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Brief paper

Graph-theoretic analysis of network input–output processes: Zero structure and its implications on remote feedback control ☆

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ABSTRACT

The control of dynamical processes in networks is considered, in the case where measurement and actuation capabilities are sparse and possibly remote. Specifically, we study control of a canonical network dynamics, when only one network component's state can be measured and only one (in general different) component can be actuated. To do so, we characterize the finite- and infinite-zeros of the resulting SISO system in terms of the graph topology. Using these results, we establish graph-theoretic conditions under which there are zeros in the closed right-half plane. These conditions depend on the length, strength, and number of the paths from the component where the input is applied to the component where the measurements are made. Then, we present the implications of these conditions on the controller design task focusing in stabilizations/destabilization of network processes under static negative feedback.

1. Introduction

Network synchronization and diffusion models are used to capture such diverse processes as vehicle coordination, biochemical reactions, and sensor-fusion algorithms (Watts & Strogatz, 1998). The temporal dynamics of these models have been extensively studied from a graph-theoretic perspective. First, emergence of synchronization has been shown under broad connectivity conditions, i.e. the models have been shown to have a stable manifold wherein all the network components' states are identical (Barahona & Pecora, 2002; Wu & Chua, 1995). Further results have been developed that tie performance characteristics (e.g., the settling rate) to features of the network's graph. As synchronization models have found wider application in engineering contexts, their design and control have also been of significant interest. Many of these studies consider decentralized control of multiple autonomous but communicating agents (e.g., vehicles), which yields a closed-loop dynamics that is a network synchronization process. In complement, several recent studies have considered topology design to shape the performance of synchronization processes (Abad Torres & Roy, 2013, 2014; Roy, Wan, & Saberi, 2009; Wan et al., 2008).

While the literature has focused on emergence and design of whole-network behaviors such as synchronization, there is a growing need to understand input–output dynamics and feedback regulation of established network processes, when measurement and actuation are available at only a few network components. This need for an input–output analysis partially stems from challenges in security and vulnerability analysis of infrastructures and other complex dynamical networks (Belykh, Belykh, & Hasler, 2004; Koh & Vinnicombe, 2012; Pasqualetti, Bicchi, & Bullo, 2009, 2011; Roy, Xue, & Das, 2012; Sandberg, Teixeira, & Johansson, 2010; Vidyasagar & Yamamoto, 2012). In these applications, an adversary can typically only make limited measurements and actuations of the dynamics, but may be able to initiate a significant propagative impact across the network (Roy et al., 2012; Sandberg et al., 2010). By the same token, system operators may often have limited measurement and actuation capabilities in reacting to threats/disturbances, and their ability to mitigate the wide-area threats via local feedback is of interest. The input–output dynamics of network processes are also germane to management and resource allocation problems in large-scale networks, where limited control resources must be placed to shape global network dynamics. A key need in these various application domains is to understand the implications of the network's graph topology on the input–output dynamics and specifically its zeros, to achieve

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simple insights into propagative impacts and enable control design.

Recently, several studies have begun to study the input–output dynamics of network synchronization and spread processes from a graph-theoretic perspective. Our work is closely aligned with the study of Briegel et al., which characterizes the zeros of a single-input–single-output (SISO) system defined on a consensus (synchronization) process (Briegel, Zelazo, Burger, & Allgower, 2011). The authors focus particularly on symmetric unweighted network topologies, and give bounds on finite-zero locations and conditions for the presence of right-half-plane zeros. Meanwhile, our previous work pursues a structural decomposition of an input–output dynamics imposed on a synchronization process (Abad Torres & Roy, 2014), and uses this decomposition to achieve simple graphical characterizations of the zeros. Motivated by vulnerability-analysis goals, control theorists have also studied robustification of synchronization processes via feedback (Vidyasagar & Yamamoto, 2012), characterized disturbance propagation (Koh & Vinnicombe, 2012), and highlighted linkages between graph connectivity and network robustness (via the presence of non-trivial zero dynamics) (Belykh et al., 2004; Pasqualetti et al., 2009, 2011).

This short paper is concerned with the input–output dynamics of a class of continuous-time linear network processes defined on a general (directed, weighted) graph, which is actuated at a single network component and measured at another component (see Section 2). For this SISO model, a full characterization of the infinite- and finite-zeros in terms of the network's graph topology and the actuation/measurement locations is undertaken (Section 4), using a structural decomposition for the input–output dynamics (reviewed in Section 3). A significant result is that networks with weak short paths as well as alternative long strong paths between the input and output have non-minimum-phase zeros. Some implications on feedback control of the network dynamics are briefly discussed, particularly focusing on destabilization through remote feedback.

