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Brief paper

Minimum cost input/output design for large-scale linear structural systems☆

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ABSTRACT

In this paper, we provide optimal solutions to two different (but related) input/output design problems involving large-scale linear dynamical systems, where the cost associated to each directly actuated/measured state variable can take different values, but is independent of the input/output performing the task. Under these conditions, we first aim to determine and characterize the input/output placement that incurs in the minimum cost while ensuring that the resulting placement achieves structural controllability/observability. Further, we address a constrained variant of the above problem, in which we seek to determine the minimum cost placement configuration, among all possible input/output placement configurations that ensures structural controllability/observability, with the lowest number of directly actuated/measured state variables. We develop new graph-theoretical characterizations of cost-constrained input selections for structural controllability and properties that enable us to address both problems by reduction to a weighted maximum matching problem — efficiently addressed by algorithms with polynomial time complexity (in the number of state variables). Finally, we illustrate the obtained results with an example.

1. Introduction

The problem of control systems design, meeting certain desired specifications, is of fundamental importance. Possible specifications include (but are not restricted to) controllability and observability. These specifications ensure the capability of a dynamical system (such as chemical process plants, refineries, power plants, and airplanes, to name a few) to drive its state towards a specified goal or infer its present state. To achieve these specifications, the selection of where to place the actuators and sensors assumes a critical importance. More often than not, we need to consider the cost per actuator/sensor, that depends on its specific functionality and/or its installation and maintenance cost. The resulting placement cost optimization problem (apparently combinatorial) can be quite non-trivial, and currently applied state-of-the-art methods typically consider relaxations of the optimization problem, brute force approaches or heuristics, see for instance Begg and Liu (2000), Chmielewski, Peng, and Manthanwar (2002), Fahroo and Demetriou (2000), Frecker (2003) and Padula and Kincaid (1999).

An additional problem is the fact that the precise numerical values of the system model parameters are generally not available for many large-scale systems of interest. A natural direction is to consider structural systems (Dion, Commault, & der Woude, 2003) based reformulations, which we pursue in this work. Representative work in structural systems theory may be found in Lin (1974), Liu, Slotine, and Barabási (2011), Murota (2009), Reinschke (1988), Ruths and Ruths (2014), and Siljak (2007) in the context of (structural) controllability and observability studies in complex networks. The main idea is to reformulate and study an equivalence class of systems for which system-theoretic properties are investigated on the basis of the location of zeros and (possibly) nonzeros of the state space representation matrices. Properties such as controllability and observability are, in this framework, referred to as *structural controllability* and

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structural observability, respectively. In addition, controllability and observability properties hold for almost all possible of real matrices satisfying the mentioned pattern (Dion et al., 2003).

In this context, consider a given (possibly large-scale) system with autonomous dynamics

$$\dot{x} = Ax, \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the state and A is the $n \times n$ dynamics matrix. Suppose that the sparsity pattern, i.e., location of zeros and (possibly) nonzeros, of A is available, but the specific numerical values of the remaining elements are not known. Subsequently, let $\bar{A} \in \{0, 1\}^{n\times n}$ be the binary matrix that represents the structural pattern of A, i.e., it encodes the sparsity pattern of A by assigning 0 to each zero entry of A and 1 otherwise.

Hereafter, we introduce two different (but related) input/output design problems involving large-scale linear dynamical systems, where the cost associated to each directly actuated/measured state variable can take different values, but is independent of the input/output performing the task. These costs can capture the specific functionality required from an actuator and/or its installation and maintenance cost, regarding the actuation of a specific state variables. Under these conditions, we first aim to determine and characterize the input/output placement that incurs in the minimum cost while ensuring that the resulting placement achieves structural controllability/observability as presented in $\mathcal{P}_2$. Further, we address a constrained variant of the above problem, in which we seek to determine the minimum cost placement configuration, among all possible input/output placement configurations that ensures structural controllability/observability, with the lowest number of directly actuated/measured state variables (Pequito, Kar, & Aguiar, 2013a) as stated in $\mathcal{P}_1$.

Problems statement

Given the structure of the dynamics matrix $\bar{A} \in \{0, 1\}^{n\times n}$ and a vector c of size n, where the entry $c_i \geq 0$ denotes the cost of directly actuating the state variable i, determine the sparsity of the input matrix $\bar{B}$ that solves the following optimization problems

$$\begin{aligned}\mathcal{P}_1 : \quad &\min_{\bar{B}\in\{0,1\}^{n\times n}} \quad \|\bar{B}\|_c \\ &\text{s.t.} \quad (\bar{A}, \bar{B}) \text{ structurally controllable} \\ &\qquad\quad \|\bar{B}\|_0 \leq \|\bar{B}'\|_0, \text{ for all} \\ &\qquad\quad (\bar{A}, \bar{B}') \text{ structurally controllable,}\end{aligned} \tag{2}$$

and

$$\begin{aligned}\mathcal{P}_2 : \quad &\min_{\bar{B}\in\{0,1\}^{n\times n}} \quad \|\bar{B}\|_c \\ &\text{s.t.} \quad (\bar{A}, \bar{B}) \text{ structurally controllable}\end{aligned} \tag{3}$$

where $\|\bar{B}\|_c = c^T\bar{B}\mathbf{1}$, $\|\bar{B}\|_0$ denotes the zero (quasi) norm corresponding to the number of nonzero entries in $\bar{B}$, and $\mathbf{1}$ the vector of ones with size n. Notice that a solution to $\mathcal{P}_1$ or $\mathcal{P}_2$ may consist of columns with all zero entries, that can be disregarded when considering the deployment of the inputs required to actuate the system. Notice that in the worst case scenario, taking the identity matrix as the input matrix we obtain structural controllability, which justifies the dimensions chosen for the solution search space.

