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Brief paper

Unknown input observer for linear time-delay systems[☆]

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ABSTRACT

This paper investigates an unknown input observer design for a large class of linear systems with unknown inputs and commensurate delays. A Luenberger-like observer is proposed by involving only the past and actual values of the system output. The required conditions for the proposed observer are considerably relaxed in the sense that they coincide with the necessary and sufficient conditions for the unknown input observer design of linear systems without delays.

1. Introduction

Time delay systems are widely used to model many applications, ranging from chemical and biological process to sampled data effects (Richard, 2003). Many results have been published to treat this kind of systems for different aspects, such as stability (Fridman, 2014), observability (Zheng, Barbot, Boutat, Floquet, & Richard, 2011) and identifiability (Zheng, Barbot, & Boutat, 2013).

The unknown input observer design for linear systems without delays has already been solved in Bhattacharyya (1978), Darouach, Zasadzinski, and Xu (1994), Yang and Wilde (1988), Hou and Muller (1992), Kudva, Viswanadham, and Ramakrishna (1980), Wang, Davison, and Dorato (1975), Hostetter and Meditch (1973). This problem becomes more complicated when the studied system involves delays, which might appear in the state, in the input and in the output. For this issue, different techniques have been proposed in the literature, such as infinite dimensional approach (Salamon, 1980), polynomial approach based on the ring theory (Emre & Khargonekar, 1982; Sename, 1997), Lyapunov function based on LMI (Darouach, 2001; Seuret, Floquet, Richard, & Spurgeon, 2007) and so on.

More precisely, Fattouh, Sename, and michel Dion (1999) proposed an unknown input observer with dynamic gain for linear systems with commensurate delays in state, input and output variables, while the output was not affected by the unknown inputs. Inspired by the technique of output injection (Krener, 1985), Hou, Zitek, and Patton (2002) solved this problem by transforming the studied system into a higher dimensional observer canonical form with delayed output injection. In Darouach (2001, 2006), the unknown input observer was designed for the systems involving only one delay in the state, and no delay appears in the input and output. The other observers for some classes of time-delay systems can be found in Conte, Perdon, and Guidone-Peroli (2003), Sename (2001), Fu, Duan, and Song (2004) and references therein.

Most of the existing works on unknown input observer are focused on time-delay systems whose outputs are not affected by unknown inputs. However, this situation might exist in many practical applications since most of the sensors involve computation and communication, thus introduce output delays. This motivates the work of this paper. Compared to the existing results in the literature, this paper deals with the unknown input observer design problem for a more general sort of linear time-delay systems where the commensurate delays are involved in the state, in the input as well as in the output. Moreover, the studied linear time-delay system admits more than one delay. As

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far as we know, there exist some methods to eliminate (or reduce the degree of) the delay, such as Lee, Neftci, and Olbrot (1982), Germani, Manes, and Pepe (2001) and Garate-Garcia, Marquez-Martinez, and Moog (2011). It has been proven in Garate-Garcia et al. (2011) that the elimination or the reduction of delay degree via a bicausal transformation with the same dimension is possible if some conditions on $A(\delta)$ and $B(\delta)$ are satisfied. Since this paper investigates the most general linear system with commensurate delays on the state, the input and the output, to impose those kinds of conditions will definitely restrict the contribution of this paper. Moreover, even for the general single delay system with **unknown input**, the problem to design an observer is still unsolved, thus the contribution of this paper does not depend on the degree of time delay involved in $A(\delta)$, $B(\delta)$, $C(\delta)$ and $D(\delta)$.

