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Second-order consensus of multi-agent systems under limited interaction ranges☆

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ABSTRACT

This paper studies a distributed linear consensus protocol for second-order multi-agent systems under limited agent interaction ranges. In particular, two agents can interact with each other only if their distance is within a certain range. Under a *linear* consensus protocol with the relative state feedback, we derive a sufficient condition on the initial states and the interaction range to achieve the second-order consensus by using a Lyapunov functional approach. Finally, simulation examples are included to validate the theoretical result.

1. Introduction

Multi-agent systems have become a hot research topic over last decades due to its wide application in natural and artificial systems, such as bird flocks, bacteria colonies, power systems, sensor networks, robotic teams and social groups (Chen, Lu, Yu, & Hill, 2013; Olfati-Saber, Fax, & Murray, 2007). For a spatially distributed mobile system, second-order consensus means that all the agents converge to the same position and move with the same velocity (Liu & Jiang, 2014; Su, Chen, Wang, & Lin, 2011; Yu, Chen, & Cao, 2010).

Under a proper protocol, consensus can be achieved on condition that the switching interaction graphs are connected frequently enough (Olfati-Saber & Murray, 2004; You, Li, & Xie, 2013). However, how to control multi-agent systems to maintain a desired interaction graph is a challenging problem. Cucker and Smale (2007) proposed a second-order system for flocking, where the agent interaction is modeled as a decreasing function of the distance between agents, and provided sufficient conditions to guarantee an asymptotic flocking. Several works extended Cucker–Smale model, such as Carrillo, Fornasier, Rosado, and Toscani (2010); Cucker and Dong (2011) and Ha, Ha, and Kim (2010). Since the agent interaction in these works continuously decreases to zero as the distance tends to infinity, each agent can still interact with any other agent at any finite distance. Strictly speaking, the interaction graph is always fully connected. To solve it, we consider that two agents interact with each other only if their distance is within a certain level, which is motivated by real applications and covers Cucker–Smale model as a special case. Take the wireless communication of unmanned aerial vehicles (UAVs) as an example. If two UAVs fly too far apart, the communication between them will be blocked.

Related works concerning limited interaction ranges also include Ji and Egerstedt (2007); Su, Wang, and Chen (2010); Zavlanos, Jadbabaie, and Pappas (2007) and references therein. Their key idea is to preserve the connected interaction links by defining a potential function, which results in that the agent input tends to infinity if the agent approaches to its neighbor's interaction range. This usually requires a nonlinear control protocol, and is hard to implement in practice because of control constraint and actuator saturation (Meng, Zhao, & Lin, 2013). Differently, we consider a simple *linear* consensus protocol which is easy to implement by using the relative state feedback, and the control signal will not grow to infinity at any time.

In comparison, the novelty and contribution of this work lies in the use of a distributed *linear* consensus protocol under a limited agent interaction range, and we need to overcome two additional challenges. Firstly, the discontinuities of the agent interaction has to be addressed by the non-smooth analysis tool, which is

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fundamentally different from Cucker and Smale (2007). Secondly, the linear consensus protocol cannot maintain connectivity of the connected link. In fact, some connected links may be lost and rebuilt, which will not occur under the *nonlinear* protocol using the connectivity preserving technique. A preliminary version of this paper was presented in Ai, You, and Song (2014).

The rest of this paper is organized as follows. In Section 2, we propose a novel agent interaction model for the multi-agent systems. In Section 3, we present the main result on the sufficient consensus condition. In Section 4, simulation examples are given to validate the theoretical result. Finally, conclusion is drawn in Section 5.

2. Problem formulation

Consider a multi-agent system consisting of N identical agents with the second-order dynamics

$$\dot{p}_i(t) = q_i(t), \qquad \dot{q}_i(t) = u_i(t), \qquad t \in [0, +\infty), \tag{1}$$

where $p_i(t), q_i(t) \in \mathbb{R}^n$ denote the ith agent's position and velocity at time t respectively, and $u_i(t) \in \mathbb{R}^n$ is the control input. The objective is to achieve second-order consensus (Yu et al., 2010), which is defined as that

$$\lim_{t\to\infty} \left\| p_i(t) - p_j(t) \right\| = \lim_{t\to\infty} \left\| q_i(t) - q_j(t) \right\| = 0.$$

2.1. Linear consensus protocol under a limited interaction range

Motivated by primitive societies, e.g. birds or fish, we consider a *linear* consensus protocol

$$u_i = \sum_{j=1}^{N} \phi\left(d_{ij}\right) k \left[\left(q_j - q_i\right) + \gamma \left(p_j - p_i\right)\right], \tag{2}$$

where the distance between agent i and j is $d_{ij} = \left\|p_j - p_i\right\|$. The gain $k > 0$ is a fixed coupling coefficient, and $\gamma > 0$ denotes the scaling factor (Ren & Atkins, 2007). As in the real application, we study a new interaction function $\phi : [0, +\infty) \to [0, +\infty)$, and is given by

$$\begin{cases} \phi(d) > 0, & d \in [0, r), \\ \phi(d) = 0, & d \in [r, +\infty), \end{cases} \tag{3}$$

where the positive real number r denotes the interaction range of each agent. Moreover, we assume that ϕ is continuously differentiable and bounded in $(0, r)$.

