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Escape time formulation of state estimation and stabilization with quantized intermittent communication ☆

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ABSTRACT

The problems of state estimation and feedback stabilization of a linear system including a communications channel are quantified as escape or survival times, which yield stochastic processes describing the time of first exit of the state estimate error or of the system output from a specific domain. The complications introduced by communications – intermittency, channel noise, quantization, etc. – are evaluated using a Markov stopping time formulation. This is compared to and contrasted with earlier analyses which considered the behavior of Kalman filters with intermittent data based on moments and conditional moments, and the evaluation of the minimal number of bits required for mean square stabilization. The main result shows the escape time is characterized by a Markov chain which is amenable to explicit analysis through the calculation of its cumulative distribution function. This is examined in the linear Gaussian and quantized linear Gaussian cases and then used to develop an approach to bitrate assignment in such communications-based control systems.

1. Introduction

We study the problems of state estimation and output feedback stabilization of a time-invariant linear system including a single communications link.

$$x_{k+1} = Ax_k + Bu_k + w_k, \quad (1)$$

$$y_k = Cx_k + v_k, \quad (2)$$

$$z_k = \gamma_k \mathcal{Q}_d(y_k). \quad (3)$$

Here, as usual, $x_k,\ u_k,\ y_k,\ w_k,\ v_k$ are the system state, input, output, process noise and measurement noise signals of dimensions $n,\ p,\ m,\ n,\ m$ respectively and $[A, B, C]$ are the system matrices of conformable dimensions. The signal z_k is the received measurement from the system and is modeled in (3) as the product of a scalar random variable, γ_k, taking values 0 or 1 and a quantized version of the system output y_k. As in Shi, Epstein, and Murray (2010) and Sinopoli et al. (2004), the intermittency of the communication channel is modeled by the $\{\gamma_k\}$ sequence. The quantization function $\mathcal{Q}_d(\cdot)$ is a subtractive dithered quantizer with finite bitrate in each channel of input y_k and will be further explicated shortly.

Definition 1 (*Escape Time*)**.** Given a closed domain $\mathcal{D} \subset \mathbb{R}^d$ and a stochastic process $\{\xi_k : k = 1, \ldots\}$ on $\mathbb{R}^d$, the escape time is defined to be

$$\tau_e = \begin{cases} \arg\min_k \xi_k \notin \mathcal{D}, & \text{or} \\ \infty, & \text{if } \xi_k \in \mathcal{D}\ \forall k. \end{cases}$$

Sometimes the escape time is called the 'first exit time', 'stopping time', 'hitting time' or 'residence time'. We shall be concerned with the escape time for the state process, x_k, state prediction error process, $\tilde{x}_{k|k-1}$, or the output process, y_k, of (1)–(2) when the control input is causally computed.

Fig. 1 shows two simulations of the Kalman state prediction error $\tilde{x}_{k|k-1}$ for a scalar system with a magnitude bound placed at 60 and differing rates, P_γ, of packet arrival. In the left graph, the expected value of the conditional error covariance – the focus of Sinopoli et al. (2004) – is finite, while in the right it is not. Note the similarities between the figures except for the time scales. Escape time would correspond to the first achievement of the bound 60. Our interest is in characterizing this escape time cumulative distribution function.

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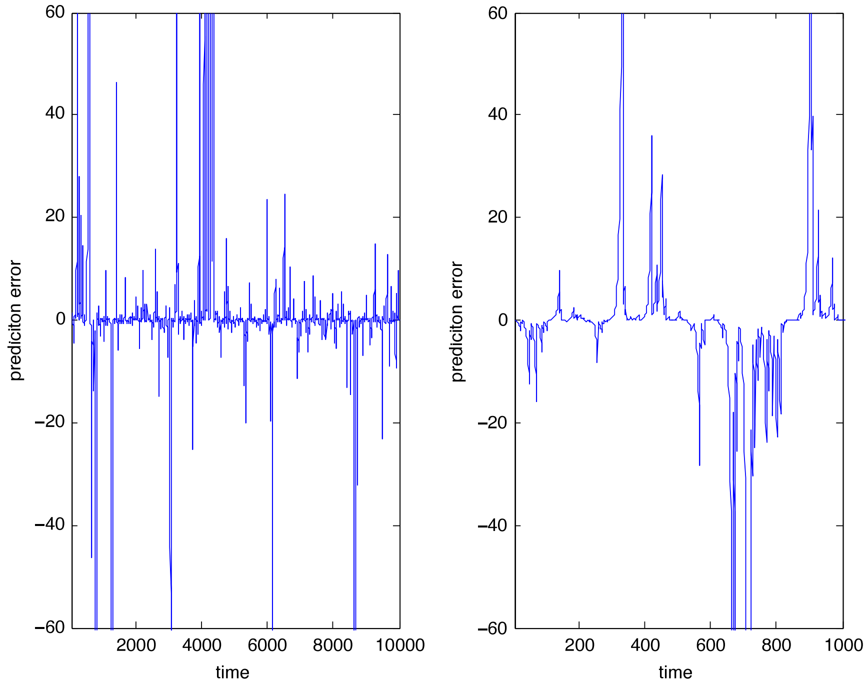

