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Brief paper

A robust globally convergent position observer for the permanent magnet synchronous motor[☆]

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ABSTRACT

Position observers are essential components in the design of sensorless controllers for electric motors. Their design is complicated by the fact that the motor dynamics is highly nonlinear and contains some uncertain parameters. In this paper a new robust, nonlinear, *globally convergent* position observer for surface-mount permanent magnet synchronous motors is proposed. A key feature of the proposed observer is that it requires only knowledge of the stator resistance and inductance with the mechanical parameters and the magnetic flux constant being unknown. Thanks to a new reparameterization of the motor dynamics only two parameters are estimated with a regressor consisting of filtered voltage and current, which is persistently exciting in normal motor operation. Realistic simulations and *experiments* show that the proposed observer outperforms other existing designs from the point of view of robustness and convergence rate.

1. Introduction

In this paper we are interested in the problem of designing robust observers for the estimation of the position of surface-mount permanent magnet synchronous motors (PMSM). Providing a satisfactory solution to this task is essential for the design of sensorless controllers to which – given its great practical importance – an enormous amount of work has been devoted. See Acarnley and Watson (2006), Ichikawa, Tomita, Doki, and Okuma (2006) and Nam (2010) for a recent review of the literature.

It has long been recognized that implementing a copy of the electrical dynamics of the PMSM generates a consistent estimate of the rotor flux—modulo the knowledge of the *flux initial conditions*. As is well-known, the rotor position is directly obtained from the estimated rotor flux via some simple trigonometric relations. A large amount of the literature reported on this problem has, therefore, concentrated on implementing means to compensate for the bias in the flux estimation induced by the imprecise knowledge of its initial conditions. In Ortega, Praly, Astolfi, Lee, and Nam (2011) a very simple, gradient descent-based procedure to carry-out this task was proposed. The analysis, done for the full nonlinear model, proves global (under some conditions, even exponential) convergence of the estimates to a *residual set* and yields very simple robust tuning rules—see also Malaize, Praly, and Henwood (2012) for a simpler stability analysis and the proof of global convergence under (a kind of) *persistent excitation* (PE) condition. In Lee, Nam, Ortega, Praly, and Astolfi (2010) this position observer was combined with an *ad-hoc* linear speed estimator and a standard field-oriented controller, which are often used in applications, yielding very encouraging experimental results. See also Dib, Ortega, and Malaize (2011), Shah, Espinosa, Ortega, and Hilairet (2014) and Tomei and Verrelli (2011) where this observer is also used.

The large majority of the observers proposed in the literature, including the one in Ortega et al. (2011), are designed under the

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assumption that all electrical parameters of the motor are constant, and exactly known. As is well-known (Acarnley & Watson, 2006; Nam, 2010), these parameters are usually uncertain and time-varying. Hence, a question of great practical interest is to design an observer which is robust *vis-à-vis* these parameters. Simulation and experimental studies have shown that the performance of the observer of Ortega et al. (2011) significantly degrades in the face of uncertainty in the *magnetic flux constant*. This sensitivity is understood because the gradient – that defines the aforementioned correction term in the observer – explicitly depends on this parameter. In Romero, Ortega, Hernandez, Devos, and Malrait (2014) an adaptive version of the flux observer of Ortega et al. (2011) that estimates the *stator resistance* was proposed. Global boundedness of all signal and convergence to a residual set of the flux estimation error was established assuming known the rotor speed and all electrical parameters, including the magnetic flux constant, hence still suffering from the aforementioned lack of robustness problem. To overcome this drawback, in Henwood, Malaize, and Praly (2012) a far more complex – and without stability proof – Kazantzis–Kravaris–Luenberger observer that explicitly estimates the magnetic flux constant is proposed.

A new approach for flux observation in PMSMs was advocated in Bobtsov, Pyrkin, and Ortega (2014). Using a novel representation of the PMSM dynamics and some suitable filtering the problem is recast as classical linear regression estimation problem, whose global convergence is ensured by the ubiquitous PE condition. The approach of Bobtsov et al. (2014) suffers, however, from the important drawback that the generation of the linear regression requires the knowledge of the rotor speed. Even though simulations using an estimate of the speed generated from the flux observer itself reveal its robustness *vis-à-vis* this assumption, from the theoretical viewpoint it is desirable to remove this assumption.

In this paper we propose a new flux observer that does not require the measurement of rotor speed. Towards this end, we take a different approach with the following innovations and contributions.