Notice that in problems $\mathcal{P}_1$ and $\mathcal{P}_2$, some solutions may comprise one nonzero entry in a column; in other words, solutions in which an input actuates one state variable, which we refer to as *dedicated inputs*. Additionally, if a solution $\bar{B}^*$ is such that all its nonzero columns consist of exactly one nonzero entry, then it is referred to as a *dedicated solution*, otherwise it is referred to as a *non-dedicated solution*. For instance, in the context of leader-selection problems, it corresponds to determining which agents should receive input signals from an external source. If the signals are crafted for a specific agent, then the input is dedicated, as it is common in peer-to-peer communication schemes. Alternatively, if the signal is broadcasted to a collection of (at least two) agents, the input is not dedicated, since a collection of individuals receive the same signal. In addition, observe that in $\mathcal{P}_1$ there is a restriction of obtaining a solution with the minimum number of state variables that need to be directly actuated in order to achieve structural controllability. Without such restriction, i.e., by possibly actuating more state variables, we may obtain a lower cost placement achieving structural controllability, hence, the interest in studying $\mathcal{P}_2$. Nonetheless, the constrained scenario in $\mathcal{P}_1$ may be desirable, for instance, in multi-agent networks in an environment where communication (of the input signal) is very expensive in comparison with actuation cost of a specific agent, or a collection of state variables for dynamical systems at large.

Finally, note that the solution procedures for $\mathcal{P}_1$ and $\mathcal{P}_2$ also address the corresponding structural observability output matrix design problem by invoking the duality between observability and controllability in linear time-invariant (LTI) systems (Hespanha, 2009).

Recently, the I/O selection problem have received increasing attention in the literature: the *minimal controllability problem*, i.e., the problem of determining the sparsest input matrix that ensures controllability of a given LTI system (Olshevsky, 2014; Ramos, Pequito, Kar, Aguiar, & Ramos, 2014), and in Clark, Alomair, Bushnell, and Poovendran (2014), Clark and Poovendran (2011), Lin, Fardad, and Jovanović (2014), Pasqualetti, Zampieri, and Bullo (2014), Summers, Cortesi, and Lygeros (2015) and Tzoumas, Rahimian, Pappas, and Jadbabaie (2015) the configuration of actuators is sought to ensure certain performance criteria, for instance, by optimizing properties of the controllability Grammian.

Alternatively, I/O selection problem for structural linear systems has also been addressed in Commault and Dion (2013), Dion et al. (2003), Liu et al. (2011), Pequito et al. (2013a); Pequito, Kar, and Aguiar (2013b,c, 2016, 2015), Ruths and Ruths (2014) and references therein, just to name a few. In particular, in Pequito et al. (2016), the structural version of the minimal controllability problem, or the *minimal structural controllability problem*, was shown to be polynomially solvable; an improvement on the computational complexity was analyzed in detail for several subsystems in Assadi, Khanna, Li, and Preciado (2015). Notice that this is a particular instance of $\mathcal{P}_1$ and $\mathcal{P}_2$ when the costs are uniform, i.e., each variable incurs in the same (non-zero) cost.

The solution proposed in Pequito et al. (2016) provides useful insights, but is not sufficient to address the problems $\mathcal{P}_1$ and $\mathcal{P}_2$ with non-uniform cost. Nonetheless, the characterizations obtained in Pequito et al. (2016) were used to obtain some preliminary results on problems $\mathcal{P}_1$ and $\mathcal{P}_2$ in Pequito et al. (2013a,c), respectively. These preliminary results are based on analyzing the intrinsic properties of the class of all minimal subsets of state variables that need to be actuated by dedicated inputs to ensure structural controllability; in particular, the proposed solution provided algorithmic solutions with computational time complexity $\mathcal{O}(n^{3.5})$, as a result of evaluating n maximum matchings using the Hungarian algorithm (Cormen, Stein, Rivest, & Leiserson, 2001). In addition, in Olshevsky (2015) the problem $\mathcal{P}_1$ is addressed for a specific binary actuation cost structure $c \in \{0, \infty\}^n$, and a solution with computational time complexity $\mathcal{O}(n+m\sqrt{n})$ is proposed, where m denotes the total number of nonzero entries, and $\mathcal{O}(n^{2.5})$ in general. Similarly, although (Olshevsky, 2015) provides useful insights to address $\mathcal{P}_1$, it is not sufficient to address the problems $\mathcal{P}_1$ with non-uniform cost, as well as $\mathcal{P}_2$.

The main contributions of this paper are as follows: by presenting new graph-theoretical characterizations of cost-constrained input selections for structural controllability and results on the