Automatica 61 (2015) 134–145

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

# Robust tracking of piecewise linear trajectories with binary sensor networks[☆]

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ARTICLE INFO

*Article history:*
Received 1 July 2014
Received in revised form
17 March 2015
Accepted 29 June 2015
Available online 27 August 2015

*Keywords:*
Tracking
Localization
Binary sensor network

ABSTRACT

We present a novel approach to the problem of tracking objects moving along smooth trajectories using a network of simple inexpensive binary sensors. Specifically, we consider object trajectories that can be well-approximated by piecewise linear curves and sensors that can only detect whether an object is in their sensing range. We start by considering objects moving along straight-line trajectories with an unknown speed and show that such objects can be tracked using the measurements of just three generically placed binary sensors whose sensing ranges intersect the trajectory. Next, we present an asymptotic analysis that shows that a trajectory consisting of a finite number of straight line segments can be recovered with high probability using an arbitrarily low spatial density of sensors in the limit when the area to be covered gets larger and larger. We also present efficient algorithms that effectively recover piecewise linear trajectories. Finally we present analysis and simulations to demonstrate the high tracking accuracy of our approach and its robustness to sensing errors.



## 1. Introduction

Localization and tracking of the path of an object are basic problems in many applications, e.g., surveillance, border monitoring, rescue, security, agriculture and many others. Fundamentally, these problems involve estimation of the position and trajectory of objects of interest using the observations of a network of sensors (Shames, Dasgupta, Fidan, & Anderson, 2012; Xu, Ding, & Dasgupta, 2013).

The reliability of a sensor's observations is a function of the strength of the sensor's interaction with the object they track (Mudumbai & Madhow, 2008), which in turn generally depends on how close the sensors are to their target. This proximity requirement is especially stringent for sensing applications where signal absorption by the medium is large. For example, the strength of gamma rays emanated by a radioactive source, in the presence of structures that act as shielding material (Kump, Bai, Chan, Eichinger, & Li, 2012), exponentially declines with distance. This suggests that in order to continuously track a moving object, it is necessary to deploy a large number of sensors throughout an area of interest in order to avoid "coverage gaps"; high tracking accuracy under this setting would require dense deployment of sensors with correspondingly high cost and complexity.

This paper considers the possibility of significantly reducing the cost and complexity of the tracking problem by exploiting the continuity and smoothness of the trajectories of real physical objects. Specifically, we consider tracking using a network of simple binary sensors. The density of sensor deployment, defined as the total number of sensors per unit area, only needs to be large enough to ensure that the speed and bearing of the object remains roughly constant over multiple sensor observations. This is much easier to satisfy than requiring that the object remain within range of at least one sensor at all times.

We also allow individual sensors to be simple, noisy and vulnerable to drift and calibration errors, and rely on statistical robustness by combining the observations of many sensors. This

☆ This work was in part supported by National Science Foundation grants CCF-0830747, EPS-1101284, ECCS-1150801 and CNS-1239509, Department of Energy grant DE-FG52-09NA29364. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Wei Xing Zheng under the direction of Editor Torsten Söderström.

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http://dx.doi.org/10.1016/j.automatica.2015.07.012
0005-1098/

is in keeping with the basic philosophy of large-scale sensor networks (Akyildiz, Su, Sankarasubramaniam, & Cayirci, 2002) that favors distributed networks of simple, inexpensive, disposable sensors to a small number of complex, expensive ones. With noisy sensors, it is also possible to radically quantize observations (Patwari & Hero, 2003) with little sacrifice in performance.

In fact, the sensors we consider are binary sensors, that can only detect if an object is within in their sensing range. Effectively, this is a case of one-bit signal quantization. The observations of binary sensors tracking a moving object continuously, can be efficiently summarized by recording the time-stamps of the instants at which a moving object enters and leaves the sensor's detection range. This in turn allows sensors to communicate their observations to a fusion center with very little overhead (just two time-stamps for each object passing through a sensor's range). Because the communication requirements are so minimal, we assume in this paper that all sensor observations are available at a centralized fusion center and therefore we do not consider the possibility of further reducing overheads using distributed algorithms.