(i) Instead of estimating the flux we propose to directly estimate the *rotor position* encrypted in the component of the flux due to the permanent magnet.
(ii) After a suitable rearrangement of the motor dynamics and filtering actions the problem of position estimation reduces to the estimation of the initial conditions of the aforementioned flux component, which are treated as *uncertain parameters* in a standard linear regression form.
(iii) The vector of unknown parameters is two-dimensional and the regressor, which consists of filtered voltage and current, is PE in normal operating conditions of the motor.
(iv) Since the rotor speed dynamic equation is not used in the observer design the construction of the regressor requires only knowledge of the stator resistance and inductance with the mechanical parameters and the magnetic flux constant being unknown. The latter feature distinguishes the proposed observer from the one reported in Ortega et al. (2011).
(v) Representative *experimental* results are presented.

The remaining of the paper is organized as follows. Section 2 presents the model of the PMSM and the assumptions needed for the observer design. Section 3 contains our main result. Some representative simulations and experimental results are presented in Section 4. The paper is wrapped-up with concluding remarks in Section 5.

Notation. For the column vector $\xi = \mathrm{col}(\xi_1, \dots, \xi_n) \in \mathbb{R}^n$, $|\xi|$ is the Euclidean norm. Given two positive integers i, j, with $n \geq j > i$, we define the column vector $\xi_{ij} := \mathrm{col}(\xi_i, \xi_{i+1}, \dots, \xi_j)$.

2. Motor model and standing assumptions

2.1. Motor model

We consider the classical, two-phase $\alpha\beta$ model of the unsaturated, *non-salient*, PMSM described by Krause (1986) and Nam (2010)

$$\begin{aligned} \dot{\lambda} &= v - Ri, \\ j\dot{\omega} &= -f\omega + \tau_e - \tau_L \\ \dot{\theta} &= \omega, \end{aligned} \tag{1}$$

where $\lambda \in \mathbb{R}^2$ is the total flux, $i \in \mathbb{R}^2$ are the currents, $v \in \mathbb{R}^2$ the voltages, $R > 0$ is the stator windings resistance, $j > 0$ the rotor inertia, $\theta \in \mathbb{S} := [0, 2\pi)$ is the rotor phase, $\omega \in \mathbb{R}$ is the mechanical angular velocity, $f \geq 0$ is the viscous friction coefficient, $\tau_L \in \mathbb{R}$ is the – possibly time-varying – load torque, τ_e is the torque of electrical origin, given by

$$\tau_e = n_p i^\top J \lambda,$$

with $n_p \in \mathbb{N}$ the number of pole pairs and $J \in \mathbb{R}^{2\times 2}$ the rotation matrix

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

For surface-mounted PMSM's the total flux verifies

$$\lambda = Li + \lambda_m \mathcal{C}(\theta), \tag{2}$$

where $L > 0$ is the stator inductance and, to simplify the notation, we defined

$$\mathcal{C}(\theta) := \mathrm{col}(\cos(n_p\theta), \sin(n_p\theta)). \tag{3}$$

Hence, a state-space model of the PMSM is given as

$$\begin{aligned} L\frac{di}{dt} &= -Ri - \lambda_m \omega \mathcal{C}'(\theta) + v \\ j\dot{\omega} &= -f\omega + \lambda_m i^\top \mathcal{C}'(\theta) - \tau_L \\ \dot{\theta} &= \omega, \end{aligned} \tag{4}$$

where $\mathcal{C}'(\theta)$ denotes its derivative and we have used (2) and the fact that

$$\mathcal{C}'(\theta) = n_p J \mathcal{C}(\theta),$$

to obtain the expression for τ_e.

2.2. Assumptions and remarks

The following *assumptions* will be invoked in the design of the position observer.

(A1) The *measurable* signals are $i(t)$ and $v(t)$.
(A2) The control signal $v(t)$ and the *unknown* external load torque $\tau_L(t)$ are such that trajectories exist for all $t \geq 0$ and are bounded.
(A3) R and L are *known* but all other parameters of the PMSM, *i.e.*, λ_m, f, j, are *unknown*.
(A4) The signals $v(t)$ and $\tau_L(t)$ are such that the rotor speed is PE, that is, there exist constants $T > 0$ and $\delta > 0$

$$\int_t^{t+T} \omega^2(\tau)d\tau \geq \delta, \quad \forall t \geq 0. \tag{5}$$

(A5) The signals $i(t)$ and $v(t)$ are *integrable*.[1]

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[1] A signal $z : \mathbb{R}_+ \to \mathbb{R}^m$ is integrable if $\int_0^\infty z_i(s)ds < \infty,\ i = 1, \dots, m$.