study on the stability analysis and the L_2–L_∞ filtering design of uncertain delayed systems and a great number of results related to these issues have been reported in [13,22,37,40] and the references therein. The obtained sufficient conditions can be divided into two categories: delay-dependent one [13,37,40,46] and delay-independent one [22]. Generally speaking, the former is less conservative than the latter when the delay is very small. And to the best of our knowledge, for the L_2–L_∞ filter design, there are two performance indices which are used to judge the conservatism of the derived conditions. One is the L_2–L_∞ performance index while the other is the maximum allowable bound of the time-delay. For a given time-delay, the smaller the L_2–L_∞ performance index, the better the conditions. For a prescribed L_2–L_∞ performance index, the larger the maximum allowable bound of the time-delay, the less conservative the conditions. Therefore, chasing the less conservative criteria of the L_2–L_∞ filter design is of much theoretical and practical value.

However, most of the results on the L_2–L_∞ filter design for time-delay systems proposed and discussed above are deterministic. In fact, time-delay system is inevitably affected by some external perturbations which in many cases are of great uncertainty and hence may be treated as stochastic system. Stochastic systems have come to play an important role in many branches of science and engineering applications. Recently, some authors have studied the robust L_2–L_∞ filter design for uncertain stochastic systems [6,12,32,36,37,39] and the references therein. For example, by using the delay decomposition approach, the problem of the L_2–L_∞ filter design for stochastic systems with time-varying delay has been considered in [32]. Although some LMI-based sufficient conditions about the robust L_2–L_∞ filtering design for uncertain stochastic time-delay systems are obtained in [36], they are delay-independent. Xia et al., in [39], have given some LMI-based delay-dependent sufficient conditions ensuring the robust control for uncertain stochastic time-delay systems.

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It is well known that many dynamical systems not only depend on the present and past states but also involve the derivative with delays as well as the functional of the past history. Neutral delay differential equations are often used to describe such systems. Many authors have considered the dynamical analysis of the neutral delay differential equations in [3,16,24,25,42] and the references therein. For example, some LMI-based conditions ensuring the stability analysis for neutral delay differential systems have been given in [3,16]. If the environmental disturbances are taken into account, such systems can be transformed into the neutral stochastic delay differential systems and some fundamental theories of neutral stochastic delay differential systems have been introduced in [27]. Since neutral stochastic delay differential systems can be extensively applied into many branches for the control field, in the past few decades, increasing attention has been devoted to the problems on the stability analysis and the L_2–L_∞ filter design for such systems via different approaches given in [3–5,15,24, 26,27,29,30,35,38] and the references therein. For example, in [4], Chen and Shen have provided some LMI-based sufficient conditions ensuring the robustly asymptotical stability in mean square moment and proved the existence of the robust L_2–L_∞ filter design for uncertain neutral stochastic systems with time-varying delay. Some LMI-based delay-dependent sufficient conditions ensuring the robustly asymptotical stability in mean square moment and the existence of the robust L_2–L_∞ filter design for uncertain neutral stochastic delayed systems were given in [29,30,35]. Although many elegant results have emerged, in our opinion, there still exist two important points waiting for further discussions on the robust L_2–L_∞ filter design for uncertain neutral stochastic delayed systems, which constitute the main focus of this paper. On one hand, the robust L_2–L_∞ filter design for our concerned problem was discussed in [4], but the obtained LMI-based sufficient conditions are delay-independent ones and thus much more conservative; on the other hand, although the delay-dependent stability criteria and some sufficient conditions to guarantee the existence of the robust L_2–L_∞ filter design for uncertain neutral stochastic delayed systems have been reported in [4,29,30,35], they cannot provide the fast convergence rate and desirable accuracy. Hence, it is necessary to consider the exponential stability and the delay-dependent robust L_2–L_∞ exponential filter design for uncertain neutral stochastic systems with mixed delays.

In this paper, with a new augmented LKF constructed, some delay-dependent robustly exponential stability criterion for uncertain neutral stochastic linear systems with mixed delays are derived in the form of LMI. And then, based on the obtained stability criterion, the delay-dependent condition for the solvability of the robust L_2–L_∞ filtering problem can be derived. The desired L_2–L_∞ filter is designed to guarantee the robustly exponential stability for the filtering error systems with a prescribed L_2–L_∞ performance level for all admissible uncertainties. As a special case, the developed method can greatly reduce the conservatism of uncertain neutral stochastic systems with one constant delay. Finally, three illustrative numerical examples are given to show the effectiveness of our obtained results.

