Automatica 61 (2015) 9–17

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Robust stabilization criterion of fractional-order controllers for interval fractional-order plants[☆]

Zhe Gao [1]

College of Light Industry, Liaoning University, Shenyang, 110036, PR China

ARTICLE INFO

Article history:
Received 2 October 2014
Received in revised form
20 February 2015
Accepted 10 July 2015

Keywords:
Fractional-order systems
Interval uncertainties
Robust stability
Value set

ABSTRACT

The robust stabilization criterion of fractional-order controllers for interval fractional-order plants is proposed in this study. For the interval uncertainties existing in the numerator and denominator of a transfer function with respect to a fractional-order plant, the vertices of the value set corresponding to the characteristic function are offered by the Minkowski Sum. This algorithm for getting the vertices avoids to calculate the redundant vertices, reducing the computational complexity. An auxiliary function reflecting the position relationship between the origin and the value set is defined to investigate the robust stability conditions. The lower and upper limits of the test frequency interval are offered to check the auxiliary function within a finite frequency. Supposing that the fractional-order controller can stabilize the nominal fractional-order plant, the stabilization criterion of the closed loop system with interval uncertainties is provided based on the auxiliary function and other two conditions. Finally, three illustrative examples with fractional-order controllers are given to validate the effectiveness of this robust stabilization criterion.

1. Introduction

Recently, the fractional-order calculus theory has drawn much attention in the control system and industrial automation fields (Efe, 2011; Monje, Chen, Vinagre, Xue, & Feliu, 2010). The fractional-order systems can describe the real-world systems better than the integer-order ones such as some physical systems with the viscoelastic and diffusive characteristics (Gabano & Poinot, 2011; Meral, Royston, & Magin, 2010). The approaches of the parameter and differentiation order estimation for fractional-order systems were addressed in Sabitier, Farges, Merveillaut, and Feneteau (2012) and Victor, Malti, Garnier, and Oustaloup (2013). Meanwhile, the introduction of fractional-order operators brings the flexibility for the controller design (Krishna, 2011). Because the PID controllers are widely used in the industrial practice, a lot of tuning methods for fractional-order PID controllers have been reported and explored (Podlubny, 1999; Yeroglu & Tan, 2011). The main methods to get the satisfactory parameters focus on the frequency response and the intelligent algorithms. The design methods of fractional-order PID controllers based on some intelligent algorithms aid to solve the control problems for complex fractional-order systems such as Sharma, Rana, and Kumar (2014) and Zamani, Karimi-Ghartemani, Sadati, and Parniani (2009). But, the performance of the control system is determined by the adopted intelligent algorithm, hence the investigations on the design of controller parameters based on intelligent algorithms focus on the improvement of intelligent algorithms themselves. Being independent of the system models, the satisfactory performances may not been achieved by intelligent algorithms. Hence, it is necessary to study the design method for fractional-order systems based on the system models (for example, see Gao, Yan, & Wei, 2014; Hamamci, 2007, 2008; Luo & Chen, 2012, and so on).

Including fractional-order systems, ensuring the stability of closed loop systems is the essential task for the controller design. Due to the existence of the power terms in the characteristic function, it is not straightforward to check the stability of a linear fractional-order system from the coefficients of the characteristic function directly. For a linear fractional-order system represented by the transfer function model, the stability condition is that all of the poles lie in the left-half plane (Matignon, 1996). The robust stability bounds of interval fractional-order systems were offered in Ma, Lu, Chen, and Chen (2014).

☆ This work was supported by the National Natural Science Foundation of China (Grant No. 61304094), and the Scientific Research Fund of Liaoning Provincial Education Department, China (Grant No. L2015194). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Hitay Ozbay under the direction of Editor Miroslav Krstic.

E-mail address: gaozhe83@gmail.com.

[1] Tel.: +86 24 62202139; fax: +86 24 62202139.

http://dx.doi.org/10.1016/j.automatica.2015.07.021
0005-1098/© 2015 Elsevier Ltd. All rights reserved.

