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Model based peer-to-peer estimator over wireless sensor networks with lossy channels ☆

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ABSTRACT

In this paper, the design methods and fundamental performance analysis of an adaptive peer-to-peer estimator are established for networks exhibiting message losses. Based on a signal state model, estimates are locally computed at each node of the network by adaptively filtering neighboring nodes' estimates and measurements communicated over lossy channels. The computation is based on a distributed optimization approach that guarantees the stability of the estimator while minimizing the estimation error variance. Fundamental performance limitations of the estimator are established based on the variance of the estimation error in relation to the message loss process. Numerical simulations validate the theoretical analysis and illustrate the performance with respect to other estimators.

1. Introduction

Monitoring physical variables is a typical task that can be efficiently performed by wireless sensor networks (WSNs). Accurate estimation of these variables is necessary in many applications, spanning from traffic monitoring and control, industrial automation, environment monitoring, to security systems (Jadbabaie, Lin, & Morse, 2003). However, nodes of WSNs are typically characterized by both limited sensing, given that measurements are noisy, and communication capabilities, message losses may be significant. Estimation algorithms must be designed to cope with these adverse conditions, while offering high accuracy of the estimates.

There are two main estimation strategies for WSNs. A traditional approach consists in letting nodes sense the environment and then report data to a central unit, which extracts the desired physical variables and sends the estimate to each local node. This approach has strong limitations: large amount of communication resources (radio power, bandwidth, routing, etc.) have to be managed for the transmission of information from nodes to the central unit and vice versa. This mode of operation reduces the nodes' lifetime as most of the energy consumption is due to communication and message forwarding to the central location. An alternative approach, which we investigate in this paper, enables each node to produce locally accurate estimates taking advantage, through communication, of neighboring nodes' local estimations and measurements. The challenge of such a distributed estimation is that local processing must be carefully characterized to avoid uncontrolled propagation of the estimation errors throughout the network and, at the same time, to guarantee good estimation performance.

The estimator presented in this paper is related to contributions on low-pass filtering by diffusion mechanisms, e.g., Carli, Fagnani, Speranzon, and Zampieri (2008), Cattivelli and Sayed (2010a,b), Hu, Xie, and Zhang (2012), Jadbabaie et al. (2003), Matei and Baras (2012), Millán, Orihuela, Vivas, and Rubio (2012), Schizas, Giannakis, Roumeliotis, and Ribeiro (2008), where each node of the network obtains the average of the initial samples collected by nodes. In Xiao and Boyd (2004), Xiao, Boyd, and Kim (2007) the authors study a distributed average computation of a time-varying signal, when the signal is affected by a zero-mean noise. Distributed filtering using model-based approaches is studied in various wireless network contexts, e.g., Hovareshti, Baras, and Gupta (2008), Luo (2005), Olfati-Saber (2007) and Shang, Ruml, Zhang, and Fromherz (2004); Shi (2008). In particular, distributed Kalman filters and more recently a combination of the diffusion mechanism with distributed Kalman filtering have been proposed,

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e.g., Hu et al. (2012), Liang, Chen, and Pan (2010) and Sijs and Lazar (2012). In Kar, Moura, and Ramanan (2012), a strategy where the estimator works at the same time as the communication update is studied. In Pasqualetti, Carli, and Bullo (2012), two iterative linear distributed estimators are proposed. The first one considers an incremental mode of cooperation, while the second considers a diffusive iterative mode, but both modes are based on the measurements only and no model of the signal is assumed. In Olfati-Saber and Jalalkamali (2012), a theoretical framework for coupled distributed estimation and motion control of mobile sensor networks for collaborative target tracking is proposed. In Ding, Wang, Dong, and Shu (2012) a distributed H_∞ state estimation for discrete time varying nonlinear systems is studied. In this estimator, sensor nodes have knowledge of the fixed network topology. In Quevedo, Ahlen, Leong, and Dey (2012) the estimation performance is studied on centralized Kalman filtering in fading wireless channels suffering message losses, where power control is considered to alleviate the fading. In Li, Shen, Liu, and Zhang (2013), non-Gaussian measurement noise is considered, where the error entropy is minimized by the estimator.

In this paper, we propose a distributed approach to estimate a time-varying multi-dimensional signal affected by unknown additive disturbances. This is in contrast to approaches where statistical models of disturbances are assumed (Chiuso & Schenato, 2011; Olfati-Saber, 2007; Zhang, Feng, & Yu, 2012), or to approaches that are focused on averaging initial samples (Carli, Como, Frasca, & Garin, 2011; Carli et al., 2008; Carli, Frasca, Fagnani, & Zampieri, 2010; Olfati-Saber & Murray, 2004; Xiao & Boyd, 2004), or to methods where the estimation variables are static parameters (Delouille, Neelamani, & Baraniuk, 2006; Kar et al., 2012; Liao & Barooah, 2013; Lopes & Sayed, 2008; Luo, 2005), or to those where only local measurements are diffused over the network (Cattivelli, Lopes, & Sayed, 2008). Compared to Carli et al. (2008, 2010), Millán et al. (2012), Olfati-Saber (2007), Olfati-Saber and Murray (2004), Xiao and Boyd (2004), we do not use the consensus algorithms in the estimator, and compared to Xiao and Boyd (2004), where the Laplacian matrix associated to the communication graph is used to design the estimator. Our estimation parameters are computed through distributed algorithms that adapt to the network topology and message losses, in contrast to Luo (2005), which rely on centralized fusion center, or to Russell, Klein, and Hespanha (2011), Xiao and Boyd (2004), Xiao et al. (2007), where the computation of estimator need the full knowledge of the communication graph. This paper is a natural extension of the distributed estimators proposed in Speranzon, Fischione, Johansson, and Sangiovanni-Vincentelli (2008), Xu, Fischione, and Speranzon (2012), which were designed for scalar signals under the assumption of perfect communication, namely no message losses among nodes. We extend and substantially generalize Speranzon et al. (2008), Xu et al. (2012) as follows: First, we consider multi-dimensional system. Second, the estimator is designed to be robust to message losses, leading to a new optimal estimator structure. Third, new analytical results are provided to demonstrate the performance achieved by the new estimator in the presence of message losses.