This paper adopts the polynomial method based on ring theory since it enables us to reuse some useful techniques developed for systems without delays. The following notations will be used in this paper. $\mathbb{R}$ is the field of real numbers. The set of nonnegative integers is denoted by $\mathbb{N}_0$. I_r means the $r \times r$ identity matrix. $\mathbb{R}[\delta]$ is the polynomial ring over the field $\mathbb{R}$. $\mathbb{R}^n[\delta]$ is the $\mathbb{R}[\delta]$-module whose elements are the vectors of dimension n and whose entries are polynomials. By $\mathbb{R}^{q\times s}[\delta]$ we denote the set of matrices of dimension $q \times s$, whose entries are in $\mathbb{R}[\delta]$. For a matrix $M(\delta)$, $\mathrm{rank}_{\mathbb{R}[\delta]} M(\delta)$ means the rank of the matrix $M(\delta)$ over $\mathbb{R}[\delta]$. $M(\delta) \sim N(\delta)$ means the similarity between two polynomial matrices $M(\delta)$ and $N(\delta)$ over $\mathbb{R}[\delta]$, i.e. there exist two unimodular[1] matrices $U_1(\delta)$ and $U_2(\delta)$ over $\mathbb{R}[\delta]$ such that $M(\delta) = U_1(\delta)N(\delta)U_2(\delta)$.

2. Problem statement

In this paper, we consider the following class of linear systems with commensurate delays:

$$\begin{cases} \dot{x}(t) = \sum_{i=0}^{k_a} A_i x(t-ih) + \sum_{i=0}^{k_b} B_i u(t-ih) \\ y(t) = \sum_{i=0}^{k_c} C_i x(t-ih) + \sum_{i=0}^{k_e} D_i u(t-ih) \end{cases} \tag{1}$$

where the state vector $x(t) \in \mathbb{R}^{n_x}$, the system output vector $y(t) \in \mathbb{R}^p$, the unknown input vector $u(t) \in \mathbb{R}^m$, the initial condition $\varphi(t)$ is a piecewise continuous function $\varphi(t) : [-kh, 0] \to \mathbb{R}^n$ ($k = \max\{k_a, k_b, k_c, k_d\}$); thereby $x(t) = \varphi(t)$ on $[-kh, 0]$. A_i, B_i, C_i and D_i are the matrices of appropriate dimension with entries in $\mathbb{R}$.

In order to simplify the analysis, let us introduce the delay operator $\delta : x(t) \to x(t-h)$ with $\delta^k x(t) = x(t-kh)$, $k \in \mathbb{N}_0$. Let $\mathbb{R}[\delta]$ be the polynomial ring of δ over the field $\mathbb{R}$, and it is obvious that $\mathbb{R}[\delta]$ is a commutative ring.

After having introduced the delay operator δ, system (1) may be then represented in the following compact form:

$$\begin{cases} \dot{x}(t) = A(\delta)x(t) + B(\delta)u(t) \\ y(t) = C(\delta)x(t) + D(\delta)u(t) \end{cases} \tag{2}$$

where $A(\delta) \in \mathbb{R}^{n_x\times n_x}[\delta]$, $B(\delta) \in \mathbb{R}^{n_x\times m}[\delta]$, $C(\delta) \in \mathbb{R}^{p\times n_x}[\delta]$, and $D(\delta) \in \mathbb{R}^{p\times m}[\delta]$ are matrices over the polynomial ring $\mathbb{R}[\delta]$, defined as $A(\delta) := \sum_{i=0}^{k_a} A_i\delta^i$, $B(\delta) := \sum_{i=0}^{k_b} B_i\delta^i$, $C(\delta) := \sum_{i=0}^{k_c} C_i\delta^i$, and $D(\delta) := \sum_{i=0}^{k_d} D_i\delta^i$.

[1] Refer to Definition 2 for the concept of unimodular matrix over $\mathbb{R}[\delta]$.

Remark 1. For the system without delay, i.e. $A(\delta) = A$, $B(\delta) = B$, $C(\delta) = C$ and $D(\delta) = D$ in (2), Hautus (1983) proposed the following unknown input Luenberger-like observer:

$$\dot{\xi} = P\xi + Qy$$
$$\hat{x} = \xi + Ky$$

and it has been proven as well the above Luenberger-like observer exists only if the following rank condition:

$$\mathrm{rank}\begin{bmatrix} CB & D \\ D & 0 \end{bmatrix} = \mathrm{rank}\begin{bmatrix} B \\ D \end{bmatrix} + \mathrm{rank} D \tag{3}$$

is satisfied.

