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Chattering-free discrete-time sliding mode control[☆]

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ABSTRACT

To avoid the chattering problem in the reaching-law-based discrete-time sliding mode control (DSMC) and the generation of over-large control action in the equivalent-control-based DSMC, a new DSMC method based on non-smooth control is proposed in this paper. Since there is no use of any switching term in the proposed DSMC, it is a chattering-free SMC method. Meanwhile, it is shown that the newly proposed non-smooth control-based DSMC can guarantee the same level of accuracy for the sliding mode motion as that of an equivalent control-based DSMC. To demonstrate the effectiveness of the proposed approach, a simulation example is presented.

1. Introduction

As a popular nonlinear control method, sliding mode control (SMC) has been widely studied in the theoretical research community (Utkin, 1992) and successfully applied in industry (Perruquetti & Barbot, 2002). The main reason is due to its many advantages, such as simple design idea and good robustness (Yu & Xu, 2002). Since more and more modern control systems are implemented by computers, the study of SMC in the discrete-time domain, i.e., discrete-time SMC, has been an important topic in the SMC literature (Yu, Wang, & Li, 2012).

The main difference between discrete-time SMC (DSMC) and continuous-time SMC (CSMC) lies in that the switching frequency of DSMC is limited, which leads to the celebrated invariance property of CSMC systems no longer holds (Drazenovic, 1969). In this case, the study of DSMC has been paid attention by many researchers and can be divided into two directions. One direction is to follow the design idea of CSMC and the switching term is still preserved, e.g., the design of DSMC law directly based on discrete-time systems (Gao, Wang, & Homaifa, 1995; Qu, Xia, & Zhang, 2014) (usually called reaching-law-based DSMC), the study of discretization effect on continuous-time SMC systems (Galias & Yu, 2007; Li, Du, & Yu, 2014; Wang, Yu, & Chen, 2009; Xia & Zinober, 2006; Yu, Wang, Galias, & Chen, 2008). The other direction is based on the equivalent control for discrete-time system, which is called equivalent-control-based DSMC (Su, Drakunov, & Ozguner, 2000; Utkin, 1994).

In the reaching-law-based DSMC, since the switching term is still employed, the chattering problem will be inevitable. In the equivalent control-based DSMC, although there is no switching term, it will generate an over-large control effort since there is no reaching process. Actually, in practice, due to the existence of disturbances, no matter which kinds of DSMC methods are employed, the sliding mode state cannot be precisely kept at zero. In such a case, the central issue is how to guarantee a smaller boundary layer for the sliding mode motion. In this regard, some improved DSMC methods have been proposed, such as disturbance observer-based DSMC (Su et al., 2000), discrete-time integral SMC (Abidi, Xu, & Yu, 2007), etc. However, these improved DSMC methods belong to the equivalent-control-based DSMC.

In this paper, we provide a new DSMC design method which is based on non-smooth control. The advantage of non-smooth control lies in its good performances such as better robustness (Bhat & Bernstein, 2000). In this new DSMC, to avoid the chatting phenomenon and the generation of overly large control action, a non-smooth term (continuous function) is employed instead of the switching term and a reaching process is added. Under the proposed DSMC, a rigorous theoretic analysis

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shows that the same accuracy for the sliding mode motion can be obtained as that of the equivalent-control-based DSMC, who provides a higher precision than that of the traditional reaching-law-based DSMC. Finally, an example is provided to demonstrate the potential of the proposed method.

2. System description and some existing results

2.1. System description

As in Wang et al. (2009) and Su et al. (2000), consider the following single-input continuous-time system with matched disturbances:

$$\dot{x} = Ax + B_u u + B_d f, \tag{1}$$

where $x \in R^n, u \in R$, and $f \in R$ are the state, input, and disturbance, and A, B_u, B_d are constant matrices of appropriate dimensions. The matching condition implies that $\operatorname{rank}[B_u, B_d] = \operatorname{rank}[B_u]$. The disturbance f is assumed to be smooth and bounded.

Assume the control law u is digitally implemented through a zero-order-holder (ZOH), i.e., $u(t) = u(k)$ over the time interval $[kh, (k+1)h)$, where h is the sampling period. Then, the continuous-time system (1) under the discrete-time control law $u(k)$ can be written in a discrete form as follows:

$$x(k+1) = \Phi x(k) + \Gamma u(k) + d(k), \tag{2}$$

where $\Phi = e^{Ah}, \Gamma = \int_0^h e^{A\tau} d\tau B_u, d(k) = \int_0^h e^{A\tau} B_d f((k+1)h - \tau) d\tau$. By Abidi et al. (2007), the system matrices of discrete-time system (2) have the following properties.

Property 2.1. (1) $\Phi = I + Ah + \frac{A^2h^2}{2!} + \cdots = I + O(h)$;
(2) $\Gamma = B_u h + \frac{AB_u h^2}{2!} + \cdots = O(h)$;
(3) $d(k) = O(h)$;
(4) *The change rate of the disturbance* $\delta(k) = d(k) - d(k-1) = O(h^2)$.