For interval fractional-order systems represented by state space models, the robust stability criteria were studied in Ahn and Chen (2008) and Ahn, Chen, and Podlubny (2007). Meanwhile, the LMI conditions for the commensurate order α satisfying the cases $1 < \alpha < 2$ and $0 < \alpha < 1$ were provided in Lu and Chen (2009, 2010), respectively. But, the controller design by solving the LMI equalities may cause the conservation for interval fractional-order systems. For the interval uncertainties existing in the parameters of characteristic function, the stability was explored for interval fractional-order systems represented by transfer function models in Petras, Chen, and Vinagre (2004) and Petras, Chen, Vinagre, and Podlubny (2004). By the zero exclusion condition, the robust stability criterion for interval fractional-order systems was proposed in Tan, Ozguven, and Ozyetkin (2009). But, a great deal of redundant vertices appear by the method proposed in Tan et al. (2009) to calculate the value set. To obtain the true vertices of the value set, the computing method of the vertices and stability criteria were proposed in Gao and Liao (2013) and Moornani and Haeri (2010a). The robust stability criterion via the value set approach was extended to fractional-order time-delay systems with retarded and neural types in Moornani and Haeri (2010b). Meanwhile, a graphical test approach was proposed to test the stability of fractional-order time-delay systems in Yu and Wang (2011). In Senol, Ates, Alagoz, and Yerolu (2014), a numerical approach was given to test the robust stability of fractional-order systems with fractional-order controllers.

In this paper, an approach to check the robust stabilization of fractional-order controllers for fractional-order plants is investigated. The interval uncertainties are concerned in the numerators and denominators of the transfer function for an interval fractional-order plant. The vertices of the value set with respect to the characteristic function of the corresponding closed loop system are determined. The advantage of this computing method of vertices is the reduction of the computational complexity by avoiding the calculation of redundant vertices. Meanwhile, a finite frequency interval is provided to replace the infinite test frequency interval, achieving the robust stabilization judgment. Based on the zero exclusion principle and an auxiliary function defined in this paper, we propose the sufficient and necessary conditions of the robust stabilization criterion of fractional-order controllers.

The rest of this paper is organized as follows. Section 2 provides the investigated interval fractional-order systems, and introduces the backgrounds of the value set. The vertices of value set with respect to the closed loop system and the sufficient and necessary conditions of the robust stabilization criterion are addressed in Section 3. In Section 4, we provide three numerical examples to illustrate the implementing steps of the robust stabilization criterion. Section 5 concludes the whole paper.

For conveniences, the following notations are adopted throughout this paper: $\arg(x)$, $\mathrm{Re}(x)$, $\mathrm{Im}(x)$ and x^* are the phase angle, the real part, the imaginary part and the conjugate value of a complex number x; j is the imaginary unit; $\mathbb{Z}^+$ is the set of positive integers; $\mathbb{Z}$ is the set of integers; $\mathbb{Z}_0^n$ is the set of integers from 0 to n; $\mathbb{R}_{\geq 0}$ is the set of non-negative real numbers; $\mathbb{R}^+$ is the set of positive real numbers; $\#M$ represents the number of entries belonging to the set M; $\arg\max\{x|f(x)\}$ returns the argument x achieving the maximum of $f(x)$.

2. Preliminaries

In this paper, we investigate the robust stability of an interval fractional-order plant $P(s)$ with the fractional-order controller $C(s) = N_c(s)/D_c(s)$. The interval fractional-order plant is represented by

$$P(s) = \frac{N_p(s)}{D_p(s)} = \frac{\sum_{j=0}^{m_p} b_j^p s^{\beta_j^p}}{\sum_{i=0}^{n_p} a_i^p s^{\alpha_i^p}}, \tag{1}$$

