Automatica 68 (2016) 277–285

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Finite-time stabilization of linear time-varying systems by piecewise constant feedback[☆]

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ARTICLE INFO

Article history:
Received 16 September 2014
Received in revised form
22 August 2015
Accepted 17 December 2015
Available online 1 March 2016

Keywords:
Finite-time stabilization
Linear time-varying systems
Piecewise constant feedback
Linear matrix inequalities

ABSTRACT

Most existing methods for finite-time stabilizing controller design of linear time-varying systems involve solving differential linear matrix equations. Due to the non-convexity of the problem, it requires a high computational burden. This paper proposes a numerical method to solve finite-time stabilization problems. Successive approximations are performed to estimate the evolution of system states. Accordingly, a gain-switched state feedback controller can be obtained by solving a sequence of linear matrix inequalities (LMIs) based optimization problems. The proposed algorithm is used to design the mass–spring system and the autopilot system of the BTT missile. Comparison with existing methods is given and the simulation results show the effectiveness of the proposed method.

1. Introduction

Finite-time stability (FTS) is a practical concept which is used to study the behavior of a system within a finite time interval. A system is said to be finite-time stable if, given a bound on the initial condition, its state does not exceed a certain threshold during a certain time interval. Typically, the feature of FTS does not guarantee stability in the sense of Lyapunov, and vice versa, however FTS is a very useful concept in many cases. For many dynamic systems the state trajectories are required to stay within a desirable operative range over a certain time interval to prevent saturation, to fulfill hardware constraints or to maintain linearity of the system. The concept of FTS was introduced in the control literature in the sixties of the last century (Dorato, 1961; Weiss & Infante, 1967) and has received much attention in the last fifteen years. Related stability criterion and stabilization methods have been studied for both linear systems (Amato, Ariola, & Cosentino, 2011; Garcia, Tarbouriech, & Bernussou, 2009; Shen, 2008) and nonlinear systems (Mastellone, Dorato, & Abdallah, 2004; Yang, Li, & Chen, 2009).

We focus our attention on linear time-varying systems in this paper, which have been studied in some references. It is well known that analysis and design of linear time-varying systems are challenging since their stability cannot be determined simply by its coefficients (for example, its eigenvalues) except for the particular case that the coefficients are periodic (see Bittanti & Colaneri, 2008; Zhou & Duan, 2012 and the references therein) or some switched systems (see Lin & Antsaklis, 2009; Sun & Ge, 2005 and the references therein). Hence there have been a lot of papers in the literature that considered linear time-varying systems in different aspects (see, for example, Kalman & Bertram, 1960; Mazenc, Malisoff, & Niculescu, 2014; Xu, Lam, & Zou, 2009; Zhou, Cai, & Duan, 2013 and the references cited there). In Zhou (in press) necessary and sufficient conditions in terms of differential Lyapunov inequalities were established for asymptotic stability, (non-uniformly) exponential stability and uniformly exponential stability of linear time-varying systems. In Abdallah, Amato, Ariola, Dorato, and Koltchinsky (2002) linear time-varying systems are considered as time-invariant systems with time-varying parametric uncertainties, accordingly, a constant-gain state feedback stabilizing controller can be obtained by solving a feasibility problem in terms of linear matrix inequalities (LMIs). In Amato, Ariola, Carbone, and Cosentino (2006) sufficient conditions for finite-time stabilization of linear time-varying systems have been provided in terms of a differential linear matrix inequality (DLMI), and in Amato, Ariola, and Cosentino (2010) and Amato, Ambrosino, Ariola, Cosentino, and Tommasi (2014) the results have been shown to be necessary and sufficient. These results have been extended also to the impulsive linear systems (Amato, Tommasi, & Pironti, 2013). However, due to the fact that we do not have a numerical technique