2.2. Problems of the existing discrete-time SMC

To design a discrete-time SMC law, as that in Su et al. (2000), a discrete-time switching function is usually chosen as:

$$s(k) = Cx(k), \tag{3}$$

where $C \in R^{1\times n}$ is a constant matrix and is chosen such that $C\Gamma$ is invertible and the matrix $(\Phi - \Gamma(C\Gamma)^{-1}C\Phi)$ has 1 zero pole and $(n-1)$ poles inside the unit disk in the complex *z-plane*. Based on matrix C and Property 2.1(4), the following assumption is imposed.

Assumption 2.1. There is a constant δ^* such that $|C\delta(k)| \le \delta^*$, $k = 0, 1, 2, \ldots,$ and the constant δ^* is in the order of $O(h^2)$.

Remark 2.1. This Assumption is satisfied if the change rate of the disturbance f (i.e., $\dot{f}(t)$) for the system (1) is bounded, whose proof can be found in Lemma 1 in Abidi et al. (2007). In practice, many kinds of disturbances satisfy this property, such as the load disturbances for permanent magnet synchronous motor (Liu & Li, 2012), etc.

2.2.1. Discrete-time SMC based on reaching process

Similar to the continuous-time SMC, a method called reaching-law-based DSMC approach is firstly introduced in Gao et al. (1995). The reaching law for sliding mode is designed as:

$$s(k+1) = s(k) - qh \cdot s(k) - \varepsilon h \cdot \operatorname{sgn}(s(k)),$$
$$\text{with } \varepsilon > 0, q > 0, 0 < 1 - qh < 1. \tag{4}$$

Under this reaching law, the sliding mode state will enter the vicinity of sliding mode surface $s(k) = 0$ with thickness Δ and stay there forever. That is, $|s(k)| \le \Delta$ for all $k > k^*$ with a constant k^*. Clearly, the size of Δ is important, which determines the accuracy of sliding mode motion and eventually the steady-state of system state. Solving Eq. (4) based on (2) and (3) leads to

$$u(k) = -(C\Gamma)^{-1}[C\Phi x(k) - (1-qh)s(k) + \varepsilon h \cdot \operatorname{sgn}(s(k)) + Cd(k)]. \tag{5}$$

Due to the unavailable information $d(k)$, as that in Su et al. (2000), the disturbance can be estimated by using the so called delay estimate method, i.e.,

$$\hat{d}(k) = x(k) - \Phi x(k-1) - \Gamma u(k-1) = d(k-1), \tag{6}$$

which yields the final realizable controller

$$u(k) = -(C\Gamma)^{-1}[C\Phi x(k) - (1-qh)s(k) + \varepsilon h \cdot \operatorname{sgn}(s(k)) + C\hat{d}(k)]. \tag{7}$$

Under this controller, it can be concluded that

$$s(k+1) = (1-qh)s(k) - \varepsilon h \cdot \operatorname{sgn}(s(k)) + C\delta(k). \tag{8}$$

By Lemma A.1 and Assumption 2.1, it can be concluded that the sliding mode state $s(k)$ will enter the following region in a finite number of steps and stay there forever

$$\{s(k) : |s(k)| \le \varepsilon h + \delta^* = O(h)\}. \tag{9}$$

Remark 2.2. The main problem of this method is to use the switching function $\operatorname{sgn}(s(k))$, which leads to the well-known chattering problem in practice. And from the control law (7), it can be found that the switching gain, i.e., the amplitude of chattering, is $(C\Gamma)^{-1}\varepsilon h$. Since $C\Gamma = O(h)$, then $(C\Gamma)^{-1}\varepsilon h$ is in the order of $O(1)$.

2.2.2. Discrete-time SMC based on equivalent control

To avoid the chattering problem, another method called equivalent control is proposed in Su et al. (2000) to design a chatter-free DSMC law. Directly solving $s(k+1) = 0$, i.e., $Cx(k+1) = 0$ leads to

$$u(k) = -(C\Gamma)^{-1}[C\Phi x(k) + Cd(k)]. \tag{10}$$

As that in (7), the disturbance $d(k)$ can be substituted by the estimate value $\hat{d}(k)$ defined in (6), which yields the final realizable controller

$$u(k) = -(C\Gamma)^{-1}[C\Phi x(k) + C\hat{d}(k)]. \tag{11}$$

Under this discrete-time controller, the dynamical behavior of sliding mode state is given by

$$s(k+1) = Cx(k+1) = C\Phi x(k) + C\Gamma u(k) + Cd(k) = C[d(k) - \hat{d}(k)] = C\delta(k). \tag{12}$$

By Assumption 2.1, $s(k)$ is bounded by

$$|s(k)| \le \delta^* = O(h^2). \tag{13}$$

Remark 2.3. Compared with the reaching-law-based DSMC, the sliding mode state $s(k)$ under the equivalent-control-based DSMC has a higher accuracy. However, the magnitude of equivalent-control-based DSMC law $u(k)$ in (11) is in the order of $O(h^{-1})$ because $(C\Gamma)^{-1} = O(h^{-1}), C\Phi = O(1)$, and $C\hat{d}(k) = Cd(k-1) = O(h)$ from Property 2.1(3). Clearly, if h is sufficiently small, the equivalent-control-based DSMC needs an over-large control input which may be undesirable due to the control saturation constraint.