Notations: In this paper, R^n and $R^{m\times n}$ denote the n-dimensional Euclidean space and the set of real $m\times n$ matrix, respectively. $|\cdot|$ presents the Euclidean norm in R^n. $(\Omega, \Im, \mathcal{P})$ is a completed probability space, where Ω is the sample space, $\Im$ is a σ-algebra of subsets of Ω, and $\mathcal{P}$ is the probability measure. For a real symmetric matrix X, $X>0$ ($X\geqslant 0$) means that X is positive definite (positive semi-definite). The asterisk "$*$" denotes a matrix that can be inferred by symmetry and the superscript "T" represents the transpose of a matrix or a vector. $\mathcal{L}_2([0,+\infty))$ is the space of square integrable functions with the norm $\|\cdot\|$. Let $C([-r,0];R^n)$ $(r>0)$ denote the family of continuous functions ϕ from $[-r,0]$ to R^n with the norm $|||\phi||| := \sup_{\theta\in[-r,0]}|\phi(\theta)|$. Denote by $L^2_{\Im_0}([-r,0];R^n)$ the family of all $\Im_0$ measurable, $C([-r,0];R^n)$-valued random process $\chi=\{\chi(\theta)\colon -r\leqslant\theta\leqslant 0\}$ such that $\mathcal{E}\{\sup_{\theta\in[-r,0]}|\chi(\theta)|^2\}<+\infty$, where $\mathcal{E}\{\cdot\}$ denotes the expectation operator. $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ denote the maximum eigenvalue and the minimum eigenvalue of a semi-positive definite matrix Q, respectively.

2. Problem statement and preliminaries

In this paper, we consider the uncertain neutral stochastic systems (Σ_0):

$$\begin{aligned} d\big[x(t)-Dx(t-\tau)\big] &= \big[A_0(t)x(t)+A_1(t)x(t-h)+B_1v(t)\big]dt \\ &\quad+\big[H_0(t)x(t)+H_1(t)x(t-h)+B_2v(t)\big]d\omega(t), \end{aligned} \tag{2.1}$$

$$\begin{aligned} dy(t) &= \big[C_0(t)x(t)+C_1(t)x(t-h)+B_3v(t)\big]dt \\ &\quad+\big[D_0(t)x(t)+D_1(t)x(t-h)+B_4v(t)\big]d\omega(t), \end{aligned} \tag{2.2}$$

$$z(t)=L_1x(t)+L_2x(t-h)+L_3v(t),\quad t\geqslant 0, \tag{2.3}$$

$$x(\theta)=\psi(\theta),\quad \theta\in[-r,0], \tag{2.4}$$

where the delays $\tau>0$, $h>0$ and $r=\max\{\tau,h\}$, $x(t)\in R^n$ is the state vector, $y(t)\in R^m$ is the measurement, $v(t)\in R^k$ is the disturbance input which belongs to $\mathcal{L}_2[0,+\infty)$, $z(t)\in R^l$ is the signal to be estimated, and $\omega(t)$ is one-dimensional Brownian motion defined on the complete probability space $(\Omega,\Im,\mathcal{P})$. $\psi(\cdot)\in L^2_{\Im_0}([-r,0];R^n)$ is the initial condition.

In (Σ_0), $A_0(t)$, $A_1(t)$, $H_0(t)$, $H_1(t)$, $C_0(t)$, $C_1(t)$, $D_0(t)$, $D_1(t)$ are appropriate dimensional matrix functions with time-varying uncertainties, this is,

$$A_i(t)=A_i+\Delta A_i(t),\qquad H_i(t)=H_i+\Delta H_i(t),$$
$$C_i(t)=C_i+\Delta C_i(t),\qquad D_i(t)=D_i+\Delta D_i(t),$$

where A_i, H_i, C_i, D_i $(i=0,1)$, D, B_k $(k=1,2,3,4)$, L_1, L_2 and L_3 are known real constant matrices of appropriate dimensions, and $\Delta A_i(t)$, $\Delta H_i(t)$, $\Delta C_i(t)$, $\Delta D_i(t)$ $(i=0,1)$ are unknown matrices which denote the time-varying parameter uncertainties. We assume that the uncertainties are norm-bounded and can be written as

$$\begin{bmatrix} \Delta A_0(t) & \Delta A_1(t) \\ \Delta C_0(t) & \Delta C_1(t) \end{bmatrix} = \begin{bmatrix} M_1 \\ M_3 \end{bmatrix} F(t)\begin{bmatrix} N_1 & N_2 \end{bmatrix}, \tag{2.5}$$

and

$$\begin{bmatrix} \Delta H_0(t) & \Delta H_1(t) \\ \Delta D_0(t) & \Delta D_1(t) \end{bmatrix} = \begin{bmatrix} M_2 \\ M_4 \end{bmatrix} F(t)\begin{bmatrix} N_1 & N_2 \end{bmatrix}, \tag{2.6}$$

where M_k, N_i $(k=1,2,3,4;\ i=1,2)$ are known real constant matrices and $F(\cdot)$ is unknown matrix function satisfying

$$F^T(t)F(t)\leqslant I,\quad \forall t\geqslant 0. \tag{2.7}$$

It is assumed that the elements of $F(t)$ are Lebesgue measurable. The parameter uncertainties $\Delta A_i(t)$, $\Delta H_i(t)$, $\Delta C_i(t)$ and $\Delta D_i(t)$ $(i=0,1)$ are said to be admissible if (2.5)–(2.7) are both satisfied.

Now, we consider the following filtering for systems (Σ_0)

$$(\Sigma_f)\quad d\hat{x}(t)=A_f\hat{x}(t)\,dt+B_f\,dy(t), \tag{2.8}$$

$$\hat{z}(t)=L_f\hat{x}(t), \tag{2.9}$$

where $\hat{x}(t)$ is the filter state, and the matrices A_f, B_f and L_f are to be determined.

Define

$$\xi(t)=\big[x^T(t)\quad \hat{x}^T(t)\big]^T,\qquad e(t)=z(t)-\hat{z}(t),$$