Fig. 1. Simulation of Kalman predictor error with $A = 1.2$, $C = 1$, $Q = 0.005$, $R = 0.001$, $P_\gamma = 0.15$ (left) and 0.1 (right), and bound level 60.

Clearly the escape time is a random variable provided the infinite value has zero probability. We have the following result applicable in the linear Gaussian case and independent of the system matrices $[A, B, C]$.

Lemma 1. *For the linear system (1), with noise process $\{w_k\}$ Gaussian, white, possessing full-rank covariance and independent from x_1, with control u_k causally computed, and for $\mathcal{D}$ compact, the escape time of x_k is almost surely finite.*

The proof is in the Appendix. The import of Lemma 1 is that it ensures that, in the linear Gaussian case or equivalent problems able to be transformed to linear Gaussian, using say Girsanov's Theorem, the finite escape of the state and/or output from any compact domain is ensured. The analysis of such processes then ought to concentrate on the description of the escape time rather than attempting to establish almost sure confinement to a compact set or characterize moment properties. This hearkens back to the escape time or residence time analysis of, say, Freidlin and Wentzell (1998), Khasminskii (1980), Meerkov and Runolfsson (1988), Varadhan (1984) and Zabzcyk (1985). These earlier treatments focus on stable continuous-time systems with small stochastic perturbations and use the Theory of Large Deviations to develop escape time characterizations as the noise power tends to zero. Our approach will maintain discrete time and deal with both stable and unstable systems with non-infinitesimal perturbations. This will not draw on Large Deviations Theory other than for comparison.

Our treatment of (1)–(3) endeavors to blend two distinct trains of research. The first is associated with the behavior of state estimators for such systems as treated in, say, Kar, Sinopoli, and Moura (2012), Mo and Sinopoli (2012), Shi et al. (2010) and Sinopoli et al. (2004) with or without control being applied. Since the system is linear and if the applied control is known, the controlled state behavior is derivable from the estimator. The second class of problems, characterized by results such as Nair and Evans (2004), Tatikonda and Mitter (2004a,b) and You and Xie (2011) concentrate on the stabilization aspects of the feedback control. The distinction between the two sets of problems in the literature rests with the description of the communications channel and the adaptation of quantization. The work in Minero, Franceschetti, Dey, and Nair (2009) and Minero, Coviello, and Franceschetti (2013) studies the earlier work in a more general case and yields necessary conditions for stabilization, recovering results of some previous works in the two approaches. In the estimator problem, the communication is taken to be intermittent – that is, the stochastic process $\{\gamma_k\}$ operates in a persistent fashion to cause arbitrarily long outages of communications – but the communication is not limited in bitrate (there is no quantizer) and full state reconstruction occurs with any successful communication packet. In earlier work on the stabilization problem, the emphasis is on the quantizer and its associated bitrate limit and the channel is assumed not intermittent, i.e. $\gamma_k = 1$ for all k, with a deterministic maximal delay and possible additive channel noise. The approach adopted in this paper is to permit both intermittency and limited bitrate, since the Markov model describing escape time applies to both. We also pose a different set of questions dealing with escape time, which we regard as being more apropos for these problems. These focus not on limiting behaviors or mean-square stabilization but on characterizing the cumulative probability distribution function (cdf) of the escape time of the system state, output or state estimate error, since in general there is no almost sure bound on these, as stated in Lemma 1.

Before launching into the analysis, it is pertinent to examine some practical sources of estimation and control problems associated with systems described by (1)–(3), since the presence of a single communications link rules out teleoperation-styled feedback control problems. Utility management of a geographically distributed system, such as a power grid or radar network, where the sensors, but not the actuators, are remotely placed and linked back to base by communications networks, is the clearest application of state estimation operating with communications limits. The study of sensor fusion and its sibling area of sensor scheduling Evans and Krishnamurthy (1998) and Evans, Krishnamurthy, Nair, and Sciacca (2005) have a long history in these arenas. Schweppe (Schweppe & Handschin, 1974) was a pioneer in the application of such methods in power system state estimation using data of variable reliability. More generally, the study of missing data has