Remark 1. It should be noticed that ϕ in (3) can be discontinuous and allows a large jump at r. An example is given in Vicsek's model (Vicsek, Czirók, Ben-Jacob, Cohen, & Shochet, 1995), i.e., $\phi(d) = 1$ if $d \in [0, r)$, and 0 otherwise. Another example is the truncated interaction function in the Cucker–Smale model, and is given as

$$\phi(d) = \begin{cases} \dfrac{K}{\left(\sigma^2 + d^2\right)^{\beta}}, & d \in [0, r), \\ 0, & d \in [r, +\infty), \end{cases}$$

where K, σ and β are positive. In Cucker and Smale (2007), there is no truncation, and $\phi(d) > 0$ for any d. This implies that u_i needs information from all the other agents, and the underly interaction graph is fully connected.

Remark 2. In Ji and Egerstedt (2007); Su et al. (2010); Zavlanos et al. (2007), nonlinear consensus protocols were given to preserve the connectivity of agents where the control signal may be very large if the distance between agents is close to r. This is in contrast with the linear protocol in (2).

Define the averaged position and velocity by

$$\bar{p}(t) \triangleq \frac{1}{N}\sum_{i=1}^{N} p_i(t), \qquad \bar{q}(t) \triangleq \frac{1}{N}\sum_{i=1}^{N} q_i(t).$$

By (2), it is clear that $\sum_{i=1}^{N} u_i = 0$, and $\dot{\bar{q}}(t) = \mathbf{0}$. Thus, the averaged velocity $\bar{q}(t)$ is invariant. Let $x_i = p_i - \bar{p}$ and $v_i = q_i - \bar{q}$, we obtain that

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= \sum_{j=1}^{N} \phi\left(\left\|x_j - x_i\right\|\right) k \left[\left(v_j - v_i\right) + \gamma \left(x_j - x_i\right)\right]. \end{aligned} \tag{4}$$

Clearly, the second-order consensus is equivalent to that

$$\lim_{t\to\infty} x_i(t) = \mathbf{0}, \qquad \lim_{t\to\infty} v_i(t) = \mathbf{0}, \quad \forall i = 1, \ldots, N.$$

2.2. Filippov solutions

As we allow ϕ to have a jump at the range r, it may introduce discontinuities to the system in (4). Hence, we study its solution in the Filippov sense (Filippov, 1988), which is an absolutely continuous function $(x(t), v(t))$ satisfying differential inclusions

$$\dot{x} = v, \quad \dot{v} \in \mathbb{K}[u] \tag{5}$$

where x, v, u denote the stack vectors of $x_i, v_i, u_i (i = 1, \ldots, N)$. $\mathbb{K}[u] = \bigcap_{\delta>0}\bigcap_{\mu(D)=0} \bar{co}\,[u\,(B\,(x, v; \delta) - \mathrm{D})]$, where $\bar{co}\,[\Lambda]$ is the convex closure of Λ, $B\,(x, v; \delta)$ denotes an open ball of radius δ at (x, v) and $\mu(D)$ is Lebesgue measure of set D.

Lemma 3. *There exists a unique Filippov solution to* (4).

Proof. Let $y = (x^T, v^T)^T$, and the subspace

$$s_{ij} = \left\{(x^T, v^T)^T \left| d_{ij} = r \right.\right\}.$$

For an arbitrary point $y^0 \in s_{ij}$, its neighborhood $B\left(y^0; \delta\right)$ is separated by the surface s_{ij} into two regions, which are denoted by B^+ and B^-. Let $\dot{y}^+, \dot{y}^-$ be the limiting values of $\dot{y}$ at y^0 from the regions B^+ and B^-, respectively. Since ϕ is continuously differentiable and bounded in $(0, r)$, we know that both $\dot{y}^+$ and $\dot{y}^-$ exist. Since these two vectors differ only in terms of $\dot{v}_i^+, \dot{v}_i^-$ and $\dot{v}_j^+, \dot{v}_j^-$, it implies that the discontinuity vector $h = \dot{y}^+ - \dot{y}^-$ is orthogonal to the normal vector $\vec{n}$ of surface s_{ij} at y^0, and its projection onto $\vec{n}$ is a zero vector. If $y^0 \in s_{ij} \cap s_{kl} \cap \ldots$, similar results hold. By Lemma 3 in Filippov (1988, p. 108), the existence and uniqueness of Filippov solution of (4) are guaranteed.

2.3. Interaction graph

We use $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ to describe the interaction graph of the multi-agent system, where $\mathcal{V}$ denotes the set of agents, and $\mathcal{E}$ denotes the interaction links between agents. Let the link weight be $a_{ij} = \phi(d_{ij})$. The adjacency matrix $A = \left[a_{ij}\right]$ is symmetric, and the Laplacian matrix is defined as $L = \Delta - A$, where Δ is a diagonal matrix with $\Delta_{ii} = \sum_{j=1}^{N} a_{ij}$. An undirected graph is called *connected* if and only if any two distinct nodes i and j in the graph can be connected via a path, which is a sequence of distinct edges, such as $(i, k_1) \in \mathcal{E}, \ldots, (k_m, j) \in \mathcal{E}$. The connectivity of an undirected graph is related to the rank of its Laplacian.

Lemma 4 (*Biggs, 1993*)**.** *$\mathcal{G}$ is connected if and only if rank$(L) = N - 1$.*