Automatica 68 (2016) 286–293

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Sliding mode control of MIMO Markovian jump systems[☆]

Jiaming Zhu[a], Xinghuo Yu[b], Tianping Zhang[a], Zhiqiang Cao[c,1], Yuequan Yang[a], Yang Yi[a]

[a] College of Information Engineering, Yangzhou University, Yangzhou, China
[b] RMIT University, Melbourne, Australia
[c] Institute of Automation, Chinese Academy of Science, Beijing, China

ARTICLE INFO

Article history:
Received 8 November 2014
Received in revised form
13 November 2015
Accepted 20 January 2016
Available online 4 March 2016

Keywords:
Sliding mode control
Markovian jump system
Linear matrix inequality (LMI)
Conditional probability
Asymptotical stability

ABSTRACT

This paper addresses the sliding mode control problem for uncertain MIMO linear Markovian jump systems. Firstly, by using the linear matrix inequality approach, sufficient conditions are proposed to guarantee the stochastically asymptotical stability of the system on the sliding surfaces. Secondly, an equivalent control based sliding mode control is proposed, such that the closed-loop system can be driven onto the desired sliding surfaces in a finite time. Finally, combining with multi-step state transition probability, the reaching and sliding probabilities are derived for situations where the control force may not be strong enough to ensure the fully asymptotical stability. Simulation results are presented to illustrate the effectiveness of the proposed design method.

1. Introduction

Nowadays, the modeling of dynamic systems subject to abrupt changes in their dynamics has been receiving a great deal of attention. These changes can be due to abrupt environmental disturbances, to actuator or sensor failure or repairs, to abrupt changes in the operation point for a nonlinear plant, etc. Such systems can be found in many fields, e.g. robotic manipulator systems, aircraft control, space stations, nuclear power plants, and wireless communication networks. The adequate operation of such devices, however, is severely compromised by the occurrence of failures, which may be intolerable in safety–critical applications, for example. Continuous Markovian jump system (MJS) is a proper model to describe these systems. A MJS is a continuous-time dynamical system with stochastic jumps, in which jumping parameter is a continuous-time, discrete-state Markov chain taking values in a finite set. A continuous-time Markov chain (CTMC) is a stochastic process that moves from state to state in accordance with a Markov chain, while its staying time in each state is exponentially distributed. Many results on MJSs have been reported in the literature (Chen, Xu, & Guan, 2003; De Farias, Geromel, Do Val, & Costa, 2000; Ji & Chizeck, 1990; Karan, Shi, & Kaya, 2006; Mahmoud & Shi, 2003; Shi, Boukas, & Agarwal, 1999; Wang, Lam, & Liu, 2004; Wu, Shi, & Gao, 2010; Xiong, Lam, Gao, & Ho, 2005; Xu, Chen, & Lam, 2003; Xu, Lam, & Mao, 2007; Zhang & Boukas, 2009), including quadratic control (Ji & Chizeck, 1990), output feedback control (De Farias et al., 2000), guarantee cost control (Chen et al., 2003), robust stabilization of MJS with uncertain switch probability (Xiong et al., 2005) and partly unknown transition probability (Zhang & Boukas, 2009), H_∞ control and filter (Xu et al., 2007), robust Kalman filter (Mahmoud & Shi, 2003), exponential filter (Wang et al., 2004), state estimation and sliding mode control of singular MJSs (Wu et al., 2010).

In the past decades, sliding mode control (SMC) has become an important method of nonlinear control, due to its inherent advantages, e.g. robustness, disturbance resistance, finite-time convergence. SMC alters the system dynamics by a discontinuous control signal that forces the system to enter and then slide along a surface, on which the system has desired properties such as stability, disturbance resistance. Most recently, significant progresses have been achieved in SMC for MJSs (Chen, Niu, & Zou, 2013a,b; Kao, Li, & Wang, 2014; Luan, Shi, & Liu, 2013; Mao, 2002; Niu & Ho, 2010; Niu, Ho, & Wang, 2007; Shi & Boukas, 1997; Shi, Xia, Liu, & Rees, 2006; Utkin & Poznyak, 2013; Wang, Liu, Yu, & Liu, 2006; Wang, Qiao, & Burnham, 2002; Wu, Shi, Su, & Chu, 2014; Yin, Shi, Liu, & Teo, 2014; Yu & Kaynak, 2009;

☆ This work is supported by National Natural Science Foundation of China under Grant Nos. 61273352, 61573307, 61473249, 61473250, 61175111, 61203195 and the Australian Research Council (Nos. DP130104765, DP140100544). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor James Lam under the direction of Editor Ian R. Petersen.