[☆] This work was supported in part by the National Natural Science Foundation of China under grant numbers 61273028, 61322305, 61573120, 61503100 and 61321062, by the Natural Science Foundation of Heilongjiang Province of China under grant number F2015007, by the Project-sponsored by SRF for ROCS, SEM, and by the Foundation for the Author of National Excellent Doctoral Dissertation of China under Grant 201343. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor James Lam under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2016.01.003

to find the optimal solution over the set of all admissible matrix-valued functions, one need to restrict the structure of the DLMI solution to make the problem computationally tractable. In Amato, Ambrosino, Ariola, and Cosentino (2009); Amato et al. (2006) the DLMI condition has been recast in terms of LMIs by dividing the finite time interval in subintervals and assuming the solutions piecewise affine. In order to reduce the possible design conservatism, the subinterval should be sufficiently small, which causes great computational burden.

This paper provides another numerical approach to solve the finite-time stabilization problem for linear time-varying systems, which does not involve differential matrix equations and is thus less demanding from the computational point of view. Successive approximations of states are used to derive new sufficient FTS conditions and a design algorithm is proposed consequently to recast the design problem as LMIs based convex feasibility or optimization problems which can be solved by some existing approaches (for example, Boyd and Vandenberghe (2004) and Teo and Clements (1985)). Two design examples are given and the simulation results validate the effectiveness of the proposed approach.

The remainder of this paper is organized as follows. The problem formulation and some preliminaries are given in Section 2. Our main results are included in Section 3. In Section 4, two design examples are given. The proposed approach is applied on the mass–spring system and the BTT missile control system and simulations are carried out there. Finally, Section 5 concludes this paper.

Notation: Throughout the paper, the notation used is fairly standard. We use A^{T} to denote the transpose of matrix A. For a symmetric matrix P, we use $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ to denote respectively its minimal and maximal eigenvalues. The symbol $\mathrm{diag}\{A_1, A_2, \ldots, A_p\}$ stands for a block-diagonal matrix whose diagonal elements are $A_1, A_2, \ldots, A_p$. I_n denotes the identity matrix with n dimensions. Finally, for a real symmetric matrix P, the notation $P > (\geq)\, 0$ is used to denote its positive (semi-positive) definiteness and the notation $P^{\frac{1}{2}}$ is used to denote the unique symmetric matrix $X > (\geq)\, 0$ satisfying $X^2 = P$.

2. Preliminaries and problem statement

We firstly recall the definition of finite-time stability (FTS) of linear time-varying systems as stated in Amato et al. (2014).

Definition 1. Given an initial time t_0, a positive scalar T, a positive definite matrix $R \in \mathbf{R}^{n\times n}$, and a positive definite matrix-valued function $\Gamma(t) : [t_0, t_0 + T] \to \mathbf{R}^{n\times n}$ such that

$$\Gamma(t_0) < R, \tag{1}$$

the linear time-varying system

$$\dot{x}(t) = A(t)x(t), \qquad x(t_0) = x_0, \tag{2}$$

where $A(t) \in \mathbf{R}^{n\times n}$, is said to be finite-time stable with respect to $(t_0, T, R, \Gamma(\cdot))$ if

$$x_0^{\mathrm{T}} R x_0 \leq 1 \Longrightarrow x^{\mathrm{T}}(t)\Gamma(t)x(t) < 1, \quad \forall t \in [t_0, t_0 + T]. \tag{3}$$

Remark 2. In practice, the matrix R can be determined according to the size of the initial conditions. Actually, for any bounded set $\Omega \subset \mathbf{R}^n$, there exists a $R > 0$ such that

$$x_0^{\mathrm{T}} R x_0 \leq 1, \quad \forall x_0 \in \Omega. \tag{4}$$

For example, if $\Omega = \mathrm{cov}\{x_i : i = 1, 2, \ldots, l\}$, where $\mathrm{cov}\{\cdot\}$ denotes convex hull of a set of vectors, then (4) holds true if and only if $x_i^{\mathrm{T}} R x_i \leq 1$, $i = 1, 2, \ldots, l$, which are equivalent to a set of LMIs

$$\begin{bmatrix} -1 & x_i^{\mathrm{T}} R \\ R x_i & -R \end{bmatrix} < 0, \quad i = 1, 2, \ldots, l,$$

by solving which R can be obtained. However, in some applications R can be determined easily according to the requirement of physical systems, as illustrated in the BTT design example to be given in Section 4.2.

