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Brief paper

Robust cooperative control of multiple heterogeneous Negative-Imaginary systems[☆]

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ABSTRACT

This paper presents a consensus-based robust cooperative control framework for a wide class of linear time-invariant (LTI) systems, namely Negative-Imaginary (NI) systems. Output feedback, dynamic, Strictly Negative-Imaginary (SNI) controllers are applied in positive feedback to heterogeneous multi-input–multi-output (MIMO) plants through the network topology to achieve robust output feedback consensus. Robustness to external disturbances and model uncertainty is guaranteed via NI system theory. Cooperative tracking control of networked NI systems is presented as a corollary of the derived results by adapting the proposed consensus algorithm. Numerical examples are also given to demonstrate the effectiveness of proposed robust cooperative control framework.

1. Introduction

Cooperative control of heterogeneous LTI systems has been widely studied in the literature and there is now a wealth of methods to handle different aspects of the nominal cooperative control problem. Robust cooperative control is however less studied due to the inherent complexities associated with robustness. For example, Cai and Hagen (2010) studies a cooperative control problem for a string of coupled heterogeneous subsystems. Such systems can arise in vehicle platoons. However, the systems considered are constrained to SISO systems (due to the mathematics of the continued fractions used) and do not allow poles on the imaginary axis, and also the graph is only restricted to string connections. On the other hand, Su and Huang (2013) solves a cooperative robust output regulation problem for a class of LTI systems with minimum phase dynamics. A combination of simultaneous high-gain state feedback control and a distributed high-gain observer is adopted to achieve cooperative output regulation under particular parameter uncertainty as well as particular external disturbances. From a different perspective, Zhu and Chen (2014) discusses a full-state feedback robust consensus protocol for heterogeneous second-order multi-agent systems. Existing published literature on robust cooperative control of heterogeneous multi-agent systems is hence restricted to either only SISO plants, or minimum phase LTI plants or full-state feedback second order plants.

NI systems theory has drawn much attention (e.g. Ferrante & Ntogramatzidis, 2013, Opmeer, 2011 and Wang, Lanzon, & Petersen, in press) since it was introduced in Lanzon and Petersen (2008). This is because there are a wide class of LTI systems with negative imaginary frequency response, for which applications can be easily found in a variety of fields including aerospace, large space structures, multi-link robotic arms usually with co-located position sensors and force actuators (Petersen & Lanzon, 2010) and nano-positioning (Mabrok, Kallapur, Petersen, & Lanzon, 2014b), etc. Also the NI systems class is invariant to additive NI model uncertainty and other type interconnections as discussed in Ferrante, Lanzon, and Ntogramatzidis (submitted for publication). Thus, result based on NI systems theory immediately yield robustness to spill-over dynamics (Lanzon & Petersen, 2008; Petersen & Lanzon, 2010; Song, Lanzon, Patra, & Petersen, 2012a).

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Nomenclature

I_n	$n \times n$ identity matrix
$\mathbf{1}_n$	$n \times 1$ vector with all elements being 1
$M > (\geq) 0$	M is a positive (semi-) definite matrix
$M < (\leq) 0$	M is a negative (semi-) definite matrix
Ker(M)	Kernel of a matrix M
Im(M)	Image of a matrix M
rank(M)	Rank of a matrix M
$\lambda_i(M)$, $\bar{\lambda}(M)$	The ith, largest eigenvalue of M
$\lambda(M)$, det(M)	Spectrum, determinant of matrix M
M^T	Transpose of matrix M
M^*	Complex conjugate transpose of matrix M
$\mathbb{R}^{m\times n}$, $\mathbb{C}^{m\times n}$	Set of $m \times n$ real, complex matrices
Re[s]	Real part of $s \in \mathbb{C}$
$\mathfrak{L}_2$	Abbreviation for $\mathfrak{L}_2[0, \infty)$
$\mathrm{Im}_{\mathfrak{L}_2}(G)$	Image of system $G(s)$ under all $\mathfrak{L}_2$ inputs
$\mathcal{RH}_\infty$	Set of real–rational stable transfer functions
$[P(s), P_s(s)]$	Positive feedback interconnection of 2 plants