E-mail addresses: qingzhu@yzu.edu.cn (J. Zhu), x.yu@rmit.edu.au (X. Yu), tpzhang@yzu.edu.cn (T. Zhang), zhiqiang.cao@ia.ac.cn (Z. Cao), yangyq@yzu.edu.cn (Y. Yang), yiyang@yzu.edu.cn, yiyangcontrol@163.com (Y. Yi).

[1] Corresponding author. Tel.: +86 10 82544790; fax: +86 10 82544799.

http://dx.doi.org/10.1016/j.automatica.2016.01.070
0005-1098/© 2016 Published by Elsevier Ltd.

Zhang, Huang, & Lam, 2003; Zhang, Wang, & Shi, 2013; Zhu, Yu, & Song, 2014,?). These include adaptive SMC for stochastic MJSs with actuator degradation (Chen et al., 2013a), SMC for stochastic MJSs with incomplete transition rate (Chen et al., 2013b), non-fragile observer-based H_∞ SMC for Ito stochastic systems with Markovian switching (Niu et al., 2007), asynchronous H_2/H_∞ filtering for discrete-time stochastic MJSs with randomly occurred sensor nonlinearities (Wu et al., 2014), observer-based H_∞ control on nonhomogeneous MJSs with nonlinear input (Yin et al., 2014), adaptive SMC with application to super-twist algorithm (Utkin & Poznyak, 2013), robust H_∞ SMC for MJSs subject to intermittent observations and partially known transition probabilities (Zhang et al., 2013), and finite-time stabilization for MJSs with Gaussian transition probabilities (Luan et al., 2013).

Aforementioned works usually assume that the strong control force is always available to overpower the stochastic uncertainties. However, in practical applications, controls are usually limited in power and sometimes insufficient. It would be beneficial to assess the risk of lowering down control force so that an economic balance between the risk and control cost can be achieved. In Zhu et al. (2014), the asymptotical stability probability was first explored for the second order MJSs under SMC where control force may not be strong enough to ensure the fully asymptotical stability. In Zhu et al. (2014), the problem on SMC of single input MJSs was studied. But it is not straightforward to extend the results for single input systems to MIMO systems as there may be strong coupling terms. Thus, the SMC scheme for MIMO MJSs (including sliding surfaces and controller design) is significantly different to the one for single input MJSs. In this paper, we explore the SMC of uncertain MIMO linear MJSs. At first, sufficient conditions are proposed in terms of LMIs to guarantee the asymptotical stability of the system on the sliding surface. Then, we derive an equivalent control based SMC paradigm, such that the closed-loop system can be driven onto the sliding surface in a finite time. Furthermore, the multi-step stochastic state transition probability function is introduced to facilitate the discussion. At last, we propose the reaching and sliding probabilities when no sufficient control is available.

The rest of this paper is organized as follows. In Section 2, the problem statement and preliminaries are presented. Next, the LMI-type sufficient conditions of asymptotical stability, the equivalent control based SMC paradigm, and the reaching probability and the sliding probability are derived for MIMO linear MJSs in Section 3. Then, the numerical simulation result is given in Section 4. A conclusion is drawn in Section 5.

Notations: $P(\cdot)$ denotes the probability of an event. $P(A|B)$ denotes the conditional probability of event A given event B. $\|\cdot\|$ denotes the Euclidean norm of a vector or the Frobenius norm of a matrix. $M > 0 (< 0)$ denotes that matrix M is a positive(negative) definite matrix. Bold $\mathbf{0}$ denotes a zero vector with compatible dimensions.

2. Problem statement and preliminaries

The MIMO linear MJS under investigation is

$$\begin{cases} \dot{X}_1(t) = (A_{11}(\eta_t) + \Delta_{11}(\eta_t))X_1(t) \\ \quad + (A_{12}(\eta_t) + \Delta_{12}(\eta_t))X_2(t), \\ \dot{X}_2(t) = (A_{21}(\eta_t) + \Delta_{21}(\eta_t))X_1(t) \\ \quad + (A_{22}(\eta_t) + \Delta_{22}(\eta_t))X_2(t) + B_2(\eta_t)U(t), \\ Y(t) = X_1(t), \\ \eta_0 = s_0, \quad t \geq 0, \end{cases} \tag{1}$$