When considering the general linear system (2) with commensurate delays which can appear in the state, in the input and in the output, the problem to design a simple unknown input Luenberger-like observer is still open. The main idea of this paper is inspired by the method proposed in Hou et al. (2002) where only linear time-delay systems without input were studied. More precisely, we first try to decompose system (2) into a simpler form provided that some conditions are satisfied, and then transform it into a higher dimensional observer normal form with output (and the derivative of the output) injection and its delay. Finally we can design an unknown input observer for the obtained observer normal form.

3. Notations and definitions

When designing an unknown input observer for time-delay systems, it is desired to use only the actual and the past information (not the future information) of the measurements to estimate the states because of the causality. Therefore, by noting $x(t; \varphi, u)$ as the solution of (2) with the initial condition φ and the input u, we have the following observability definition stated in Bejarano and Zheng (2014).

Definition 1. System (1) (or system (2)) is said to be backward unknown input observable on $[t_1, t_2]$ if for each $\tau \in [t_1, t_2]$ there exists $t_1' < t_2' \le \tau$ such that, for all input u and every initial condition φ, $y(t; \varphi, u) = 0$ for all $t \in [t_1', t_2']$ implies $x(\tau; \varphi, u) = 0$.

Concerning the above definition of backward unknown input observability, Bejarano and Zheng (2014) analyzed it by following the ideas of Silverman (1969) and Molinari (1976). Define $\{\Delta_k(\delta)\}$ as the matrices generated by the following algorithm:

$$\begin{aligned} &\Delta_0 \triangleq 0, \quad G_0(\delta) \triangleq C(\delta), \quad F_0(\delta) \triangleq D(\delta) \\ &S_k(\delta) \triangleq \begin{bmatrix} \Delta_k(\delta)B(\delta) \\ F_k(\delta) \end{bmatrix}, \quad k \ge 0 \\ &\begin{bmatrix} F_{k+1}(\delta) & G_{k+1}(\delta) \\ 0 & \Delta_{k+1}(\delta) \end{bmatrix} \triangleq P_k(\delta)\begin{bmatrix} \Delta_k(\delta)B(\delta) & \Delta_k(\delta)A(\delta) \\ F_k(\delta) & G_k(\delta) \end{bmatrix} \end{aligned} \tag{4}$$

where $P_k(\delta)$ is a unimodular matrix over $\mathbb{R}[\delta]$ that transforms $S_k(\delta)$ into its Hermite form. Moreover define $\{M_k(\delta)\}$ as follows:

$$\begin{aligned} &M_0(\delta) \triangleq N_0(\delta) \triangleq \Delta_0, \quad N_{k+1}(\delta) \triangleq \begin{bmatrix} N_k(\delta) \\ \Delta_{k+1}(\delta) \end{bmatrix}, \quad \text{for } k \ge 0 \\ &\begin{bmatrix} M_{k+1}(\delta) \\ 0 \end{bmatrix} \triangleq \begin{bmatrix} \mathcal{S}_{N_{k+1}(\delta)} & 0 \\ 0 & 0 \end{bmatrix} = \Lambda_{k+1}(\delta)N_{k+1}(\delta)\Sigma_{k+1}(\delta) \end{aligned} \tag{5}$$

where $\mathcal{S}_{N_{k+1}(\delta)} = \mathrm{diag}\{\psi_1^{k+1}(\delta), \ldots, \psi_{r_{k+1}}^{k+1}(\delta)\}$ with $\Lambda_{k+1}(\delta)$ and $\Sigma_{k+1}(\delta)$ being unimodular matrices over $\mathbb{R}[\delta]$ that transform $N_{k+1}(\delta)$ into its Smith form, and $\left\{\psi_i^{k+1}(\delta)\right\}$ are called the invariant factors of $N_{k+1}(\delta)$.

Since we are going to analyze system (2) which is described by the polynomial matrices over $\mathbb{R}[\delta]$, therefore let us give some useful definitions of unimodular and change of coordinates over $\mathbb{R}[\delta]$.