A square, real, rational, proper transfer function matrix $P(s)$ is NI if the following conditions are satisfied (Lanzon & Petersen, 2008; Mabrok, Kallapur, Petersen, & Lanzon, 2014a; Xiong, Petersen, & Lanzon, 2010): (1) $P(s)$ has no pole in Re[s] > 0; (2) $\forall \omega > 0$ such that $j\omega$ is not a pole of $P(s)$, $j(P(j\omega) - P(j\omega)^*) \geq 0$; (3) If $s = j\omega_0$ where $\omega_0 > 0$ is a pole of $P(s)$, then it is a simple pole and the residue matrix $K = \lim_{s\to j\omega_0}(s - j\omega_0)jP(s)$ is Hermitian and positive semi-definite; (4) If $s = 0$ is a pole of $P(s)$, then $\lim_{s\to 0} s^k P(s) = 0$ $\forall k \geq 3$ and $P_2 = \lim_{s\to 0} s^2 P(s)$ is Hermitian and positive semi-definite. This definition includes free body dynamics which leads to dynamical models with poles at the origin, such as $\frac{s^2+1}{s^2(s^2+2)}$. Examples of NI systems can be found in Mabrok et al. (2014a), Petersen and Lanzon (2010), and these include a single-integrator system, a double-integrator system, second-order systems such as those that arise in undamped and damped flexible structures or inertial systems, to name a few typically considered in the consensus literature. Cooperative control of multiple NI systems arises with the development of NI systems' applications where one single NI system is incapable of achieving the mission goals, for example, the load is too heavy to be carried by one multi-link robotic arm.

This paper solves the general problem, robust output feedback cooperative control of heterogeneous MIMO NI systems (possibly with poles on the imaginary axis) under external disturbances and model uncertainty. Unlike the literature, we impose no minimum phase assumption; the communication graph can be any general undirected and connected graph rather than any specific graph; we allow MIMO agents; we consider explicitly robustness to both unmodelled dynamics of arbitrary order and energy-bounded disturbances; we handle output feedback rather than full state feedback; and explicitly characterise a family of control laws that could be tuned for performance. Towards this end, NI system theory is adopted to first derive conditions for robust output feedback consensus and then transport the proposed results to cooperative tracking to obtain a robust output feedback cooperative control framework for a wide class of LTI systems.

Preliminaries of graph theory: $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ mathematically describes a graph with n nodes and l edges. An undirected and connected graph requires that there exists at least one bidirectional path in $\mathcal{E}$ connecting all nodes in $\mathcal{V}$. The incidence matrix $\mathcal{Q}$ of $\mathcal{G}$ is a $|\mathcal{V}| \times |\mathcal{E}|$ $(n \times l)$ matrix, which can be attained by first letting each edge in the graph have an arbitrary but fixed orientation and then

$$\mathcal{Q} := \begin{cases} q_{ve} = 1 & \text{if } v \text{ is the initial vertex of edge } e, \\ q_{ve} = -1 & \text{if } v \text{ is the terminal vertex of edge } e, \\ q_{ve} = 0 & \text{if } v \text{ is not connected to } e. \end{cases}$$

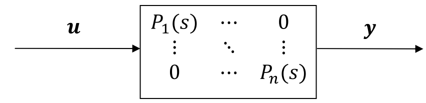

Fig. 1. Multiple heterogeneous NI plants.

For an undirected graph $\mathcal{G}$, $\mathcal{Q}$ is not unique but the corresponding Laplacian matrix is unique and given by $\mathcal{L}_n = \mathcal{Q}\mathcal{Q}^T$. Similarly, the edge-weighted Laplacian is also unique given by $\mathcal{L}_e = \mathcal{Q}\mathcal{K}\mathcal{Q}^T$, where $\mathcal{K} \geq 0$ is the diagonal edge weighting matrix. It is also shown in Bapat (2010) that rank($\mathcal{Q}$) $= n-1 =$ rank($\mathcal{L}_n$) when $\mathcal{G}$ is connected and rank($\mathcal{Q}$) $= n - 1 =$ rank($\mathcal{L}_e$) when $\mathcal{G}$ is connected and det($\mathcal{K}$) $\neq 0$. It is well-known (Ren & Beard, 2008) that $\mathcal{L}_n$ and $\mathcal{L}_e$ will both have one unique zero eigenvalue associated with the eigenvector $\mathbf{1}_n$ and all the other eigenvalues are positive and real, when det($\mathcal{K}$) $\neq 0$, $\mathcal{G}$ is undirected and connected. In this case, $\mathcal{L}_n \geq 0$, $\mathcal{L}_e \geq 0$, and

$$\mathrm{Ker}(\mathcal{L}_n) = \mathrm{Ker}(\mathcal{L}_e) = \mathrm{Ker}(\mathcal{Q}^T) = \mathrm{span}\{\mathbf{1}_n\}. \quad (1)$$