In this paper, we consider the finite-time stabilization problem of linear time-varying systems. All the involved time-varying matrices, unless otherwise stated, are assumed to be bounded. The problem can be stated as follows.

Problem 3. Consider the linear time-varying system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \qquad x(t_0) = x_0, \tag{5}$$

where $A(t) \in \mathbf{R}^{n\times n}$ and $B(t) \in \mathbf{R}^{n\times m}$ are matrix-valued functions, and $u(t) \in \mathbf{R}^m$ is the control input. Then, given an initial time t_0, a positive scalar T, a positive definite matrix $R \in \mathbf{R}^{n\times n}$, and a positive definite matrix-valued function $\Gamma(t) : [t_0, t_0+T] \to \mathbf{R}^{n\times n}$ satisfying $\Gamma(t_0) < R$, determine a time-varying gain $K(t) : [t_0, t_0+T] \to \mathbf{R}^{m\times n}$ such that, with the state feedback controller

$$u(t) = K(t)x(t), \tag{6}$$

the closed-loop system

$$\dot{x}(t) = (A(t) + B(t)K(t))x(t), \qquad x(t_0) = x_0, \tag{7}$$

is finite-time stable with respect to $(t_0, T, R, \Gamma(\cdot))$.

3. Main results

In this section, we firstly establish sufficient criteria for FTS of the linear time-varying system (2). Based on this sufficient criteria we will then derive a condition for the existence of a solution to Problem 3 and the corresponding algorithm for the controller design will also be proposed.

Theorem 4. *Given an initial time t_0, a positive scalar T, a positive definite matrix $R \in \mathbf{R}^{n\times n}$, and a positive definite matrix-valued function $\Gamma(t) : [t_0, t_0 + T] \to \mathbf{R}^{n\times n}$ such that $\Gamma(t_0) < R$, then system (2) is finite-time stable with respect to $(t_0, T, R, \Gamma(\cdot))$ if there exists a discrete time sequence*

$$\left\{ t_i \,\middle|\, \begin{array}{l} t_{i+1} = t_i + \delta_i,\ \delta_i > 0, \\ i = 0, 1, \ldots, r,\ \ t_{r+1} = (t_0 + T)^+ \end{array} \right\} \tag{8}$$

and a piecewise constant matrix-valued function $P(t) : [t_0, t_{r+1}) \to \mathbf{R}^{n\times n}$ in the form of

$$P(t) = P_i, \quad \forall t \in [t_i, t_{i+1}),\ i = 0, 1, \ldots, r, \tag{9}$$

such that the following conditions hold:

Condition 1. *For all $t \in [t_0, t_0 + T]$,*

$$\begin{cases} P(t) \geq \Gamma(t), & \forall t \in [t_0, t_0 + T], \\ P_0 < R. \end{cases} \tag{10}$$

Condition 2. *For $i = 0, 1, \ldots, r$,*

$$\int_{t_i}^{\sigma} \alpha(\tau)\mathrm{d}\tau \leq 0, \quad \forall \sigma \in [t_i, t_{i+1}), \tag{11}$$

where $\alpha(s)$ is defined by

$$\alpha(s) = \lambda_{\max}\left(P^{-\frac{1}{2}}(s)A^{\mathrm{T}}(s)P^{\frac{1}{2}}(s) + P^{\frac{1}{2}}(s)A(s)P^{-\frac{1}{2}}(s)\right). \tag{12}$$

Condition 3. *For $i = 1, 2, \ldots, r$,*

$$\int_{t_0}^{t_i} \alpha(\tau)\mathrm{d}\tau + \sum_{k=1}^{i} \ln\left(\lambda_{\max}\left(P_k P_{k-1}^{-1}\right)\right) \leq 0. \tag{13}$$