and the denominator and numerator of the fractional-order controller are $D_c(s) = s^{\alpha_{n_c}^c} + \sum_{i=0}^{n_c-1} a_i^c s^{\alpha_i^c}$ and $N_c(s) = \sum_{j=0}^{m_c} b_j^c s^{\beta_j^c}$ respectively, where the fractional-orders satisfy $\alpha_{n_p}^p > \alpha_{n_p-1}^p > \cdots > \alpha_1^p > \alpha_0^p = 0$, $\beta_{m_p}^p > \beta_{m_p-1}^p > \cdots > \beta_1^p > \beta_0^p = 0$, $\alpha_{n_c}^c > \alpha_{n_c-1}^c > \cdots > \alpha_1^c > \alpha_0^c = 0$ and $\beta_{m_c}^c > \beta_{m_c-1}^c > \cdots > \beta_1^c > \beta_0^c = 0$. Moreover, the highest orders are subject to $\alpha_{n_p}^p > \beta_{m_p}^p$ and $\alpha_{n_c}^c > \beta_{m_c}^c$ for the plant and controller respectively. The coefficients of the fractional-order plant (1) contain the interval uncertainties $b_j^p \in [b_j^{p-}, b_j^{p+}]$ and $a_i^p \in [a_i^{p-}, a_i^{p+}]$. The condition $0 \notin [a_{n_p}^{p-}, a_{n_p}^{p+}]$ is also the assumption of the fractional-order plant $P(s)$.

The characteristic function of the closed loop system in terms of plant $P(s)$ and controller $C(s)$ is a multivalued function, hence we assume that $\arg(s) \in (-\pi, \pi]$ throughout this paper to achieve the first Riemann sheet.

The characteristic function $F(s) = N_c(s)N_p(s) + D_c(s)D_p(s)$ indicates that the value set of $F(s)$ is determined by the value set of $D_p(s)$ and $N_p(s)$. By the assumption $0 \notin [a_{n_p}^{p-}, a_{n_p}^{p+}]$, the leading coefficient of the characteristic function $F(s)$ does not contain zero. Denote $[a_i^{p-}, a_i^{p+}] = \bar{a}_i^p + w_i^D \delta_i^D$ and $[b_j^{p-}, b_j^{p+}] = \bar{b}_j^p + w_j^N \delta_j^N$, where $\bar{a}_i^p = (a_i^{p-} + a_i^{p+})/2$, $\bar{b}_j^p = (b_j^{p-} + b_j^{p+})/2$, $w_i^D = (a_i^{p+} - a_i^{p-})/2$, $w_j^N = (b_j^{p+} - b_j^{p-})/2$ and $|\delta_i^D| \leq 1$, $|\delta_j^N| \leq 1$, then we have

$$F(s) = \bar{F}(s) + F_\Delta(s). \tag{2}$$

The nominal function $\bar{F}(s)$ is represented by $\bar{F}(s) = N_c(s) \sum_{j=0}^{m_p} \bar{b}_j^p s^{\beta_j^p} + D_c(s) \sum_{i=0}^{n_p} \bar{a}_i^p s^{\alpha_i^p}$. Meanwhile, the disturbance function $F_\Delta(s)$ is represented by $F_\Delta(s) = N_c(s)N_{p\Delta}(s) + D_c(s)D_{p\Delta}(s)$, where $D_{p\Delta}(s) = \sum_{i=0}^{n_p} w_i^D \delta_i^D s^{\alpha_i^p}$ and $N_{p\Delta}(s) = \sum_{j=0}^{m_p} w_j^N \delta_j^N s^{\beta_j^p}$.

Lemma 1 (*Bonnet & Partington, 2002*). *The closed loop fractional-order system is BIBO-stable if and only if $F(s)$ has no poles in $\{\mathrm{Re}(s) \geq 0\}$ on the first Riemann sheet.*

Although Lemma 1 offers an approach to determine stability for no uncertainty existing in any coefficient, it is hard to solve the poles due to the power and exponential terms in the characteristic function. Lemma 1 provides the theoretical foundation of the stability analysis for fractional-order systems, including interval fractional-order systems. For convenience, the stability investigated in this paper represents the BIBO (Bounded Input and Bounded Output) stability.

To prove the stability criterion, the Minkowski Sum is used in the following section. The properties of the Minkowski Sum are introduced as follows.

Definition 1 (*Ewald, 1996*). For two sets P and Q, the Minkowski Sum of P and Q is defined as follows

$$P + Q = \{p + q | p \in P, q \in Q\}.$$

The important property of Minkowski sum states that the Minkowski sum of two convex sets is also a convex set.

Lemma 2 (*Ziegler, 1995*). *For any two convex sets $P = \mathrm{conv}(U)$ and $Q = \mathrm{conv}(V)$, we have*

$$P + Q = \mathrm{conv}(U) + \mathrm{conv}(V) = \mathrm{conv}(U + V).$$