**Related work**. Binary sensors represent a convenient and powerful abstraction for a variety of physical sensors, and different types of binary sensors have been widely studied in the literature on localization and tracking (Aslam et al., 2003; Busnel, Querzoni, Baldoni, Bertier, & Kermarrec, 2008; Djuric, Vemula, & Bugallo, 2008; Katenka, Levina, & Michailidis, 2008; Kim, Mechitov, Choi, & Ham, 2005; La Scala, Morelande, & Savage, 2006; Lázaro, Sánchez-Fernández, & Artés-Rodríguez, 2009; Le & Kaplan, 2010; Nguyen, Nguyen, Liu, & Tran, 2007; Shrivastava, Mudumbai, Madhow, & Suri, 2009; Singh, Madhow, Kumar, Suri, & Cagley, 2007; Wang, Bulut, & Szymanski, 2010). One type of binary sensor network tries to use the geometry of the network (Aslam et al., 2003) and considers the network as a connected graph to aid detection. The work in Lázaro et al. (2009) considers heterogeneous binary sensors with different performance and cost. The purpose is to analyze the best proportion of sensors in each class to tradeoff performance with cost. Some assume probabilistic models for object movement and/or for the received signal strength (Djuric et al., 2008; Katenka et al., 2008; La Scala et al., 2006; Le & Kaplan, 2010; Nguyen et al., 2007) and develop detection algorithms based on these models. Others additionally assume that each sensor (Aslam et al., 2003) has the ability to sense if the object is approaching or moving away.

Many of the existing studies rely on the density of the sensors being large enough to essentially guarantee blanket coverage of the region of interest (Shrivastava et al., 2009; Wang et al., 2010). The idea is that if the density is high enough, every movement of the object lies in the sensing range of several sensors and the object's position and trajectory can be pinned down with a higher degree of accuracy because the intersection of sensing ranges of multiple sensors is much smaller than the sensing area of any single sensor. Clearly, a higher density implies that the sensing ranges of more sensors intersect which results in a smaller intersecting area or a more accurate estimate. In fact, as shown in Shrivastava et al. (2009) the estimation accuracy is of the order $1/\rho$, where $\rho$ is the sensor density assuming all sensors have the same sensing range.

**Contributions**. In this paper, we ask the following fundamental question: how many binary sensors are needed to track an object moving in an unknown piecewise linear trajectory *with unknown speed*? We are motivated by (a) the observation that inertial and other dynamical constraints restrict most trajectories to be smooth, and (b) smooth trajectories can be arbitrarily well approximated by piecewise linear trajectories. These assumptions are reasonable in some applications. Consider an example of monitoring radioactive material smuggling at a sea port of entry which consists of a number of piecewise linear roads from unloading cargoes to the exit of the port. Our main contributions are as follows.

- We show that three generically placed sensors can uniquely determine almost all linear trajectories that intersect with their sensing range. This naturally extends to piecewise linear trajectories that are trajectories containing a finite number of connected line segments.
- We also show that sensors deployed randomly with vanishingly small densities can uniquely determine a straight line trajectory with high probability. More generally, for smooth trajectories, the required sensor density for accurate tracking is determined only by the number of piecewise linear segments needed to sufficiently approximate the trajectory, and this density can be much smaller than what is required to achieve blanket coverage.
- The geometric problem of determining the points of intersection of a straight-line trajectory with the coverage area of a binary sensor involves solving a set of non-linear equations. However, we present a reformulation of the tracking problem that involves a constrained set of linear equations and allows a simple least-squares solution.
- Based on the above reformulation into linear equations, we present a practical, computationally efficient algorithm to solve the piecewise linear trajectory estimation problem.
- We conduct an extensive robustness analysis of the algorithm under departures from the idealizing assumptions. Specifically we show that the algorithm can tolerate uncertainty in the sensing ranges. *Further we show that it can track objects that do not travel on a precise piecewise linear trajectory*.
- Finally, we present an extensive set of simulations to illustrate the performance of our proposed approach and its robustness to sensor errors and uncertainties.

Our presentation in this paper proceeds in a step-by-step fashion. First we consider the simple and idealized setting of tracking objects moving in straight-line trajectories with known speeds and homogeneous noise-free sensors. While these assumptions are of course unrealistic, this initial analysis allows us to present key geometric intuitions concisely, and also to present a simple trajectory estimation algorithm that involves solving a set of linear equations with constraints. We then show that this algorithm can be used to estimate arbitrary trajectories that can be approximated with piecewise linear segments, and extended to remove restrictions of known target speed and ideal sensors. Practical sensors may deviate from the model of an ideal binary sensor in many different ways e.g. measurement noise, calibration error, non-circular sensing range and possibility of sensor failure i.e. misses or false positives; our analysis shows the robustness of our approach to calibration error, and we present simulations to demonstrate that we are able to tolerate other types of disturbances.

The organization of the paper is as follows. Section 2 describes the problem formulation. Section 3 considers a target moving with known speed, and describes a simple algorithm to estimate piecewise linear trajectories. Section 4 takes these basic results and extends them to the case where the target speed is unknown. Section 5 addresses robustness issues associated with uncertainties in the sensing range of the sensors. It also shows that the algorithm can track a source that does not travel on an exact piecewise linear trajectory. Section 6 quantifies the required density of sensors needed to track linear trajectories. Section 7 presents simulation results. Section 8 concludes.

## 2. Problem setup

Consider a region of interest $\mathcal{D} \subset \mathbb{R}^2$ over which we would like to track a moving object, and a binary sensor network of $N$ sensors which are deployed over the region.