where $X(t) = [X_1^T(t), X_2^T(t)]^T$ is the system state, $X_1 \in R^{(n-m)}$, $X_2 \in R^m$; $U(t) \in R^m$ is the control input; $Y(t) \in R^{(n-m)}$ is the system output; $A_{ij}(\eta_t), i,j = 1,2$, $B_2(\eta_t)$ are stochastic coefficient matrices with compatible dimensions, $\Delta_{ij}(\eta_t), i,j = 1,2$ are stochastic uncertain matrices and $\{\eta_t, t \in [0,T]\}$ is a finite-state Markovian process having a state-space $S = \{1,2,\ldots,\upsilon\}$, $\det(B_2(j)) \neq 0$, $j \in S$, generator (q_{ij}) with transition probability from mode i at time t to mode j at time $t+\delta$, $i,j \in S$,

$$P_{ij}(\delta) = P(\eta_{t+\delta} = j | \eta_t = i) = \begin{cases} q_{ij}\delta + o(\delta), & \text{if } i \neq j, \\ 1 + q_{ii}\delta + o(\delta), & \text{if } i = j, \end{cases} \tag{2}$$

where

$$q_{ii} = -\sum_{m=1, m\neq i}^{\upsilon} q_{im}, \; q_{ij} \geq 0, \quad \forall i,j \in S, \; i \neq j, \tag{3}$$

$\delta > 0$ and $\lim_{\delta\to 0} o(\delta)/\delta = 0$.

Assumption 1. The uncertain matrices satisfy

$$\|\Delta_{ij}(k)\| \leq \delta_{ij}(k), \quad i,j = 1,\ldots,2, \; k = 1,\ldots,\upsilon, \tag{4}$$

where $\delta_{ij}(k)$ are known constants.

Definition 1 (*Zhu et al., 2014*)**.** The system (1) is called mean-square stable, if for each $\varepsilon > 0$, there exists $\delta > 0$, such that

$$\sup_{t_0 \leq t < \infty} \mathbf{E}\|X(t)\|^2 < \varepsilon, \quad \text{for all } \|X(t_0)\| < \delta. \tag{5}$$

In addition, the system (1) is called asymptotically mean-square stable, if it is mean-square stable and

$$\lim_{t\to\infty} \mathbf{E}\|X(t)\|^2 = 0, \quad \text{for all } \|X(t_0)\| < \delta. \tag{6}$$

Furthermore, if Eq. (6) holds for arbitrary positive constant δ, then the system (1) is called globally asymptotically mean-square stable.

The sliding surfaces are defined as

$$S(X, \eta_t) = C_1(\eta_t)X_1 + X_2 = \mathbf{0}. \tag{7}$$

Denotes $\mathscr{L}$ as the weak infinitesimal operator of the process $\{X_1(t), \eta_t, t \geq 0\}$ at the point $\{t, X_1, j\}$. Substituting (7) and $\eta_t = j$ into system (1) yields

$$\begin{cases} \dot{X}_1 = \bar{A}_{11}(j)X_1 + \bar{A}_{12}(j)S, \\ \mathscr{L}S = \bar{A}_{21}(j)X_1 + \bar{A}_{22}(j)S + B_2(j)U(t), \\ \eta_0 = s_0, \quad t \geq 0, \end{cases} \tag{8}$$

where $\eta_t = j, j = 1,\ldots,\upsilon$, and

$$\begin{aligned} \bar{A}_{11}(j) &= A_{11}(j) + \Delta_{11}(j) - (A_{12}(j) + \Delta_{12}(j))C_1(j), \\ \bar{A}_{12}(j) &= A_{12}(j) + \Delta_{12}(j), \\ \bar{A}_{21}(j) &= C_1(j)(A_{11}(j) + \Delta_{11}(j)) \\ &\quad - C_1(j)(A_{12}(j) + \Delta_{12}(j))C_1(j) + (A_{21}(j) + \Delta_{21}(j)) \\ &\quad - (A_{22}(j) + \Delta_{22}(j))C_1(j) + \sum_{i=1}^{\upsilon} \alpha_{ji}C_1(i), \\ \bar{A}_{22}(j) &= C_1(j)(A_{12}(j) + \Delta_{12}(j)) + (A_{22}(j) + \Delta_{22}(j)). \end{aligned} \tag{9}$$

An important step for analyzing the probability problems of MJSs under SMC is to derive multi-step state transition probability. Some events are defined as follows.

$B:$ The initial condition is $\eta_0 = s_0$. (10)
$A_j: \; \eta_t = j, 0 \leq t \leq t_r$. (11)
$A^0(t):$ The stochastic process parameter does not jump,
$\quad \eta_\tau = s_0, \; 0 \leq \tau \leq t$, (12)
$A_j^k(t):$ The stochastic process parameter jumps k times