2. Problem formulation

We are concerned about the input–output behavior of a dynamical-network process that is actuated at a single network component, and measured at a single component (which in general may be remote from the stimulation location). Formally, a network with n components, labeled $1, 2, \ldots, n$, is considered. Each component is assumed to have a scalar state $\tilde{x}_i$ associated with it. These states evolve according to the differential equations:

$$\dot{\tilde{x}} = A\tilde{x} + e_i\tilde{u} \tag{1}$$

where $\tilde{x} = \begin{bmatrix}\tilde{x}_1 & \cdots & \tilde{x}_n\end{bmatrix}'$ is the full state of the network, e_i is 0–1 indicator vector with ith entry equal to 1, $\tilde{u}$ is a scalar input signal at a single network component i, and the state matrix A of the network dynamics is called the *graph matrix*. We assume that the off-diagonal entries of the state matrix A are nonnegative, and the diagonal entries are negative and satisfy $A_{i,i} \leq -\sum_{j=1,j\neq i}^{n} A_{i,j}$. Dynamical models of this form are commonly used to represent both natural and engineered network processes (e.g., circuit dynamics, fluid-flow systems, satellite alignment). The special case that $A_{i,i} = -\sum_{j=1,j\neq i}^{n} A_{i,j}$ has been particularly well studied, as a canonical model for synchronization or consensus or diffusion. We stress that the matrix A need not be symmetric. The state of one component, say component n WLOG, is measured. Thus, the observation or output $\tilde{y}$ of the dynamical network is given by:

$$\tilde{y} = e_n'\tilde{x}. \tag{2}$$

Eqs. (1) and (2) together specify a SISO network process.

A weighted digraph G with n vertices is associated with the network dynamics, where each vertex $i = 1, 2, \ldots, n$ in the graph corresponds to the network component i. Formally, an arc (directed edge) is drawn from vertex i to vertex j in the graph (i, j distinct) if and only if $A_{j,i} \neq 0$, and is assigned a weight of $A_{j,i}$. The vertices corresponding to the input and output network components are referred to as the input and output vertices. The state matrix $-A$ can be viewed as (the transpose of) a Laplacian or grounded Laplacian matrix associated with the graph, per our definition.

Our goal is to characterize the zeros of the SISO system (1) and (2) in terms of the digraph G and the input/output locations. Specifically, we seek graph-theoretic conditions under which the infinite zeros (asymptotes of the positive root-locus branches) are in the left- or right-half plane. Also, graph-theoretic characterizations of the number and locations of the finite zeros are sought. These results then imply myriad limits on feedback control (e.g., on perfect tracking) imposed the graph topology, as we will discuss briefly. As a particular application, we will draw on graph-theoretic conditions for nonminimum-phase dynamics to determine when static negative feedback $\tilde{u} = -K\tilde{y}$ yields instability.

3. Background

In Abad Torres and Roy (2014), we used the special coordinate basis (SCB) for linear systems (Sannuti & Saberi, 1987) to obtain some preliminary structural results on the zeros of the SISO network model ((1) and (2)), which are foundational to the graph-theoretic results developed here. The SCB is a convenient tool for the graph-theoretic analysis of zeros, because it provides an explicit matrix-algebraic characterization of a system's zero structure. Specifically, the SCB expresses a linear system as two feedback-interconnected subsystems: (1) an integrator chain that is directly driven by the system input and directly impacts the output, and (2) a zero dynamics that is neither directly driven by the input nor directly affects the output. The transformation of (1) and (2) into the SCB and resultant structural analyses are summarized in the following list (Abad Torres & Roy, 2014). The development uses standard terminology related to finite and infinite zeros, and zero dynamics, please see Chen, Lin, and Shamash (2004), Kouvaritakis and Edmunds (1979) and Sannuti and Saberi (1987).

(1) The relative degree (number of infinite zeros) is given by $n_d = d + 1$, where d is the distance from the input to the output vertex in G (see also Briegel et al., 2011 and Reinschke, 1988). In the SCB formulation, the states associated with the vertices in the shortest directed input–output path form the chain of n_d integrators. Let us call this path the *special input–output path*.

(2) The dimension of the zero dynamics (number of finite zeros) is $n_a = n - d - 1$. The states of the zero dynamics can be defined via a transform of the states corresponding to vertices that are not on the special input–output path. We define a set V_1 containing these n_a vertices. We also use the notation G_1 for the induced subgraph of G on V_1. WLOG, the vertices on the special input–output path are labeled $n - d, n - d + 1, \ldots, n$, where vertex $n - i$ is at a distance i to the output, while the vertices in V_1 are labeled $1, 2 \ldots, n_a$.

(3) The network's finite-zero dynamics is given by:

$$x_0 = A_{aa}x_0 + \left(\sum_{i=0}^{n_d-1} A_{aa}^{n_d-i-1}A_{n_{ad}}Z_{nd}^{-1}e_{n_d-i}\right)\tilde{y} \tag{3}$$

where $A_{aa} = A_{n_a} - \Delta$ and $\Delta = A_{n_{ad}}Z_{nd}^{-1}Z_{n_{ad}}$. The matrix A_{n_a} is a principal submatrix of the A formed by the rows and columns corresponding to the vertices in V_1. The matrix $A_{n_{ad}}$ is an off-diagonal submatrix of A, while Z_{nd}^{-1} and $Z_{n_{ad}}$ can be computed