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Full Length Article

A computational method for solving a class of singular boundary value problems arising in science and engineering

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ABSTRACT

This note deals with a new computational method for solving a class of singular boundary value problems. The method is based upon Bernstein Polynomials. The properties of Bernstein Polynomials, together with Bernstein operational matrix for differentiation formula, are presented and utilized to reduce the given singular boundary value problems to the set of algebraic equations. The proposed method is applied to solve some test problems with the comparison between the computational solutions and the exact solutions. The results indicate that the proposed algorithm provides high reliability and good accuracy.

1. Introduction

In this paper, we consider the class of singular boundary value problems (SBVPs) of the form

$$y''(x)+\frac{\alpha}{x}y'(x)+p(x)y(x)=r(x), \quad 0<x\le 1 \quad \textit{and} \quad \alpha>0 \tag{1}$$

with the boundary conditions

$$y(0)=a_1, \quad y(1)=b \quad \text{(or)} \quad y'(0)=a_2, \quad y(1)=b \tag{2}$$

where $p(x)$ and $r(x)$ are analytic in $x\in(0,1)$ and a_1, a_2, b and α are finite constants. Problem (1) has singularity at the initial point $x=0$. We note that the main difficulty arises in the singularity of the equations at $x=0$.

The numerical treatment of singular boundary value problems has always been a difficult and challenging task due to the singular behavior that occurs at a point.

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In recent years, strenuous action and interest have been investigating singular boundary value problems (SBVPs) and a number of methods have been proposed. The singular boundary value problems arise frequently in many branches of applied mathematics, mechanics, nuclear physics, atomic theory and chemical sciences. Hence, the singular boundary value problems have attracted much attention and have been investigated by many researchers. Kadalbajoo and Agarwal treated the homogeneous equation using Chebyshev polynomial and B-spline [1]. Kanth and Reddy solved a particular singular boundary value problem by applying higher order finite difference method [2].The same authors investigated by using the cubic spline [3] method. Mohanty et al. [4] introduced accurate cubic spline method for solving the singular boundary value problems. Variational iteration method (VIM) was introduced by Wazwaz [5] for solving nonlinear singular boundary value problems.

The authors [6,7] elaborately discussed singular boundary value problems and solved by the methods based on reproducing kernel space. Galerkin and Collocation methods are presented for solving two-point boundary value problems by Mohsen and El-Gamel [8]. Secer and Kurulay [9] described the Sinc-Galerkin method for singular Dirichlet-type boundary value problems. Rashidinia et al. [10] proposed a reliable parametric spline method.

The purpose of this note is to develop a reliable operational matrix method for singular boundary value problems. The aim of the present paper is to apply Bernstein operational matrix of differentiation to propose a reliable numerical technique for solving SBVPs.

With the advent of computer graphics, Bernstein polynomial restricted to the interval $x \in [0,1]$becomes important in the form of Bezier curves [11,12]. Bernstein polynomials have many constructive properties such as the positivity, the continuity, recursive relation, symmetry and unity partition of the basis set over the interval. For this reason, Bernstein operational matrix method is a new and rising area in applied mathematical research which has gained considerable attention in dealing with differential equations. Optimal stability of the Bernstein basis was discussed by Farouki and Goodman [13]. Bhatta and Bhatti [14] have been used modified Bernstein polynomials for solving Korteweg-de Vries (KdV) equation. Chakrabarti and Martha [15] described a method for Fredholm integral equations. Bhattacharya and Mandal [16] presented Bernstein polynomials method for Volterra integral equations. Yousefi and Behroozifar [17] found the operational matrices of integration and product of B-polynomials. The same authors [18] introduced the Bernstein operational matrix method (BOMM) for solving the parabolic type partial differential equations (PDEs). Isik et al. [19] have demonstrated a new method to solve high order linear differential equations with initial and boundary conditions. Ordokhani et al. [20] introduced Bernstein polynomial for solving differential equations. Singh et al. [21] established the Bernstein polynomials have been first orthonormalized by using Gram–Schmidt ortho normalization process and then the operational matrix of integration has been acquired. Doha et al. [22,23] have implemented and proved new formulas about derivatives and integrals of Bernstein polynomials and solving high even-order differential equations by Bernstein polynomial based method. Yousefi et al. [24] described the Ritz-Galerkin method for solving an initial boundary value problem that combines Neumann and integral condition for the wave equation. Bhattacharya et al. [25] implemented the algorithm for integro differential equations. Shen et al. [26] suggested a boundary knot method (BKM) to solve Helmholtz problems with boundary singularities. Lin et al. [27] used the efficient boundary knot method to obtain the solution of axisymmetric Helmholtz problems. Recently, Pirabaharan et al. [28] introduced a Bernstein operational matrix method for boundary value problems.

The objective of this paper is to give the computational method based on the Bernstein operational matrix for differentiation for the class of singular boundary value problems (SBVPs). The proposed algorithm converted to the given SBVP to the system of algebraic equations with unknown coefficients and solving them by using mathematical tools like MATLAB easily.

The outline of the paper is as follows. In section 2, we review the basic properties of Bernstein polynomials and we develop the Bernstein operational matrix for derivative. Section 3 implemented and presented the methods of solution for the singular boundary value problems. In Section 4, some numerical examples are presented to show the efficiency and the applicability of the suggested algorithm. Conclusion is given in Section 5.

2. Properties of Bernstein polynomials [29]

The Bernstein basis polynomials of degree n are defined in [0, 1] as [17,18]

$$b_i^n(x) = \binom{n}{i} x^i (1-x)^{n-i}, \quad i = 0, 1, 2, \ldots, n \tag{3}$$

These Bernstein polynomials form a complete basis on [0, 1]. Using the recursive definition to generate these polynomials

$$b_i^n(x) = (1-x)\, b_i^{n-1}(x) + x\, b_{i-1}^{n-1}(x), \quad i = 0, 1, 2, \ldots, n$$

where $b_{-1}^{n-1}(x) = 0$ and $b_n^{n-1}(x) = 0$.

Since the power basis forms a basis for the space of polynomials of degree less than or equal to n, any nth degree Bernstein polynomial can be expressed in terms of the power basis. This can be computed by using the binomial expansion,

one can illustrate that $$b_i^n(x) = \sum_{j=1}^{n} (-1)^{j-1} \binom{n}{i} \binom{n-1}{j-i} x^j, \quad i = 0, 1, \ldots, n \tag{4}$$

By using the property, the dual basis is defined as follows:

$$\int_0^1 b_i^n(x)\, d_j^n(x) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \textit{for} \quad i, j = 0, 1, 2, \ldots, n.$$

A function $f(x)$, square integrable in the interval [0, 1], may be written in terms of Bernstein basis [17,18].

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