The remainder of the paper is organized as follows. In Section 2, we define the estimator structure and the message loss model. In Section 3, we characterize the estimator design method when message losses are present and propose an estimation algorithm. In Section 4, we establish bounds on the estimation error variance of the proposed algorithm. In Section 5, we report the computational complexity of the proposed algorithm. Numerical simulations are reported in Section 6 to illustrate the performance of the proposed algorithm compared to others in literatures. Conclusions are drawn in Section 7.

1.1. Notation

Let $\mathbb{E}x$ and Cov$x$ denote the expectation and the covariance matrix of stochastic variable x, respectively, whereas $\mathbb{E}_{\mathbf{y}}[x(y)]$ and $\mathrm{Cov}_{\mathbf{y}}[x(y)]$ denote the expectation and the covariance matrix taken with respect to the probability density function of y, respectively. $\|\cdot\|$ is the spectral norm of a matrix, or ℓ^2-norm of a vector, while $\|\cdot\|_F$ is Frobenius norm of a matrix. For matrix A, let $\ell_m(A)$ and $\ell_M(A)$ denote the minimum and maximum eigenvalue (with respect to the absolute value of the real part), respectively. tr A is the trace of A. Let $\circ$ and $\otimes$ denote the Hadamard (element-wise) product and the Kronecker product for matrices, respectively. Let † be the Moore–Penrose pseudo-inverse for matrices (Horn & Johnson, 1985). For matrix A, $A \geq 0$ ($A \leq 0$) is equivalent to A ($-A$) being positive semidefinite. Let $\mathbf{I}$ and $\mathbf{1}$ be the identity matrix and the vector $(1, \dots, 1)^T$, respectively, whose dimensions may be indicated by a subscript. Let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

2. Problem formulation

Suppose a multi-dimensional signal needs to be estimated by a WSN. The signal is modeled by a linear system with unknown additive disturbance

$$x(t+1) = Ax(t) + w(t), \tag{1}$$

for $t \in \mathbb{N}_0$, where $x(t) \in \mathbb{R}^n$ is system state at time t, and $A \in \mathbb{R}^{n\times n}$ represents its dynamics. $w(t) \in \mathbb{R}^n$ models the disturbance in the state, whose ℓ^2-norm is bounded by Δ for all the time (Li & Jia, 2010; Zhang, Xia, & Shi, 2009). Note that the unknown disturbance is commonly addressed by H_∞ filtering (Ding et al., 2012; Li & Jia, 2010; Simon, 2006; Zhang et al., 2009). Suppose we place N sensor nodes at static positions in the space. These nodes can communicate to their neighbors. At each time, node i takes a noisy measurement of the system as

$$y_i(t) = C_i x(t) + v_i(t), \tag{2}$$

where $y_i(t) \in \mathbb{R}^{m_i}$ is the measurement of $x(t)$, and $C_i \in \mathbb{R}^{m_i\times n}$. $v_i(t) \in \mathbb{R}^{m_i}$ has a Gaussian distribution with zero mean and diagonal covariance R_i, and $\mathbb{E}[v_i(t)v_j^T(t)] = \mathbf{0}$ for all $t \in \mathbb{N}_0$, $i \neq j$. We assume that node i knows A and C_i.

The communication network is modeled by a graph, in which $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, N\}$ is the vertex set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. The set of neighbors of node $i \in \mathcal{V}$ plus node i is denoted as $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\} \cup \{i\}$. Namely $\mathcal{N}_i$ is the set containing the neighbors that a node i can have, including itself. Let N_i be the cardinality of set $\mathcal{N}_i$.

In each time interval, every node takes the measurement of signal (1) before broadcasting its previous estimates and measurements to its neighbors. After receiving messages from neighbors, each node computes the latest estimate of the signal. However, the wireless communication may be lossy because of bad channel conditions, caused by radio interference and/or transmission conflicts (hidden nodes or exposed nodes). Let $\varphi_{i_{j|t}}$ for $(i, j) \in \mathcal{E}$ denote a binary random variable, modeling the message loss on the edge (i, j) at time t from node j to i. We assume that $\varphi_{i_{j|t}}$ has the probability mass function:

$$\Pr(\varphi_{i_{j|t}} = 1) = p_{ij}, \qquad \Pr(\varphi_{i_{j|t}} = 0) = 1 - p_{ij} = q_{ij}, \tag{3}$$

where $p_{ij} \in (0, 1)$ denotes the successful message reception probability, and consequently $q_{ij} \in (0, 1)$ is the message loss probability. The message loss process is assumed to be independent among links, and independent from past message losses. These assumptions are natural when the coherence time of the wireless channel is small with respect to the typical communication rate of messages over WSNs (Stüber, 1996).