Note also that, for an undirected and connected graph $\mathcal{G}$, any row removal of $\mathcal{Q}$ or column removal of $\mathcal{Q}^T$ yields a full row rank $\mathcal{Q}$ or a full column rank $\mathcal{Q}^T$ respectively by inspecting the relation of Laplacian matrix with $\mathcal{Q}$ and the property of positive semi-definite matrices with a kernel dimension of 1 (Golub & Van Loan, 2012).

2. Robust output feedback consensus

In this section, we will consider robust output feedback consensus for multiple heterogeneous NI systems under $\mathfrak{L}_2$ external disturbance and additive SNI model uncertainty (as would arise in spill-over dynamics for truncated order flexible structures). Two cases will be discussed to cover all the heterogeneous cases. First of all, let us begin with the problem formulation with the following notation: $\max_{i=1}^n\{a_i\}$ is the maximum value of a_i, $i \in \{1, \ldots, n\}$ and $\operatorname{diag}_{i=1}^{n}\{A_i\}$ is a block-diagonal matrix with A_i, $i \in \{1, \ldots, n\}$ on the diagonal. A square, real, rational, proper transfer function matrix $P_s(s)$ is SNI if the following conditions are satisfied: (1) $P_s(s)$ has no pole in Re[s] ≥ 0; (2) $\forall \omega > 0$, $j(P_s(j\omega) - P_s(j\omega)^*) > 0$. Examples of SNI systems include $\frac{1}{s+a}$ where $a > 0$, $\frac{a}{s^2+bs+c}$ where $a, b, c > 0$ or non-minimum phase systems such as $\frac{1-s}{2+s}$. See Lanzon and Petersen (2008), Petersen and Lanzon (2010) for further examples.

For multiple heterogeneous NI systems (in general MIMO) with $n > 1$ agents, the transfer function of agent $i \in \{1, \ldots, n\}$ is given as

$$\hat{\boldsymbol{y}}_i = \hat{P}_i(s)\hat{\boldsymbol{u}}_i, \quad (2)$$

where $\hat{\boldsymbol{y}}_i \in \mathbb{R}^{m_i\times 1}$ and $\hat{\boldsymbol{u}}_i \in \mathbb{R}^{m_i\times 1}$ are the output and input of agent i respectively. In order to deal with the consensus of different dimensional inputs/outputs, $\hat{P}_i(s)$ can be padded with zeros up to $m = \max_{i=1}^n\{m_i\}$ and the locations of padding zeros depend on which output needs to be coordinated, for instance, $P_i(s) = \begin{bmatrix} \hat{P}_i(s) & 0 \\ 0 & 0 \end{bmatrix}$ has dimension of m such that the first m_i outputs are to be coordinated, or $P_i(s) = \begin{bmatrix} 0 & 0 \\ 0 & \hat{P}_i(s) \end{bmatrix}$ also has dimension of m, but now the last m_i outputs are to be coordinated instead. Accordingly, the input $\hat{\boldsymbol{u}}_i$ and output $\hat{\boldsymbol{y}}_i$ are extended to be $\boldsymbol{u}_i = \begin{bmatrix} \hat{\boldsymbol{u}}_i^T & 0 \end{bmatrix}^T$ or $\begin{bmatrix} 0 & \hat{\boldsymbol{u}}_i^T \end{bmatrix}^T \in \mathbb{R}^{m\times 1}$, and $\boldsymbol{y}_i = \begin{bmatrix} \hat{\boldsymbol{y}}_i^T & 0 \end{bmatrix}^T$ or $\begin{bmatrix} 0 & \hat{\boldsymbol{y}}_i^T \end{bmatrix}^T \in \mathbb{R}^{m\times 1}$, respectively. Note that interleaving zero rows and corresponding columns within $\hat{P}_i(s)$ is also permissible. It can be easily seen that the above manipulation would preserve the NI property by checking the definition. Therefore, without loss of generality, the overall plant can be described as Fig. 1: